A dessert talk on:

strong interactions, pairing and spin polarons in imbalanced Fermi gases

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Work done in Utrecht in collaboration with
Georg Bruun (Copenhagen and Trento) and Henk Stoof (Utrecht)

Fermi systems: pairing and superfluidity

- Strongly-correlated Fermi gases:
  - neutron stars (huge and hot)
  - quark-gluon plasma and high-T_c superconductors
  - atomic gases (tiny and ultracold)

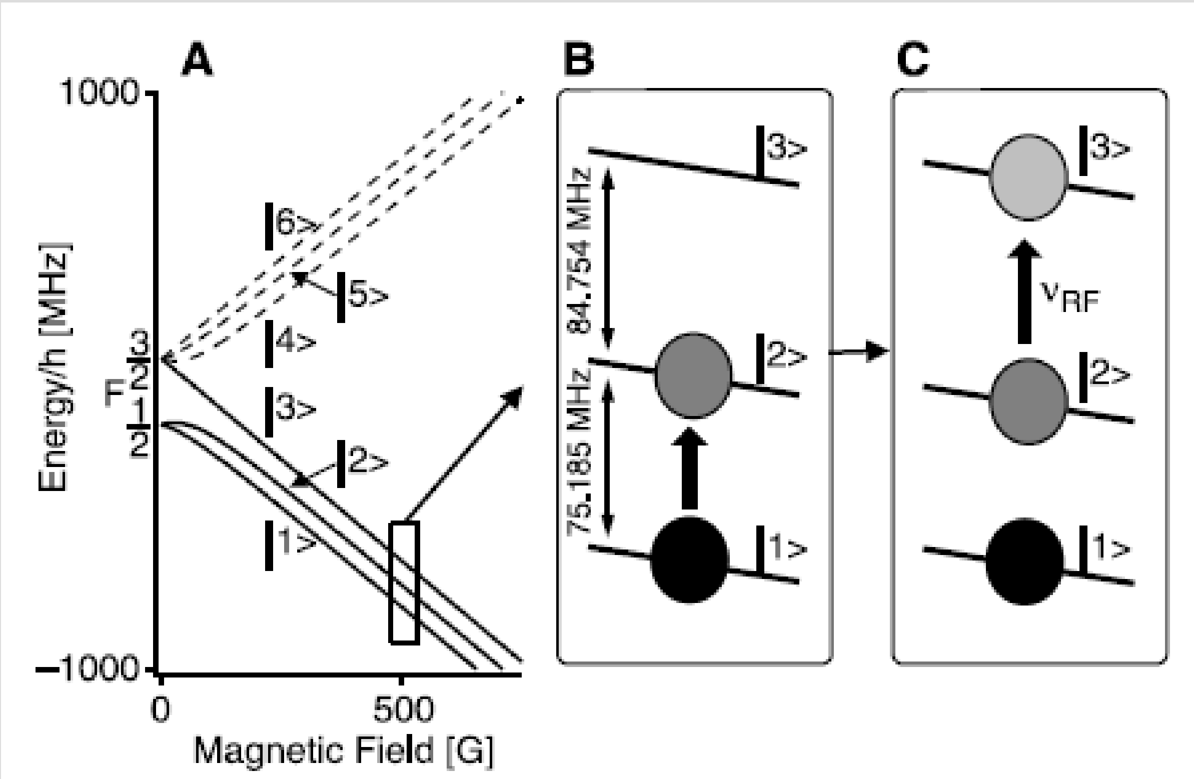
- In BCS theory pairs condense as they form, but when interactions are strong $T^{pairing} > T_{superfl.}$

Radio-frequency spectroscopy

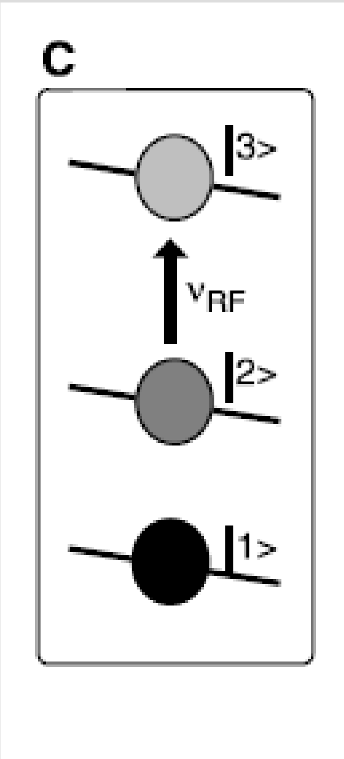
a versatile tool to probe interactions,
onset of pair correlations and presence of molecules.

C. Regal et al. (Nature 2003), C. Chin et al., (Science 2004).

Level scheme in ^{6}Li:



Early evidence for pairing

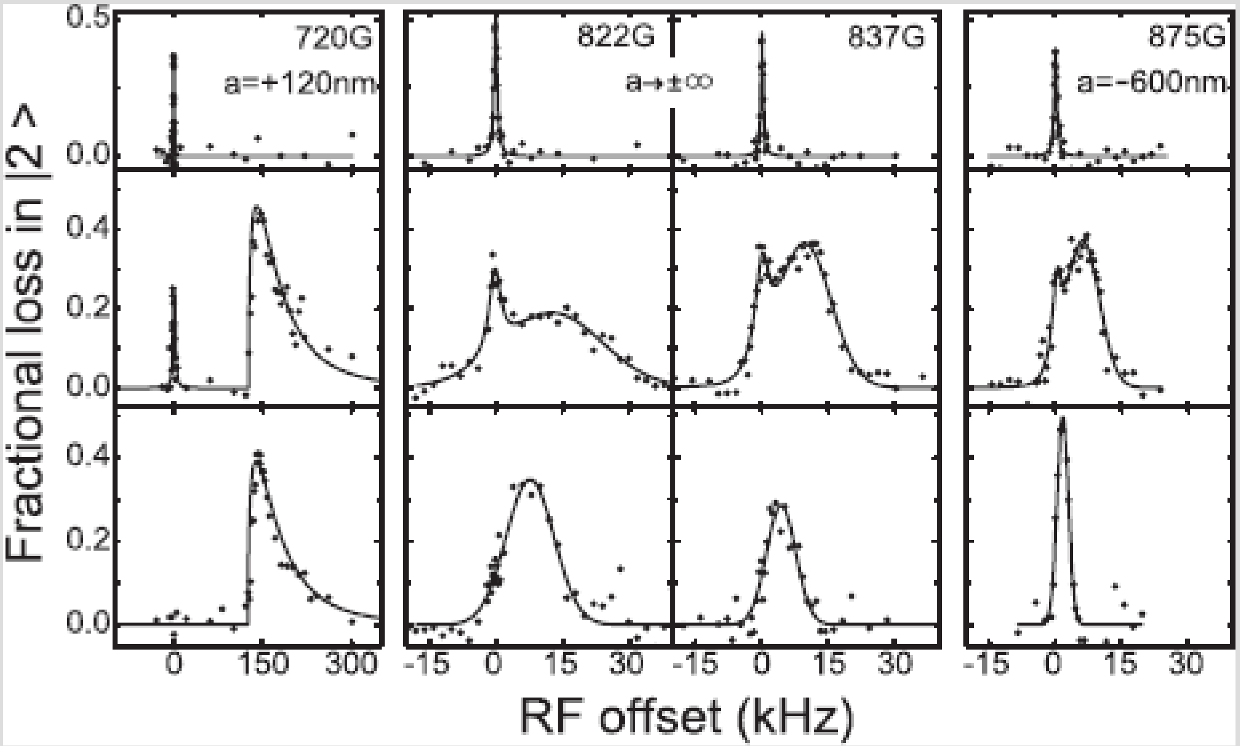


BEC Unitarity BCS

$T >> T_F$

$T \sim 0.5 T_F$

$T << T_F$



C. Chin et al., (Science 2004).

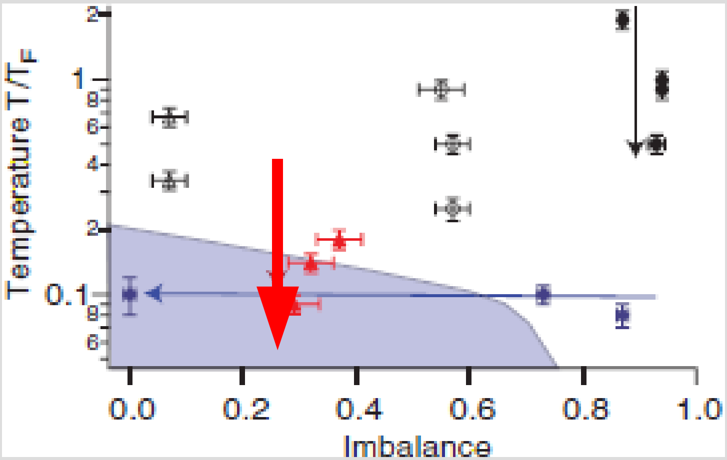
Pairing Without Superfluidity: The Ground State of an Imbalanced Fermi Mixture

C. H. Schunck,* Y. Shin, A. Schirotzek, M. W. Zwierlein,† W. Ketterle

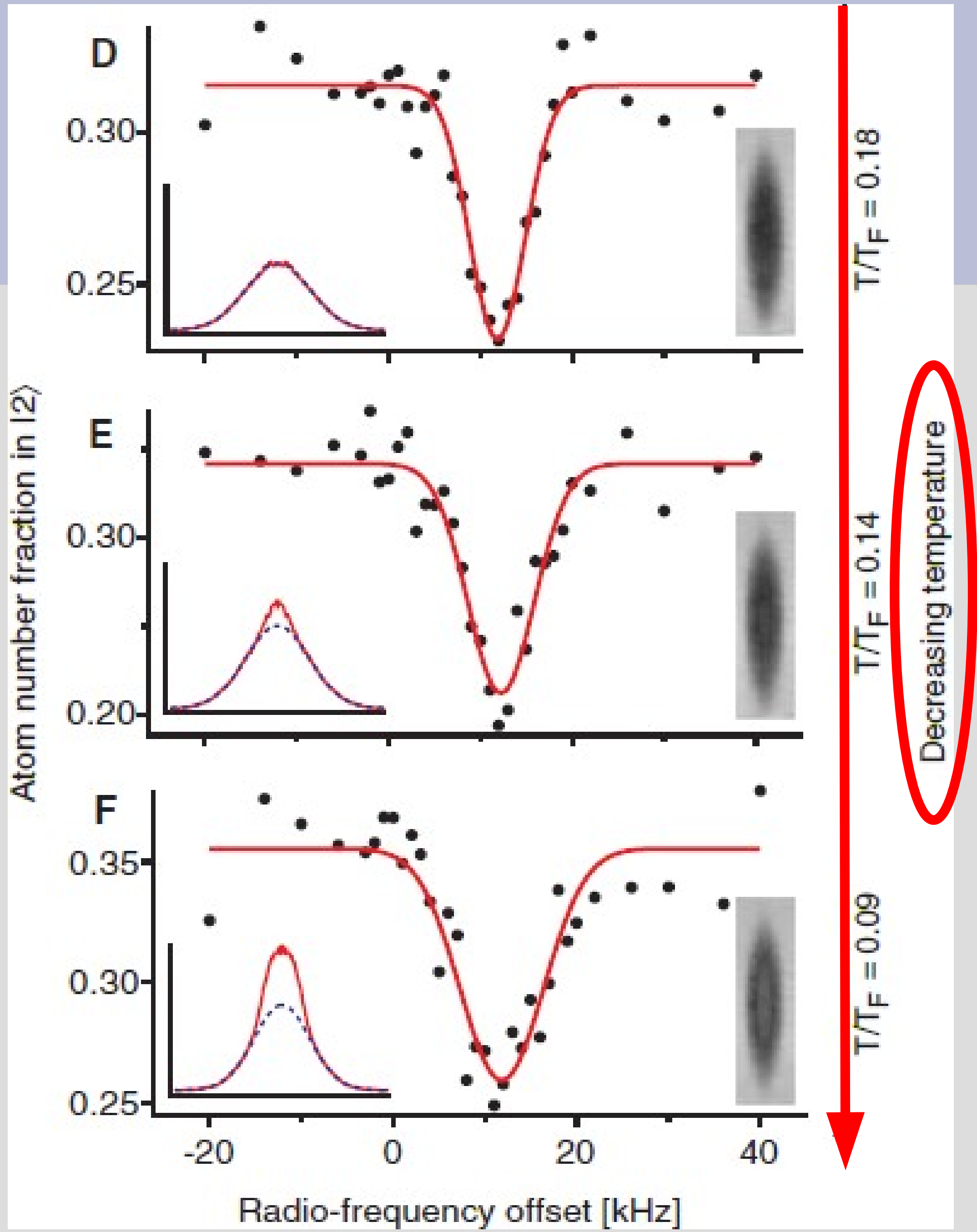
Science (2007)

RF spectra do not reveal the normal-SF transition

(while density profiles do)



Imbalance=$(N_1-N_2)/(N_1+N_2)$



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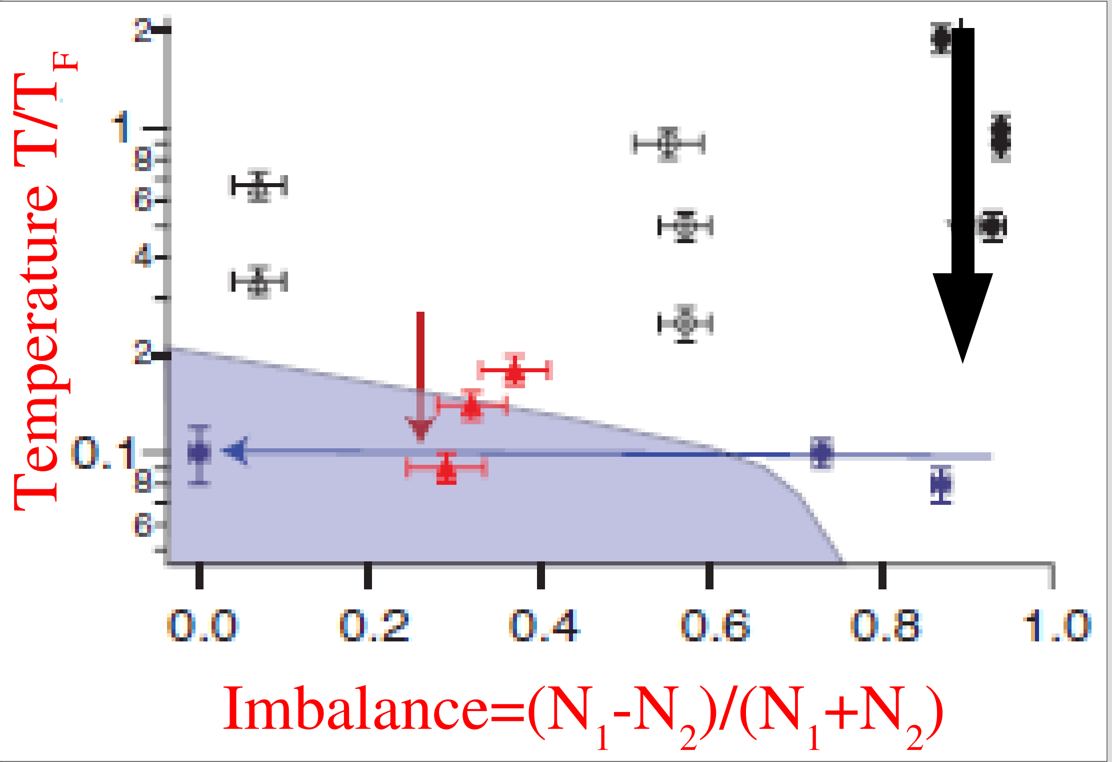
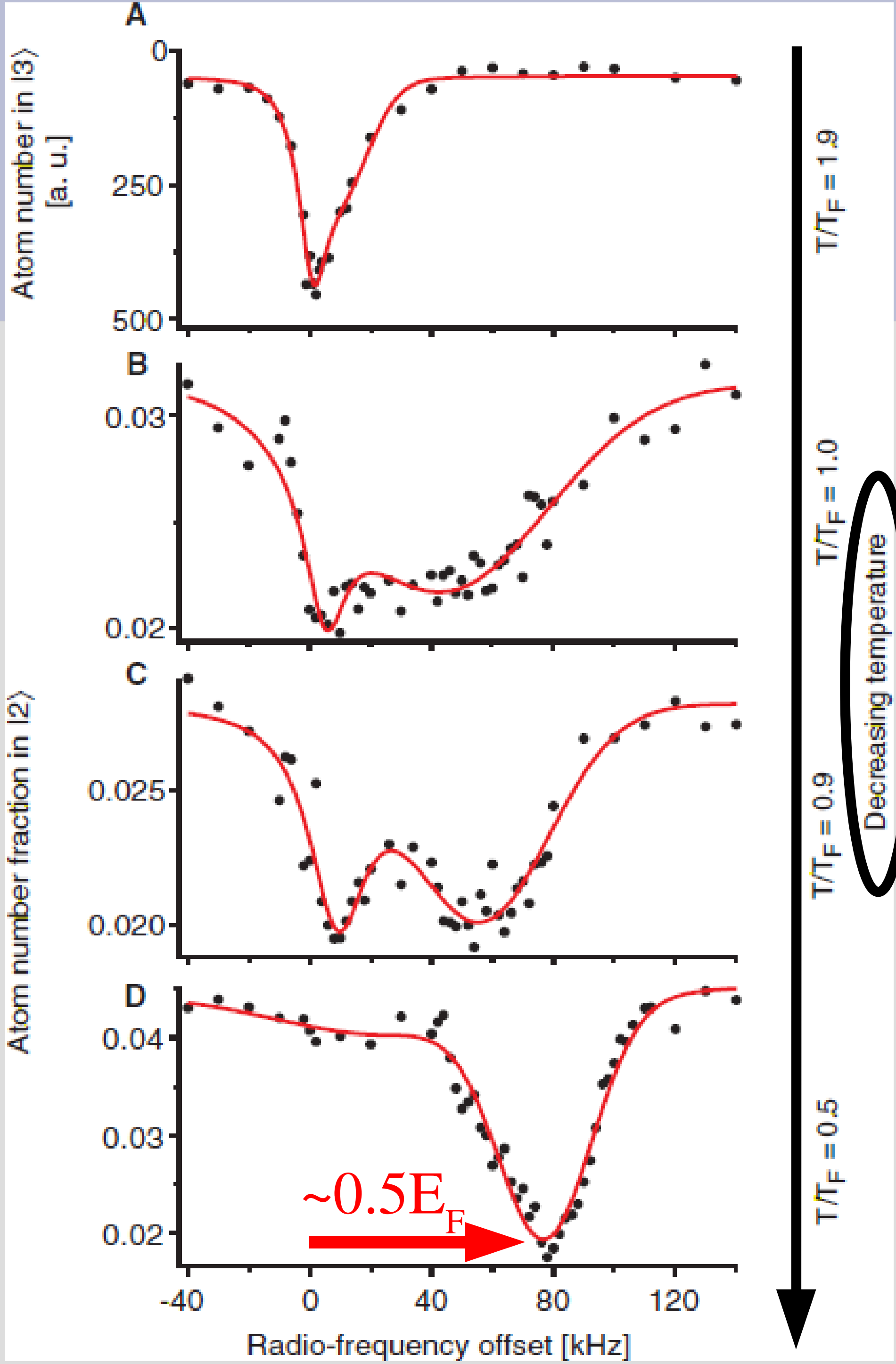
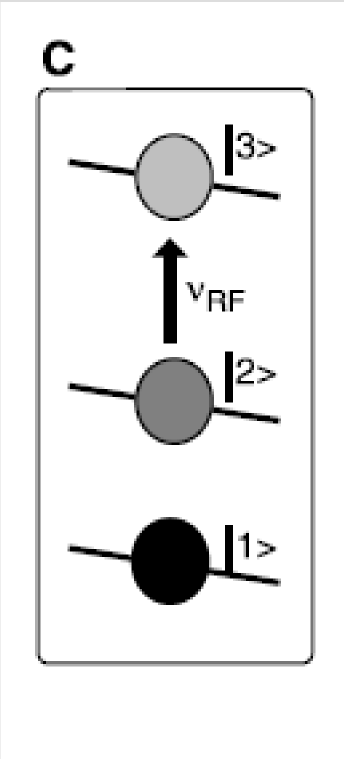




Large shift in the peak position signals strong interactions in the gas

Linear response theory



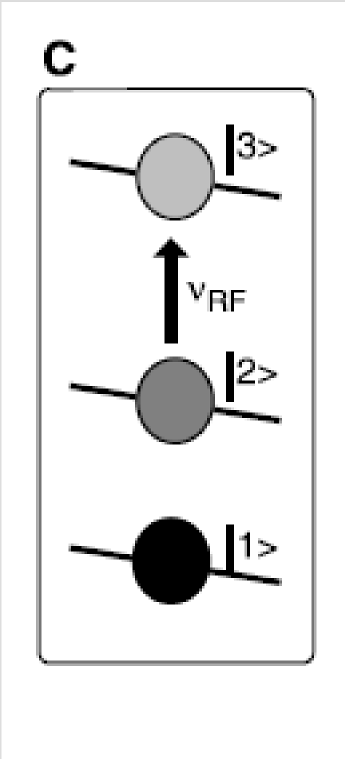
$$H_{\rm rf} = \frac{\Omega}{2}\int d\mathbf{r}\left[e^{-i\omega t}\psi_3^\dagger(\mathbf{r},t)\psi_2(\mathbf{r},t) + \text{h.c.}\right]$$

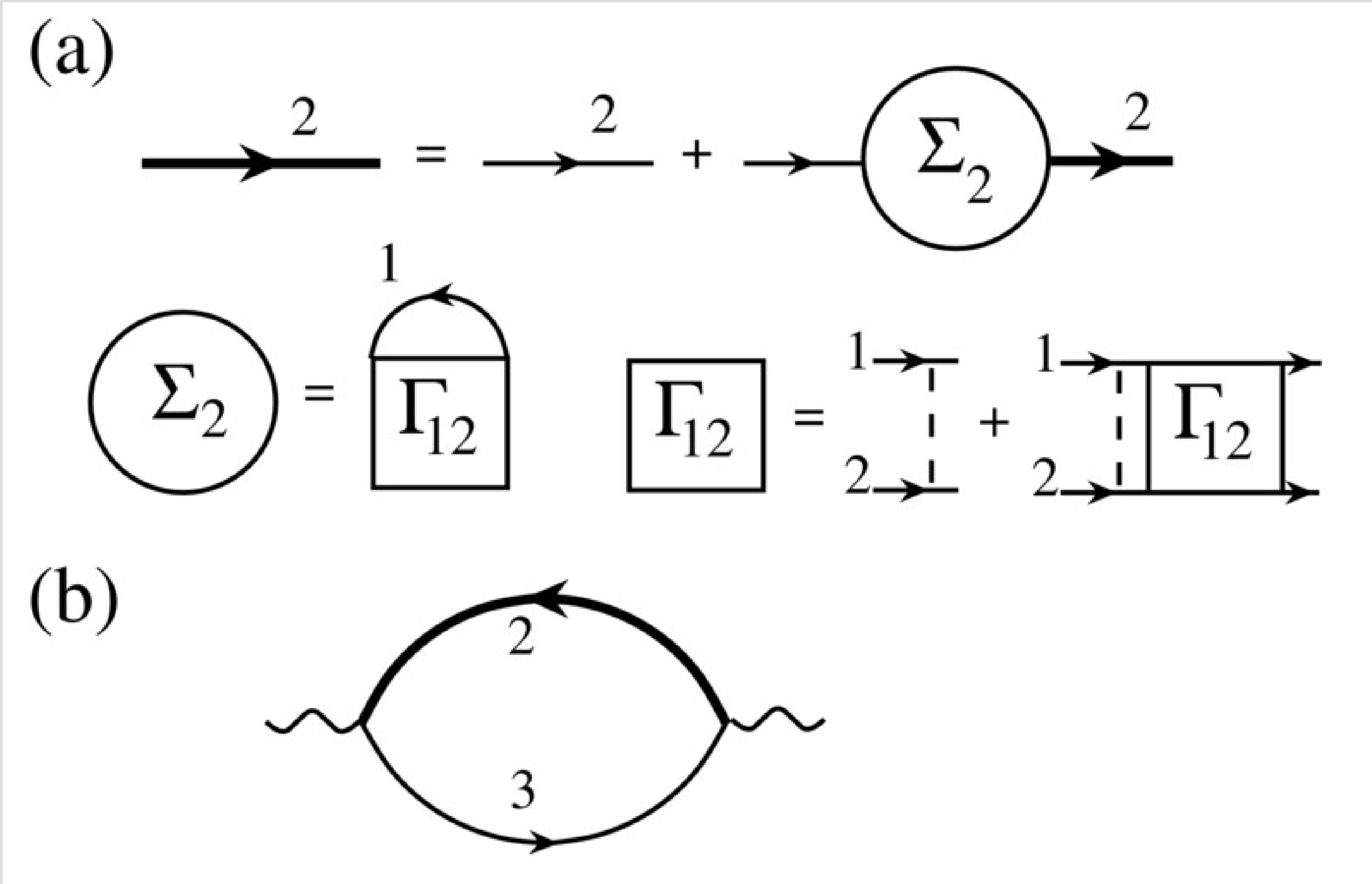
$$\text{Transition rate: } R(\omega) \propto -\int d\mathbf{r}d\mathbf{r}'\,\text{Im}\mathcal{D}(\mathbf{r},\mathbf{r}',\omega)$$

$$\mathcal{D}(\mathbf{r},\mathbf{r}',\omega) = \text{F.T.}\Big\{-i\theta(t-t')\langle[\psi_3^\dagger(\mathbf{r},t)\psi_2(\mathbf{r},t),\psi_2^\dagger(\mathbf{r}',t')\psi_3(\mathbf{r}',t')]\rangle\Big\}.$$

{...}: retarded 2-3 flip correlation function

T-matrix and self-energy in the ladder approx.





(we take into account the strong 1-2 interaction, while we neglect the weaker 1-3)

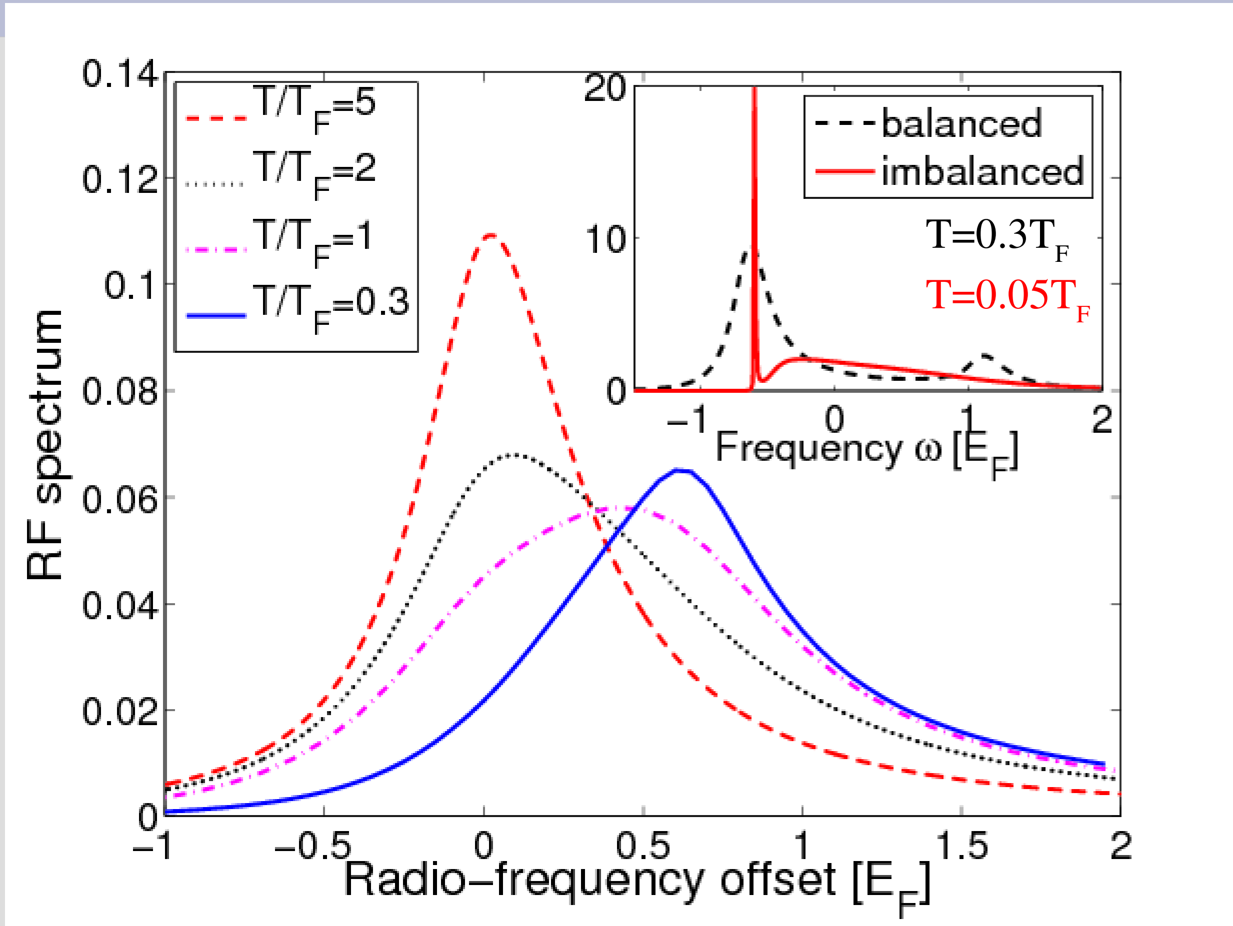
Spectral function and spectrum

(no trap, unitarity)

$$\mathrm{Im}\mathcal{D}(\omega) = -\mathcal{V}\int \frac{d\mathbf{k}}{(2\pi)^3} A_2(k, \xi_{2k}-\omega) f(\xi_{2k}-\omega)$$

$$\xi_{2k} = k^2/2m - \mu_2$$

$$\text{Spectral function: } A_2(k,\omega) = -2\mathrm{Im}G_2(k,\omega)$$



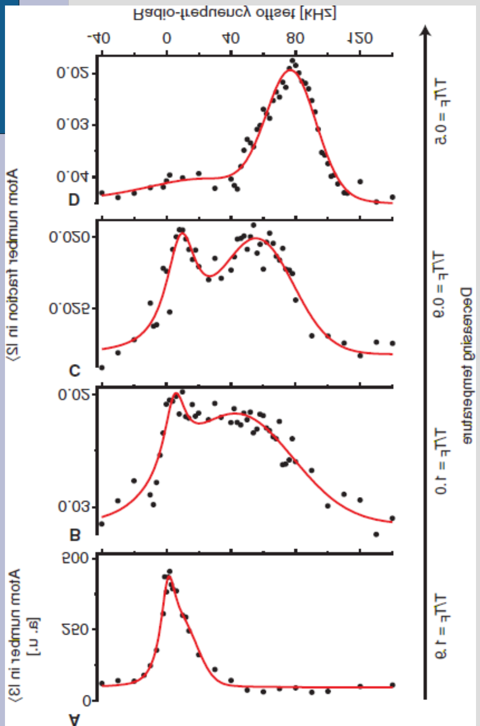
Pairing without superfluidity (*pseudogap* regime)

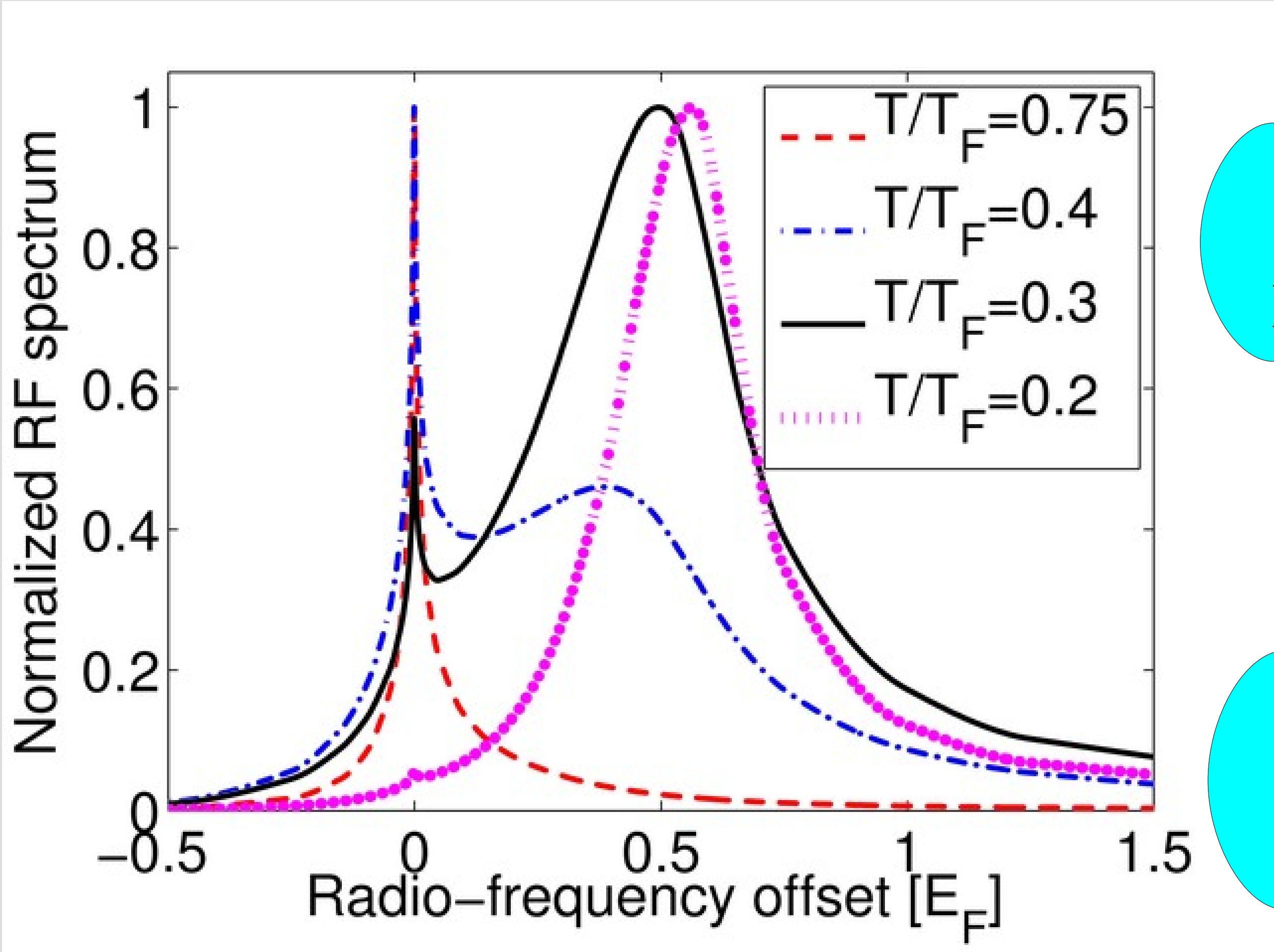
Inset: spectral function $A_2(k=0,\omega)$.

Main: spectrum for $n_1=n_2$.

No double peaks in the spectrum, even in the pseudogap regime!

Spectrum of a
trapped sample at unitarity





no fitting parameters

peak position independent of imbalance

N_2/N_1=5%

Number of "pairs" δn_σ in the ladder (NSR) approx.

Thermodynamic potential: $\Omega = \Omega_0 + \Delta\Omega$

NSR approx: $\Delta\Omega = \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \frac{\ln(1 - T\Pi^+) - \ln(1 - T\Pi^-)}{e^{\beta\omega} - 1}.$

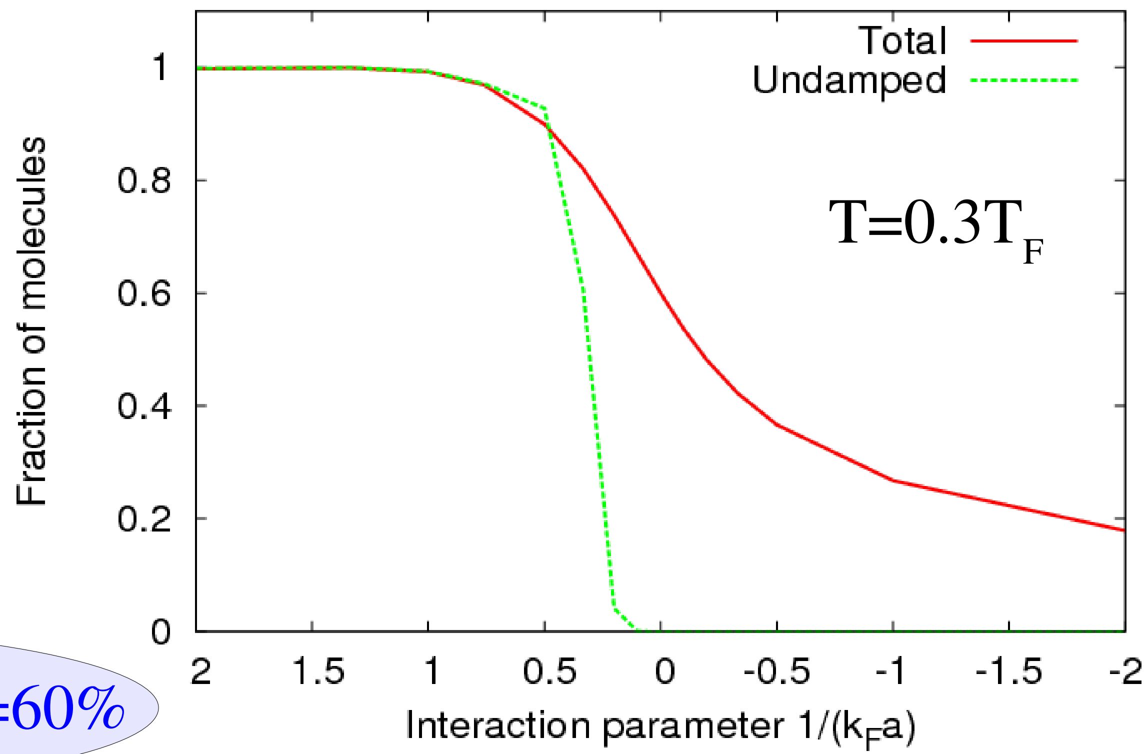
$$n_\sigma = -\partial_{\mu_\sigma}\Omega =$$

$$= n_\sigma^0 + \delta n_\sigma$$

$$n_\sigma^0 = \int \frac{d^3k}{(2\pi)^3}\left[e^{\beta(k^2/2m-\mu)}+1\right]^{-1}$$

$$\delta n_\sigma = -\partial_{\mu_\sigma}\delta\Omega$$

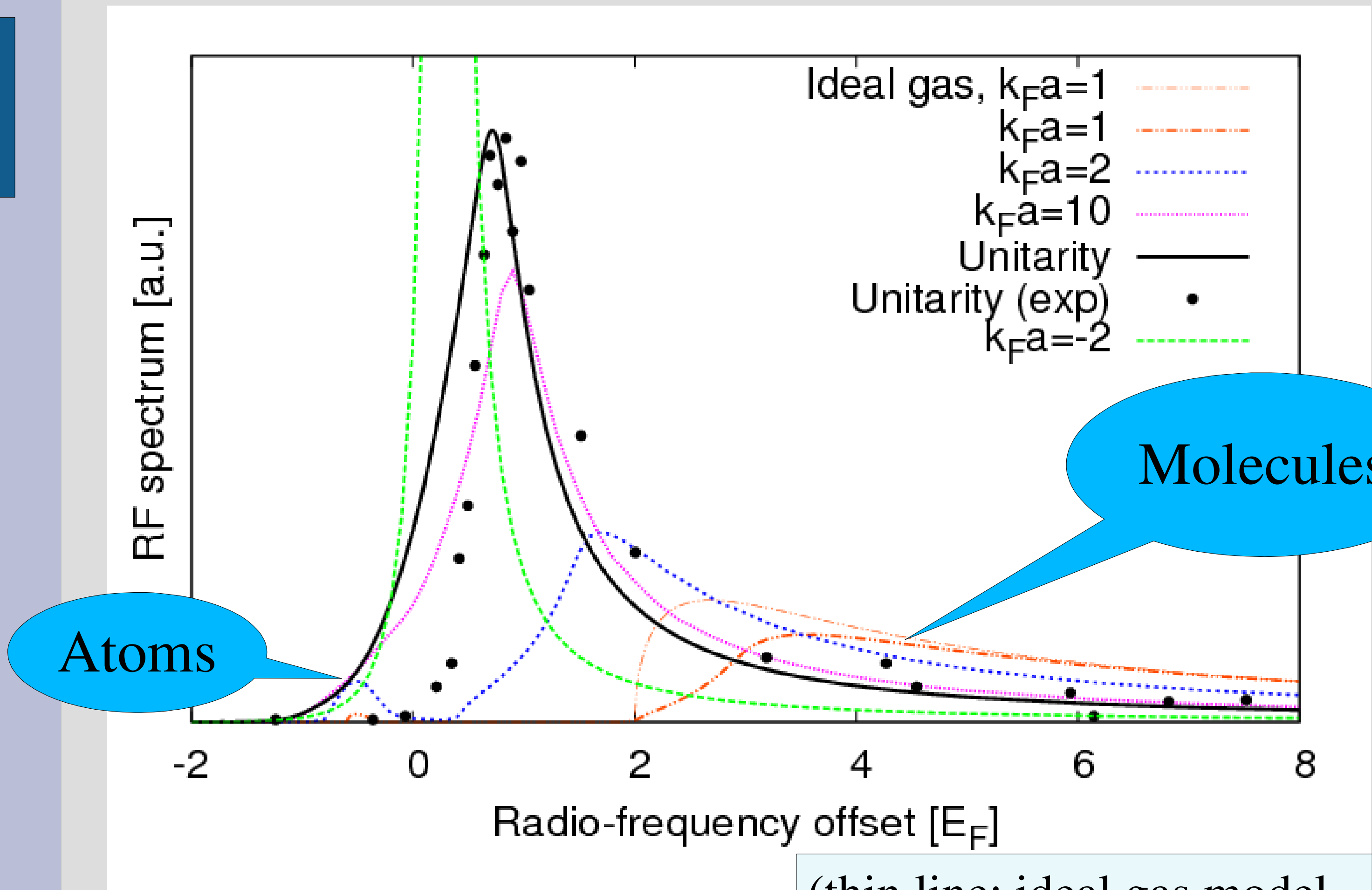
$(\sigma = 1,2)$



At $T=T_c$ and unitarity, $\delta n_\sigma=60\%$

Spectrum across the crossover

(no trap, $n_1 = n_2$, $T = 0.3T_F$)



(thin line: ideal gas model,
Yukawa molecular function)

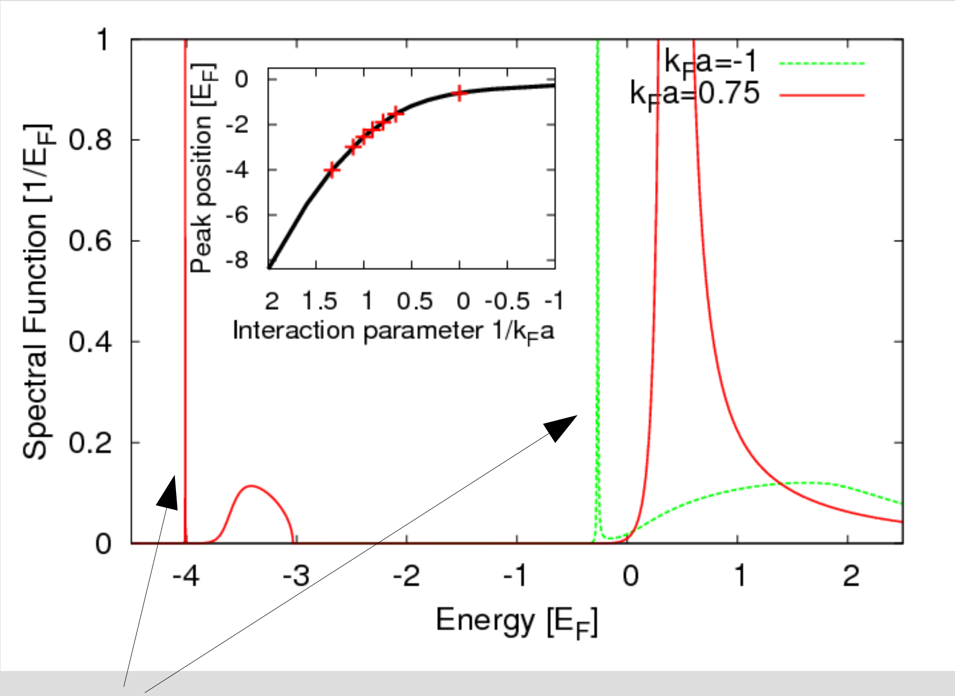
1) Stable molecules appear when a gap opens in A_2

2) Atoms coexist with molecules for $T < E_b$

Binding energy $E_b \sim \frac{1}{(ma)^2}$

Exp: Schunck et al., arXiv:0802.0341

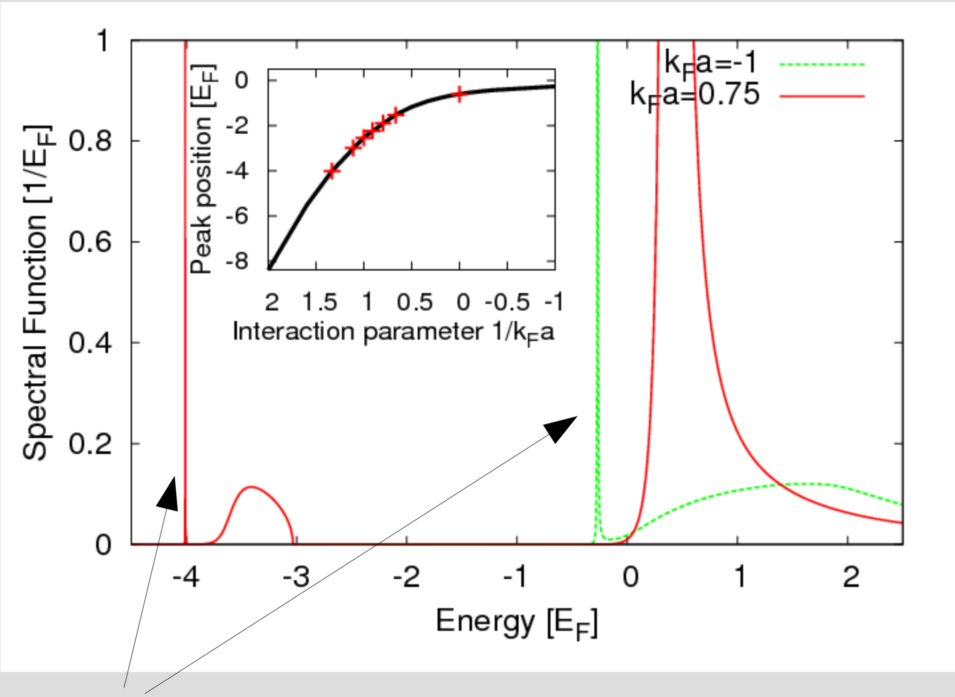
An impurity in a Fermi sea: spectral function and spectrum

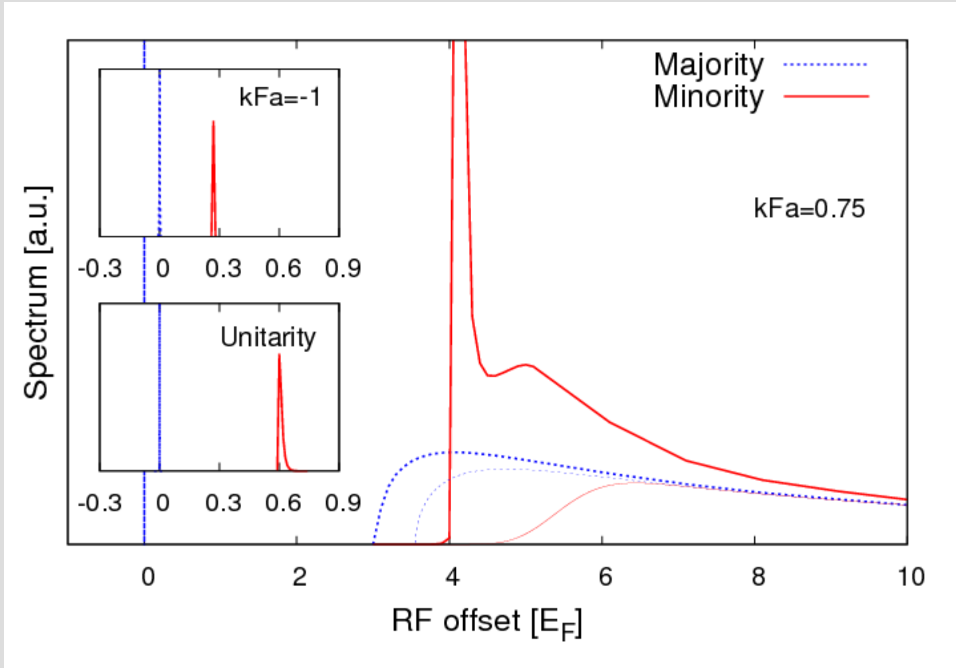


Narrow feature: a "polaron" (quasiparticle, MB), which disappears in the BEC limit (2B).

Red crosses: MC results by Lobo et al. (PRL 2006) and Prokof'ev&Svistunov (PRB 2008).

An impurity in a Fermi sea: spectral function and spectrum





Narrow feature: a "polaron" (quasiparticle, MB), which disappears in the BEC limit (2B).

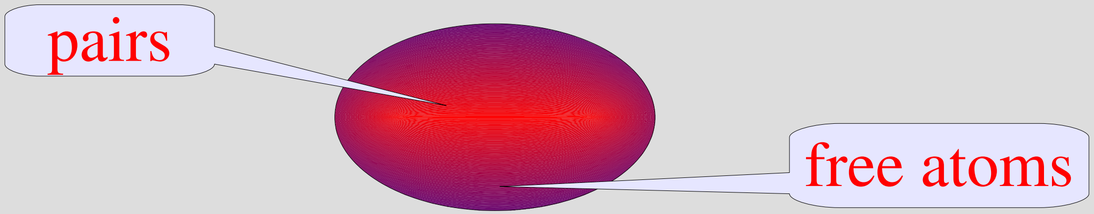
(thin lines: ideal gas model, Yukawa molecular function)

The RF spectrum gives useful information on both the quasiparticle energy and the molecular wavefunction

Red crosses: MC results by Lobo et al. (PRL 2006) and Prokof'ev&Svistunov (PRB 2008).

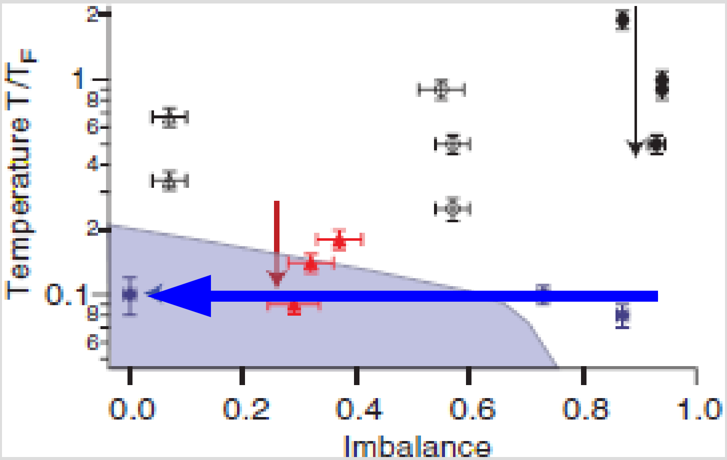
Conclusions

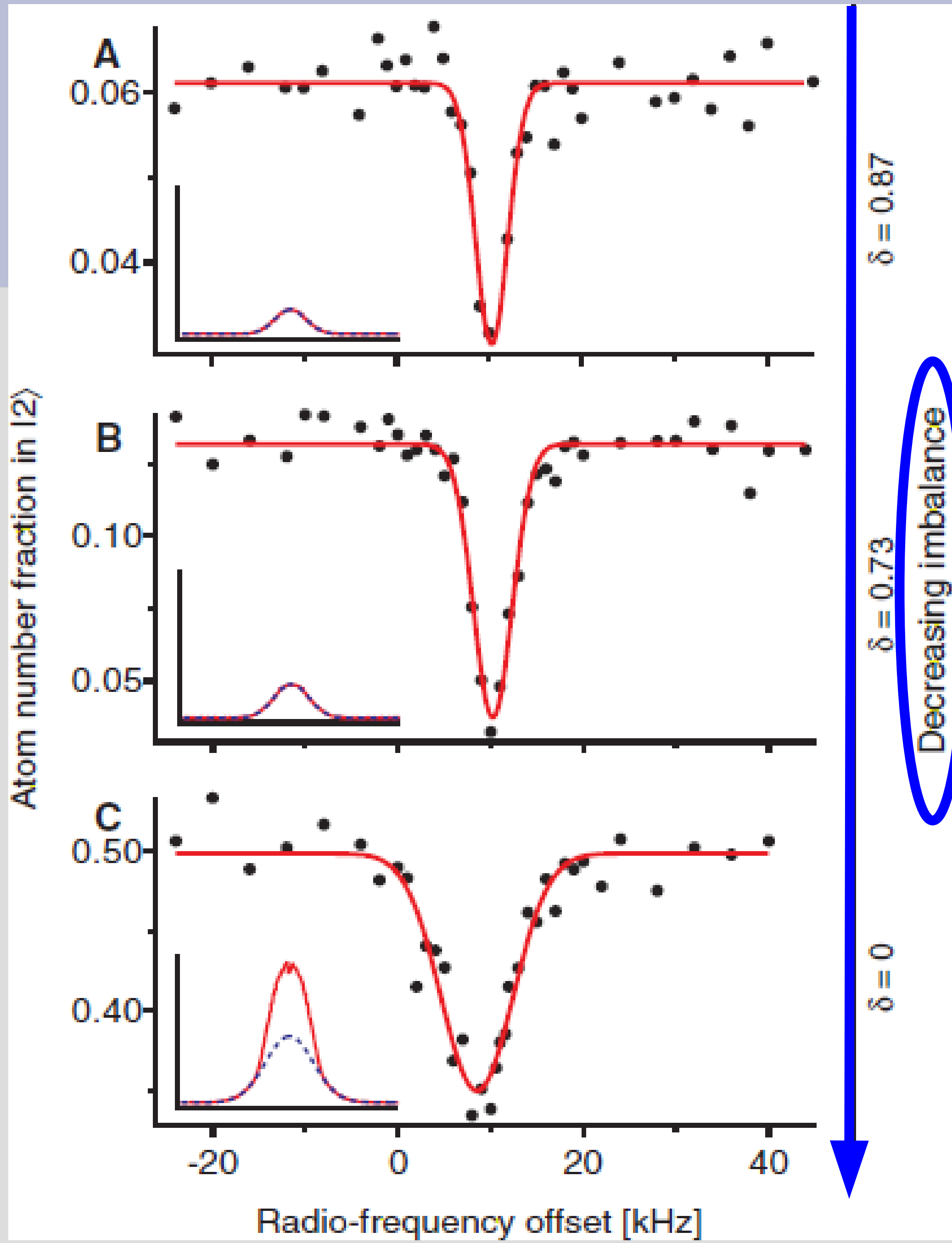
- The spectrum of a homogeneous gas at unitarity has a single peak at any imbalance (the double-peak structure comes from the trap)



- On the BEC side of the resonance $(0 < k_F a < 3)$ unpaired atoms coexist locally with molecules
- An impurity in a Fermi sea: a polaron (MB) at unitarity, a molecule in the BEC regime (2B).

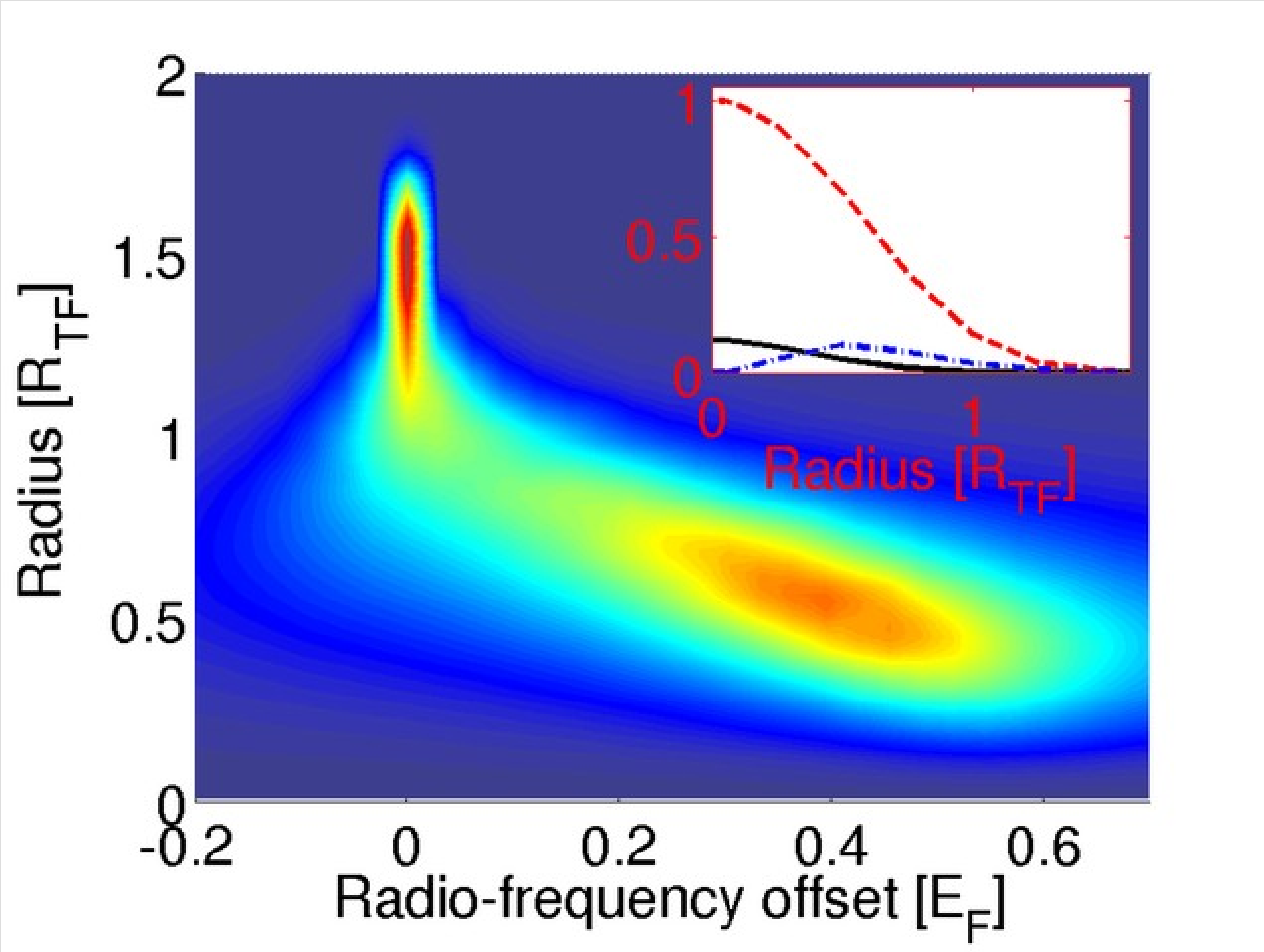
Refs.: Massignan, Bruun and Stoof, PRA 77 (2008) and 78 (2008).





Imbalance=$(N_1-N_2)/(N_1+N_2)$

Integrand of the trapping average



$n_1(r)$

$n_{٢}(r)$

$r^2 n_2(r)$

$T=0.4T_F$

Imbalance=0.9

Improvements

- The pairing peak appears at a lower T and is too shifted: include the 1-3 interaction and the Aslamazov-Larkin correction

  (Baym et al., Punk et al.: shift ~ $\frac{1}{a_{12}} - \frac{1}{a_{13}}$)

- In the ladder approx. T_C is too high, include exchange interactions (Gorkov correction)