Static properties of positive ions in atomic Bose-Einstein condensates

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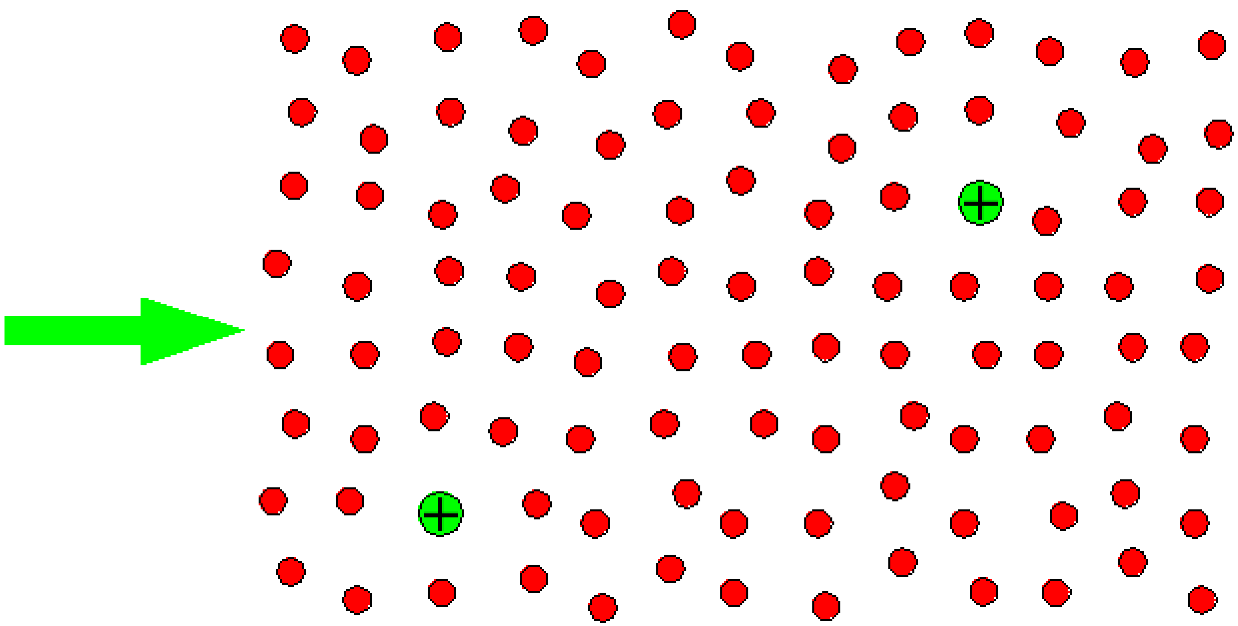
Uniform BEC + few ions

(density n_0)



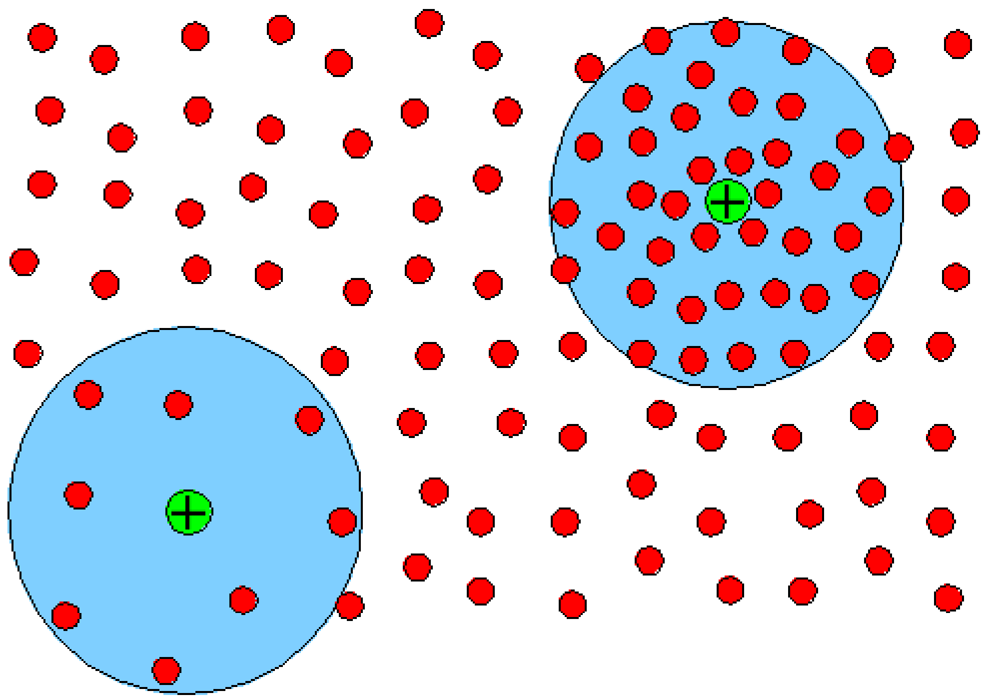
$$E_{es}=\frac{e}{4\pi\epsilon_0 r^2}$$

$$V(r)=-\alpha\frac{E_{es}^2}{2} \quad\rightarrow\quad V(r)=-\frac{\tilde{\alpha}e_0^2}{2}\frac{1}{r^4}$$ (+repulsive core)

$$\tilde{\alpha}=\frac{\alpha}{4\pi\epsilon_0}$$

[Exp: Pisa, PRA **66** 043409 (2002)]

At equilibrium,
two possible outcomes



$N - N_0 > 0$

$N - N_0 < 0$

[R. Côté et al., PRL **89** 093001 (2002)]

What are we looking for?

- States at fixed $\mu = U_0 n_0 > 0$, i.e. in the continuum

  [no capture of atoms into bound levels]

- Determine the number of excess atoms around a single static ion

$$\Delta N = N - N_0 = \int d^3\mathbf{r}\,[n(\mathbf{r}) - n_0]$$

- For given inner b.c. and chemical potential, many solutions are possible: which is the relevant one?

Thermodynamics

1) $F = E - \mu_a N \;\rightarrow\; N = -\frac{\partial F}{\partial \mu_a} \;\rightarrow\; \Delta N = -\frac{\partial (F - F_0)}{\partial \mu_a}$

$\mu_a = \frac{\partial \varepsilon}{\partial n_a}$

2) Add 1 ion and ΔN atoms, $\delta \mu_a = 0 \;\rightarrow\; \frac{\partial \mu_a}{\partial n_a} \cdot \Delta N + \frac{\partial \mu_a}{\partial n_i} \cdot 1 = 0$

Dilute gas, few ions: $\varepsilon = \frac{1}{2} U_{aa} n_a^2 + U_{ai} n_a n_i$

$U_{jl} = \frac{2 \pi \hbar^2 a_{jl}}{m_{jl}}$

reduced mass

a: atoms
i: ions

Dilute limit: $\Delta N = -\frac{U_{ai}}{U_{aa}} = -\frac{m_{aa}}{m_{ai}} \frac{a_{ai}}{a_{aa}}$

$$\text{GP eq.: } \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ai}(r) + U_{aa}|\psi|^2\right]\psi = \mu_a \psi$$

$$E_{kin} \sim E_{pot} \rightarrow \beta_4 = \sqrt{\frac{\tilde{\alpha}}{a_0}\frac{m}{m_e}}$$

$$\beta_4^{Rb} \sim 7.2\times 10^3 a_0$$
$$\beta_4^{Na} \sim 2.6\times 10^3 a_0$$

$$E_{kin} \sim E_{VdW} \rightarrow \beta_6$$

(β_4, β_6 set the scale for $|a_{ai}|$, $|a_{aa}|$)

$$|\Delta N| \sim \left|\frac{a_{ai}}{a_{aa}}\right| \sim \frac{\beta_4}{\beta_6} \sim 10 \div 100$$

large cloud $\rightarrow m_{ai} \sim m_a$

$$E_{kin} \sim E_{int} \rightarrow \xi = \frac{1}{\sqrt{8\pi a_{aa} n_0}}$$

Asymptotics

$\delta\psi = \psi - \psi_0$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + 2U_0 n_0\right]\delta\psi = -V(r)\psi_0 \;\rightarrow\; \text{Finite range V:}\;\; \delta\psi \propto \frac{e^{-k_\xi r}}{r}$$

$$\left(k_\xi = \sqrt{2}/\xi\right)$$

Repulsive hard-core, $\xi \gg R$: $\Delta N = -\frac{1}{2}\frac{R}{a_{aa}}$ ← in accord with TD result

Power law V: $n_{TF}(r) = n_0\left(1 + \frac{(\xi\beta_4)^2}{r^4}\right)$

relevant:

- low n_0 $(\xi \gg \beta_4)$ → Yukawa
- high n_0 $(\xi \ll \beta_4)$ → TF

Attractive square well

Nonlin. diff. eq., many solutions for given n_0

# of nodes of $\psi_S^{(E=0)}$

At low density there are

2 sols. with ν_S nodes
2 sols. with $\nu_S - 1$ nodes
⋮
1 sol. with 0 nodes

i.e. $\mathbf{2}\nu_\mathbf{S}+\mathbf{1}$ possible solutions.

Low asymptotic density

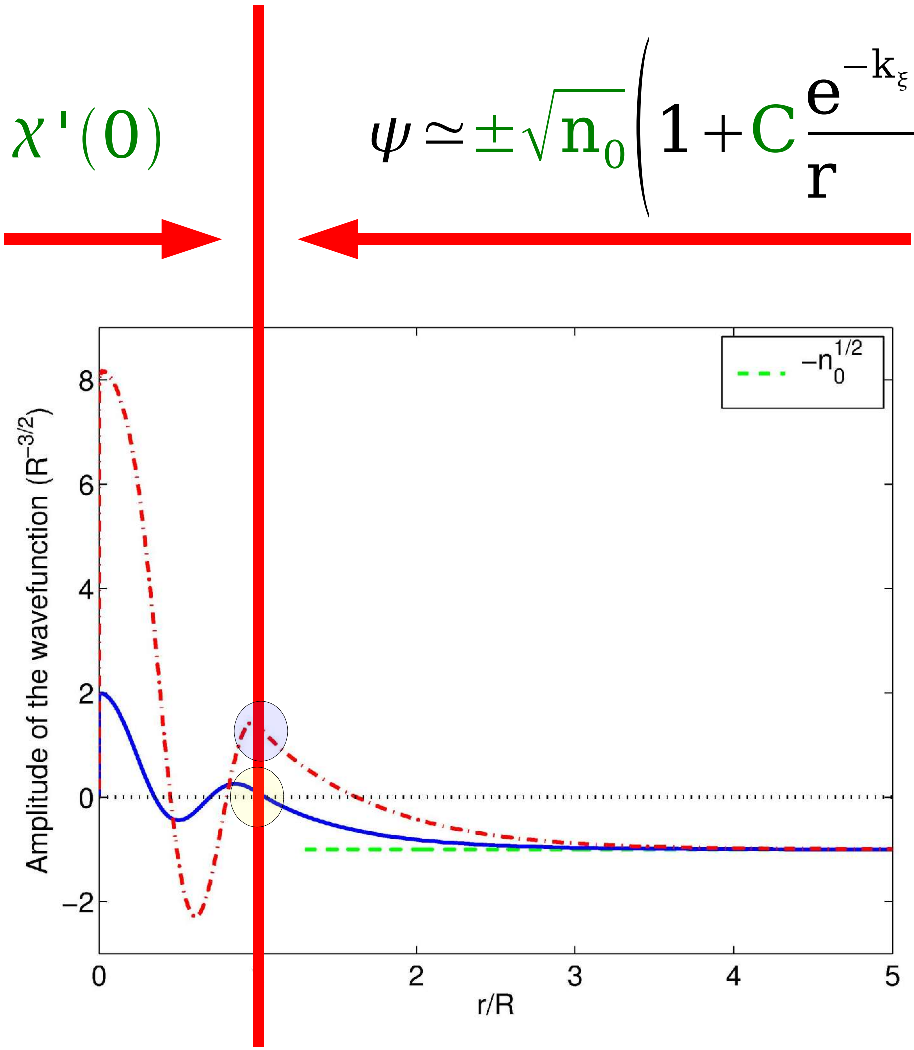
(here $\nu_S = 3$)

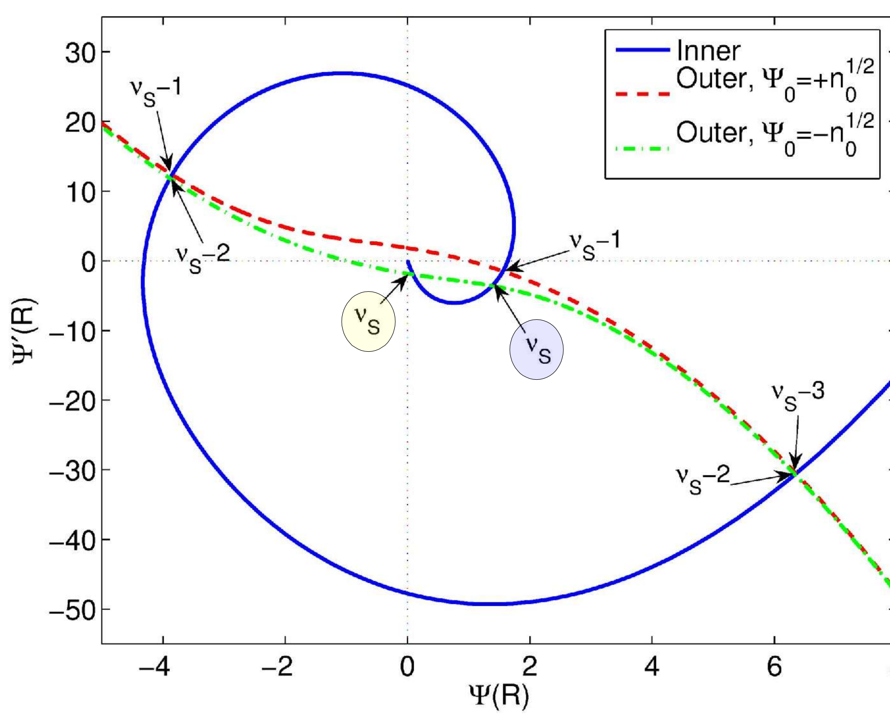
$\chi = r\,\psi$

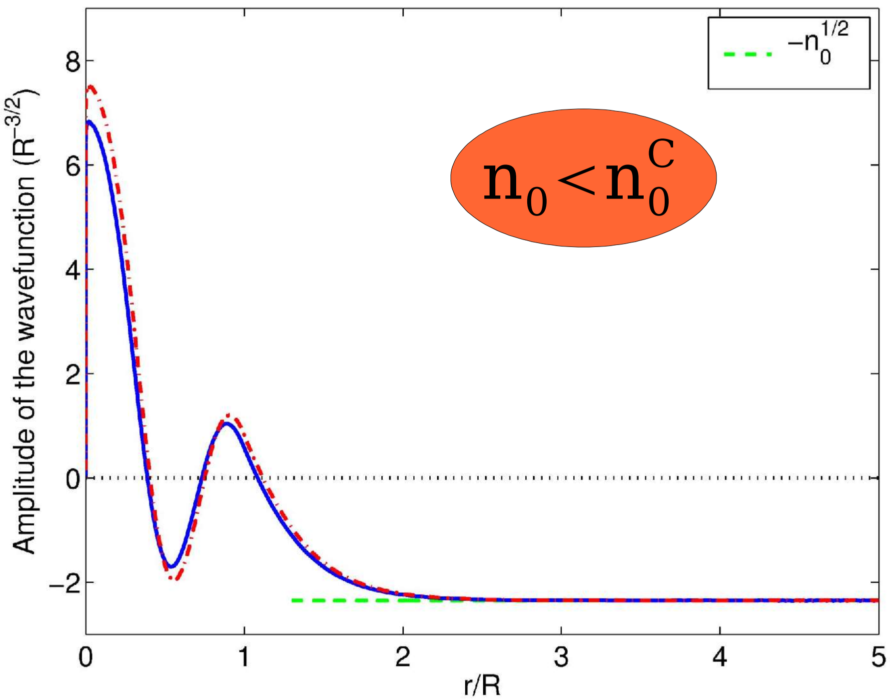
$\chi'(0)$

$$\psi \simeq \pm\sqrt{n_0}\left(1 + C\,\frac{e^{-k_\xi r}}{r}\right)$$

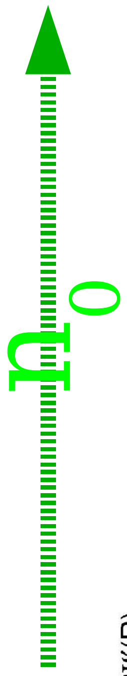
$\psi(R) - \psi'(R)$ plane

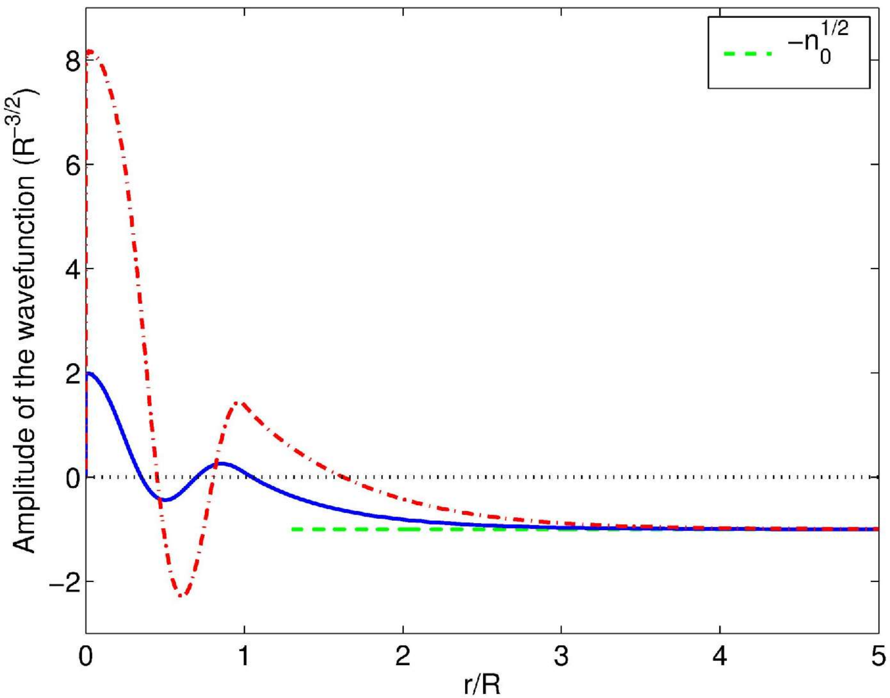


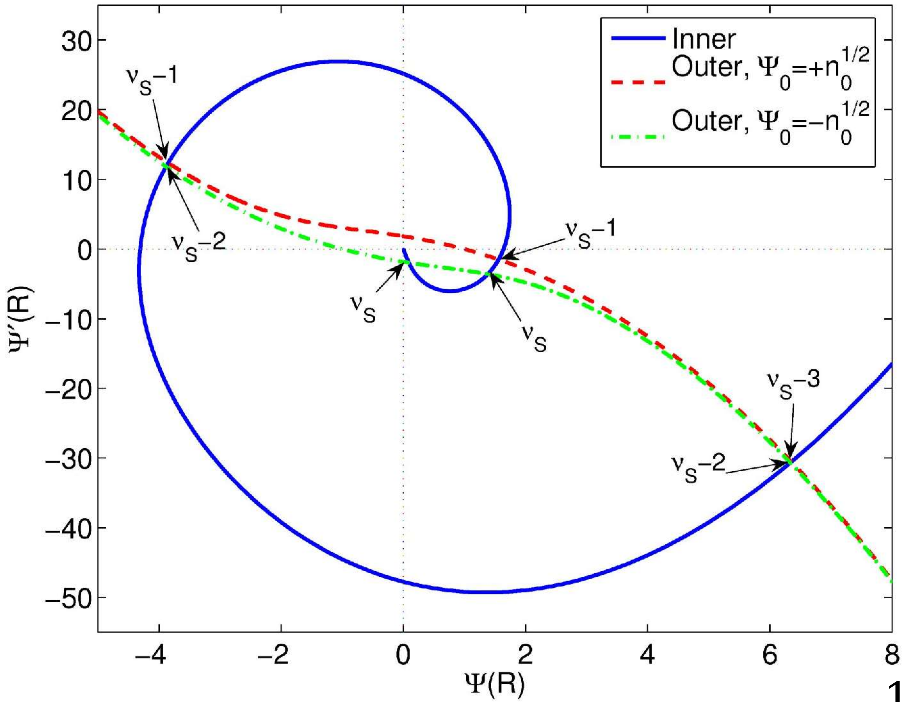


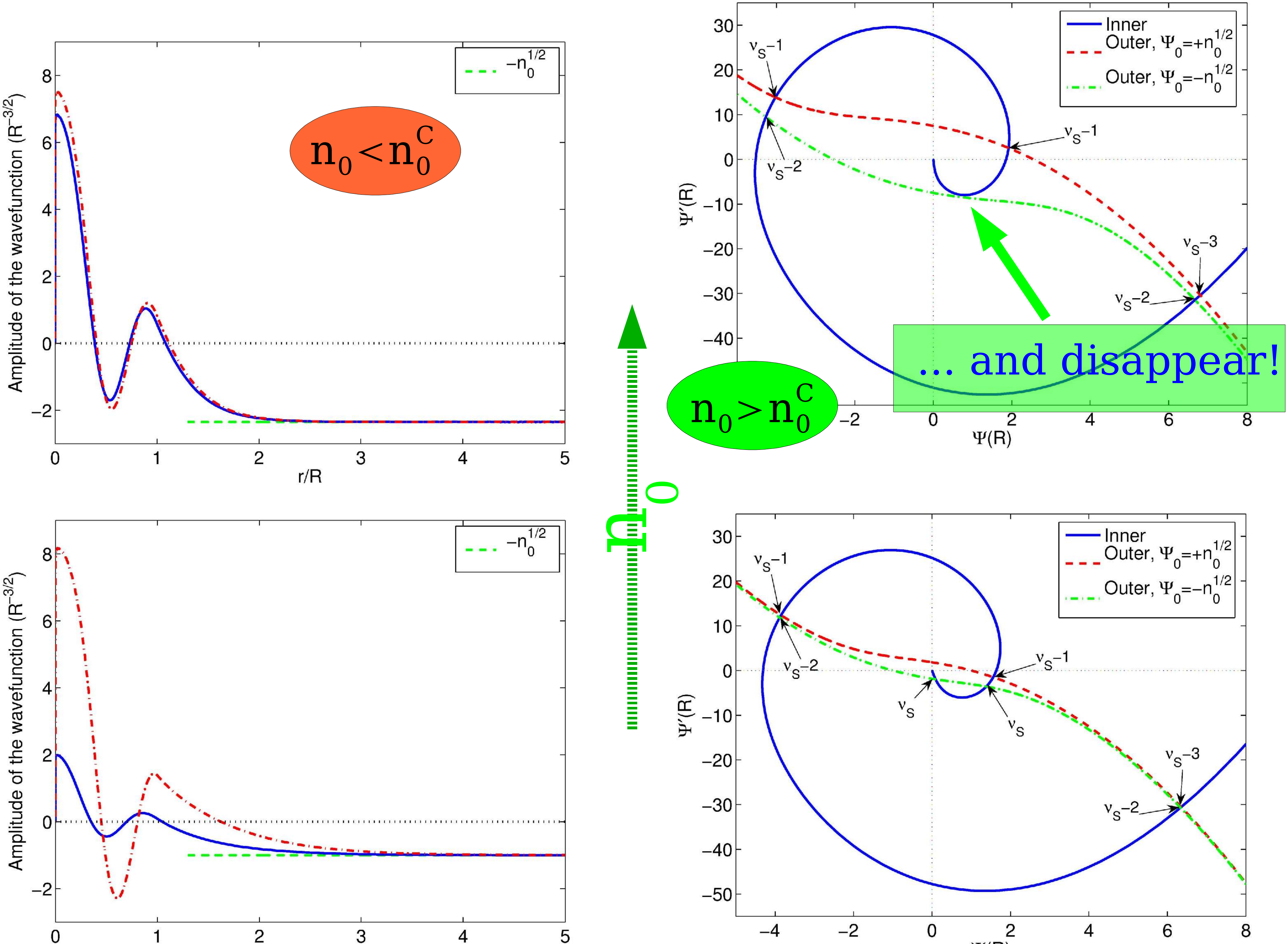


Increasing asympt. density n_0, solutions merge

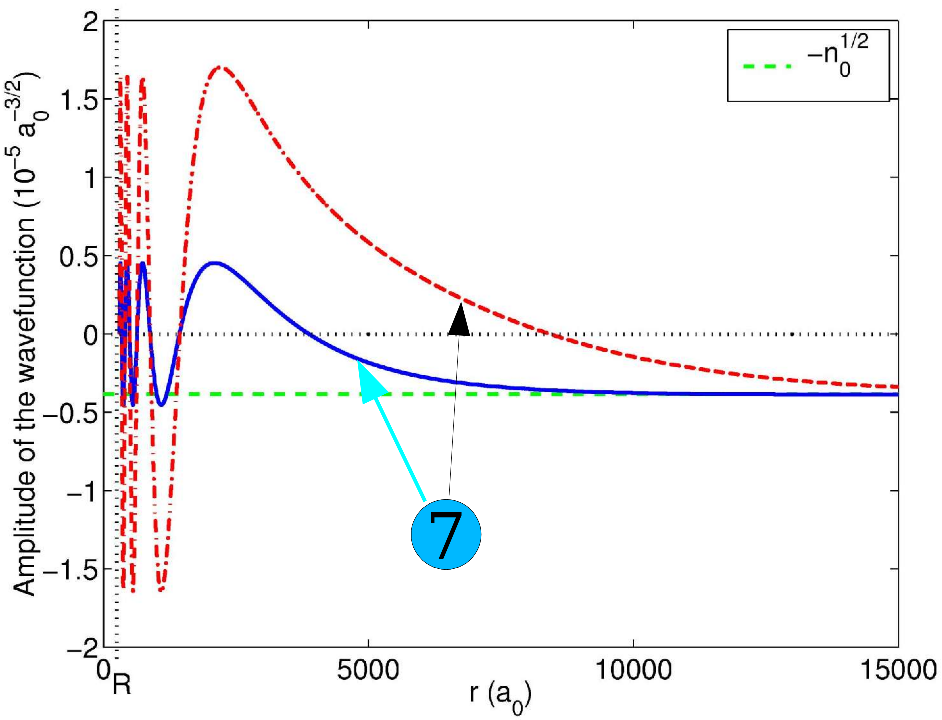




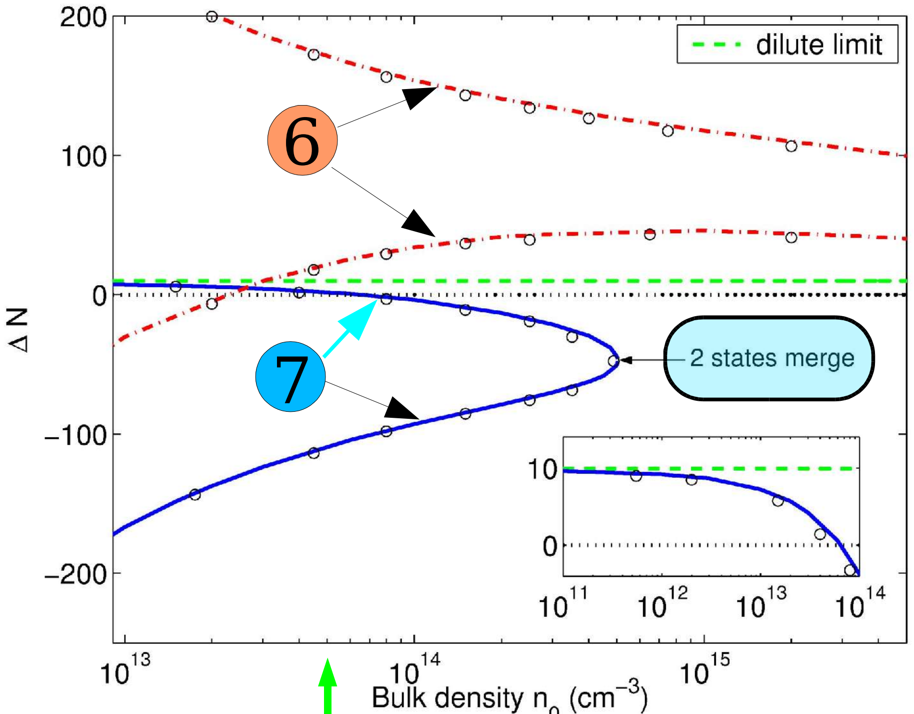



Amplitude of the wavefunction ($R^{-3/2}$)
r/R
$-n_0^{1/2}$
$n_0 < n_0^C$
$n_0 > n_0^C$
n_0
$\Psi'(R)$
$\Psi(R)$
Inner
Outer, $\Psi_0=+n_0^{1/2}$
Outer, $\Psi_0=-n_0^{1/2}$
ν_S-1
ν_S-2
ν_S-3
ν_S
... and disappear!

Ionic potential



(^{87}Rb, core $R=300a_0 \rightarrow \nu_S=7$)



$\xi = \beta_4$

(dilute limit valid for $\xi \gg \beta_4$)

Summary

- The structure around the ion can be very large, involving up to a few hundred atoms.
- Even in absence of inelastic processes, the evolution of the system with density cannot be continuous (sudden jumps in ΔN).
- New experimental work?

[more details on: PRA **71**, 023606 (2005)]