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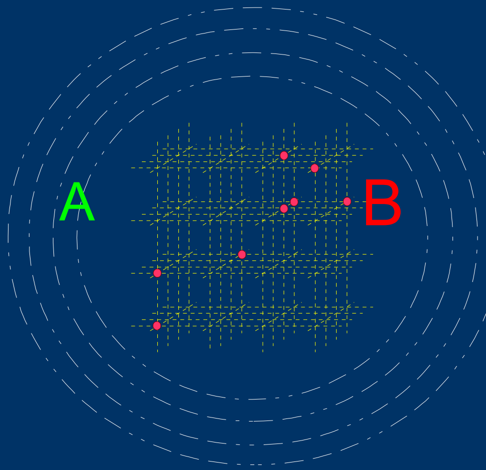
Trapped impurities in a lattice: confinement induced resonances and Anderson localization



Quantum Simulation with ultracold atoms

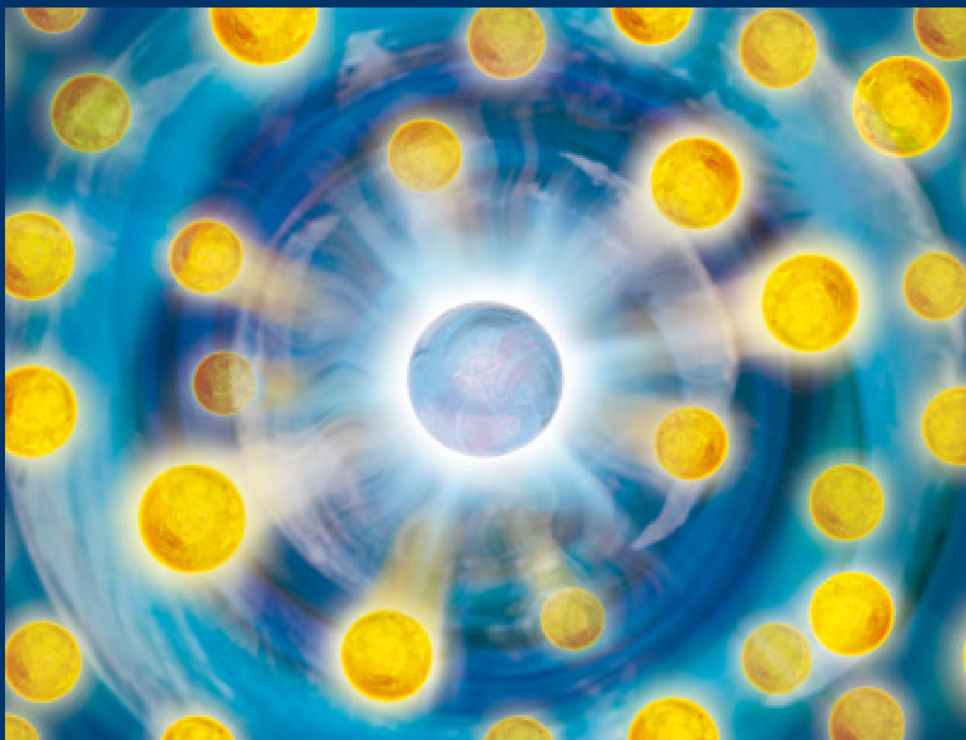
Interactions
Temperature

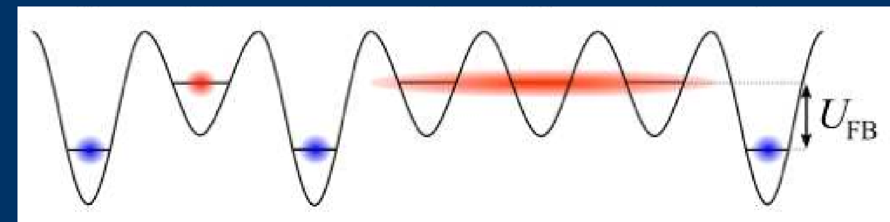
Periodic potentials
Physical dimension
Exotic couplings
Dynamics

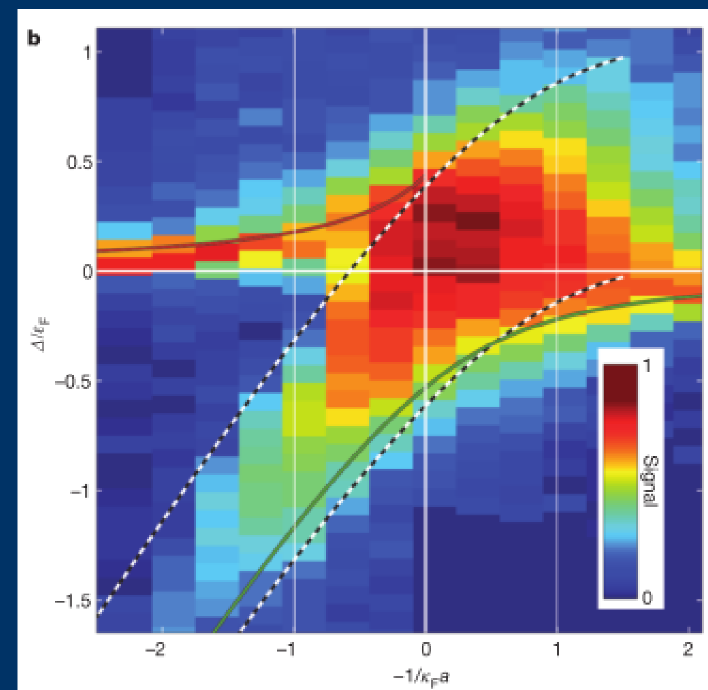
Impurities

Disorder

Weak admixtures: mobile impurities







Shift of the Mott-SF transition: Hamburg, LENS, MPQ

Strong interactions in free space: MIT, Innsbruck, Cambridge

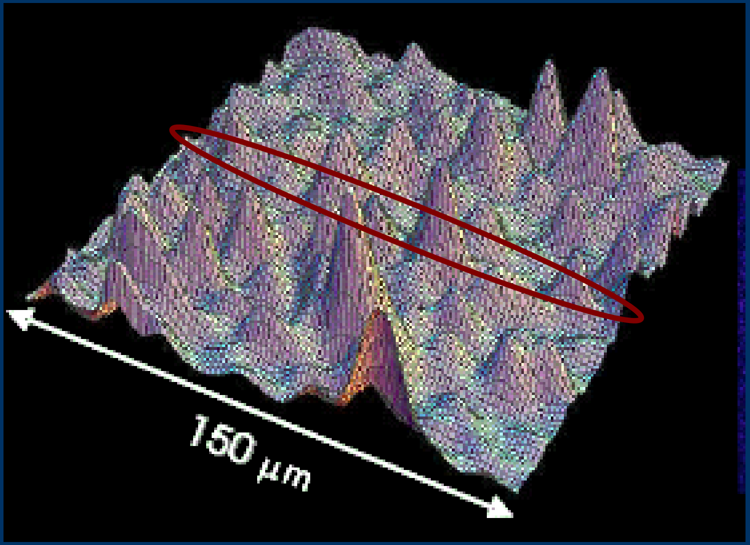
Static disorder: Anderson localization

- **What**: quantum transition due to static disorder
  - massive number of localized states
  - no diffusion in an infinite medium
- **When**: $\lambda > \frac{1}{n\sigma}$ (Ioffe-Regel criterion)
- **Where**: cold atoms! (no decoherence)

But …, how to produce disorder?

Speckle potentials

1D potentials $(d \sim 5-20\,\mu m)$*,*
cigar-shaped condensates,
*up to 30*1 wells occupied*



Disorder related effects:

- collective oscillations
- fragmentation
- suppression of diffusion

Problems towards strong localization:

- classical trapping
- large length scale for the disorder

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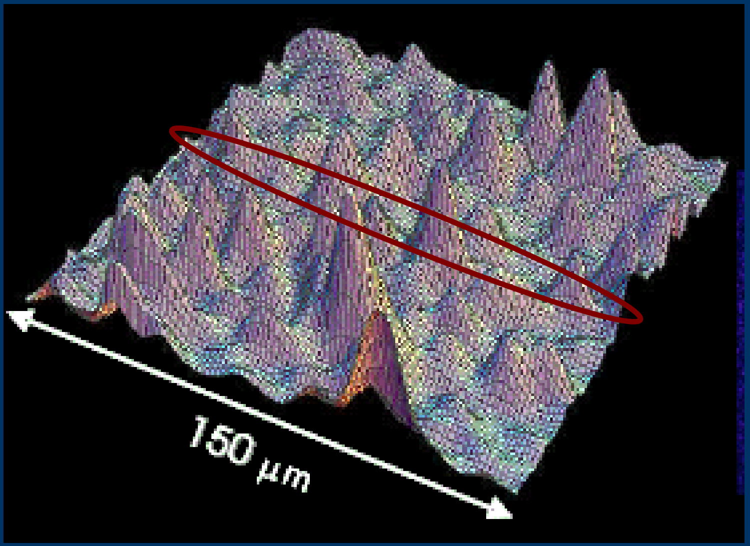

Disorder related effects:
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- fragmentation
- suppression of diffusion

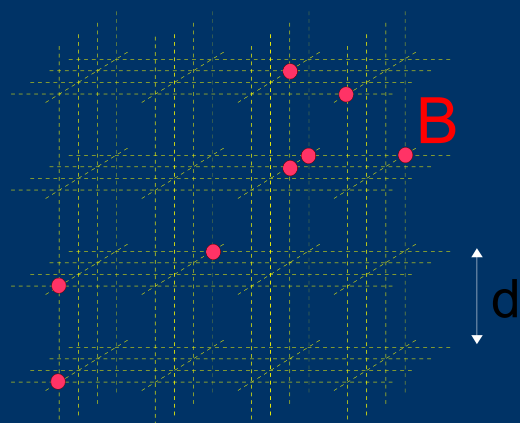
Problems towards strong localization:
- classical trapping
- large length scale for the disorder

So ..., how to produce **stronger and better disorder**?

Static and disordered gas

Set of particles (**B**) **trapped** in a deep lattice,
filling factor << 1 ----> **random potential**

each particle in the ground state of the local well



$$d = \lambda/2 \sim 0.5\,\mu m$$

1D: Gavish & Castin, PRL 2005
3D: Massignan & Castin, PRA 2006
2D: Antezza, Castin & Hutchinson, PRA 2010

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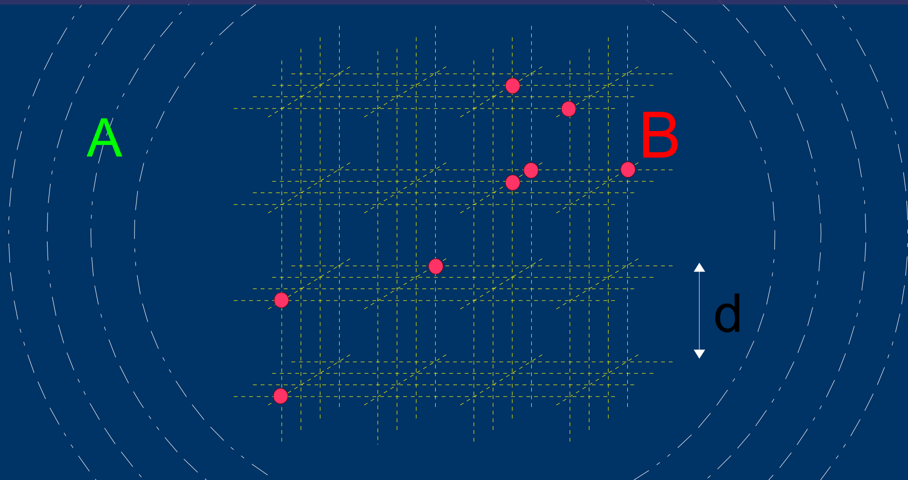

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Matter wave **A** interacts with **B** particles but does **NOT** feel the lattice

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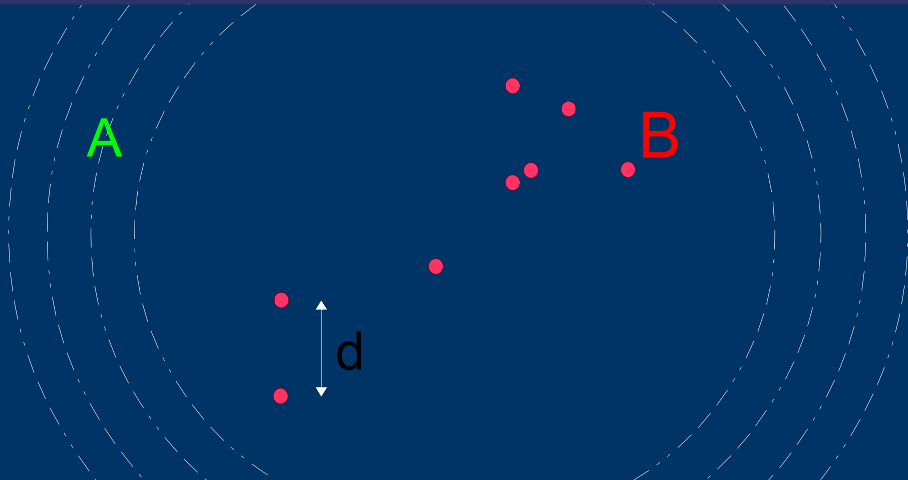


$$d = \lambda/2 \sim 0.5\,\mu m$$

Matter wave **A** interacts with **B** particles but does **NOT** feel the lattice

Advantages of this scheme

Very small correlation length for the disorder ($\xi \sim d$)

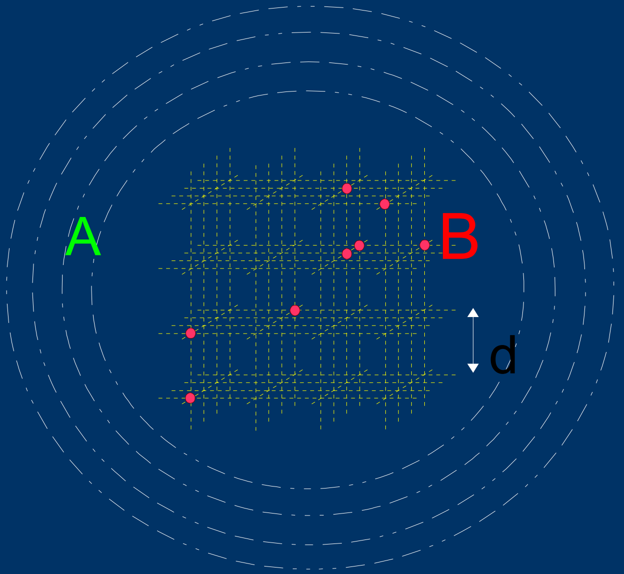
No classical localization in potential minima

Unitarity limited A-B interaction

Exact numerical analysis

Conditions for deep trapping

$$V_0^B \gg E_r^B$$



$$Elastic\ scattering\ if: \frac{\hbar^2 k^2}{2m_A} \ll \hbar\omega$$

$$\text{Fluorescence rate: } \Gamma^B_{\text{fluo}} = \Gamma_B \frac{V_0^B}{\omega_L - \omega_B} \frac{3k_L^2}{2m_B\omega}$$

(blue detuning)

Γ_B: spontaneous emission rate

Examples with $V_0^B = 50E_r^B$

^{87}Rb and $\lambda_L = 779$nm: one gets $t_{\text{tunnel}} \approx 0.7s$ and $\Gamma^B_{\text{fluo}} = 3/s$

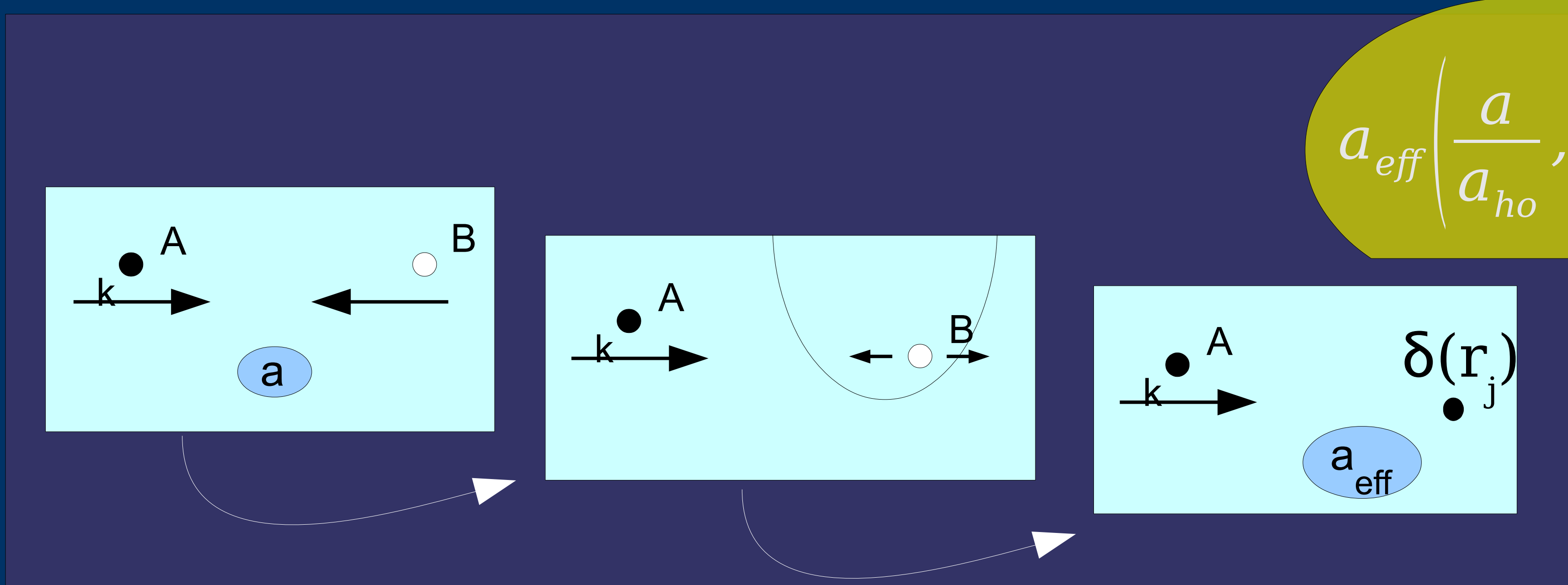
^{40}K and $\lambda_L = 765.5$nm: one gets $t_{\text{tunnel}} \approx 0.3s$ and $\Gamma^B_{\text{fluo}} = 5/s$

Many trapped impurities

Deep lattice --> static and independent scatterers

$$V(\vec{r}_A)=g_{eff}\sum_{j=1}^{N}\delta(\vec{r}_A-\vec{r}_j)\,\partial_{|\vec{r}_A-\vec{r}_j|}|\vec{r}_A-\vec{r}_j| \qquad g_{\mathrm{eff}}=\frac{2\pi\hbar^2 a_{\mathrm{eff}}}{m_A}$$

$$a_{eff}\left(\frac{a}{a_{ho}},\frac{m_B}{m_A}\right)$$

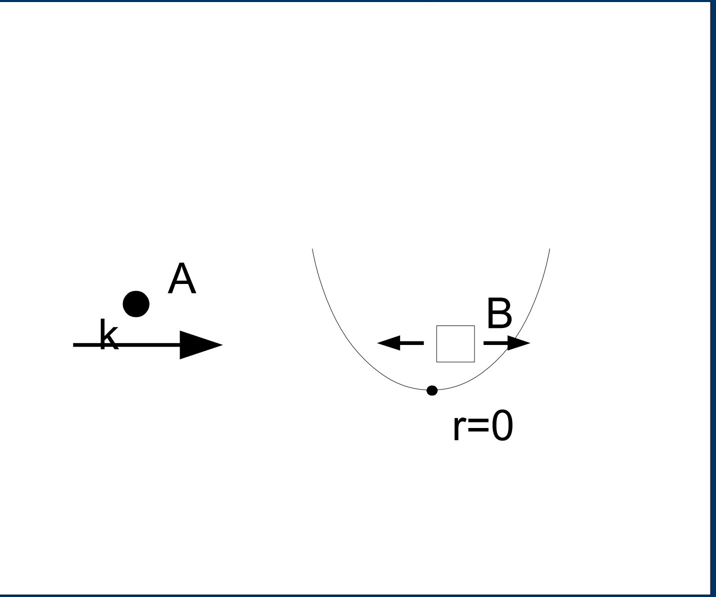


(Massignan & Castin, PRA 2006)

2 body problem

A free, B trapped

$$H = H_0$$



$$s\text{-wave sol. of } H_0: \ \psi_0 = \frac{\sin(k r_A)}{k r_A} \cdot \phi_0(r_B)$$

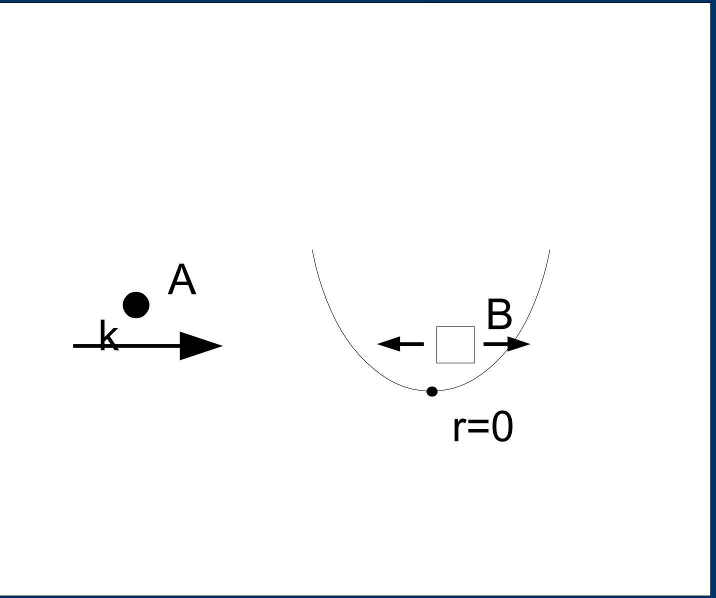
(Massignan & Castin, PRA 2006)

2 body problem

A free, B trapped

$$H = H_0 + V$$

Contact pot.



$$s\text{-}wave\ \ sol.\ \ of H_0:\ \ \psi_0 = \frac{\sin(k r_A)}{k r_A} \cdot \phi_0(r_B)$$

$$At\ \ fixed\,\vec{R}\,,\ \ \psi(\vec{r}_A,\vec{r}_B) \underset{r\to 0}{=} \psi_{reg}(R)\left(1 - \frac{a}{r}\right) + o(1)$$

$$s\text{-}wave\ \rightarrow\ \psi_{reg}(\vec{R}) = \psi_{reg}(R)$$

$$V\psi(\vec{r}_A,\vec{r}_B) = g\cdot\delta(\vec{r})\frac{\partial}{\partial r}\Big(r\psi(\vec{r}_A,\vec{r}_B)\Big) = g\cdot\delta(\vec{r})\psi_{reg}(R)$$

$$G=\frac{1}{E+i0^{+}-H_0}$$

$$\psi\left(\vec{r}_A,\vec{r}_B\right)=\psi_0\left(\vec{r}_A,\vec{r}_B\right)+g\int d\vec{\rho}\, G\left(\vec{r}_A,\vec{r}_B;\vec{\rho},\vec{\rho}\right)\psi_{reg}(\rho)$$

$$\vec{r}\rightarrow 0: \quad \psi_{reg}(R)=\psi_0(R,R)+g\int d\rho\,\hat{O}(R,\rho)\psi_{reg}(\rho)$$

$$\psi_{reg}=\frac{I}{I-g\hat{O}}\psi_0$$

$$G=\frac{1}{E+i0^{+}-H_0}$$

$$\psi(\vec{r}_A,\vec{r}_B)=\psi_0(\vec{r}_A,\vec{r}_B)+g\int d\vec{\rho}\, G(\vec{r}_A,\vec{r}_B;\vec{\rho},\vec{\rho})\psi_{reg}(\rho)$$

$$\vec{r}\rightarrow 0:\quad \psi_{reg}(R)=\psi_0(R,R)+g\int d\rho\, \hat{O}(R,\rho)\psi_{reg}(\rho)$$

$$\psi_{reg}=\frac{I}{I-g\hat{O}}\psi_0$$

$$G=\sum\frac{|\vec{k}_A,\vec{n}><\vec{k}_A,\vec{n}|}{E-E_{\vec{k}_A,\vec{n}}}$$

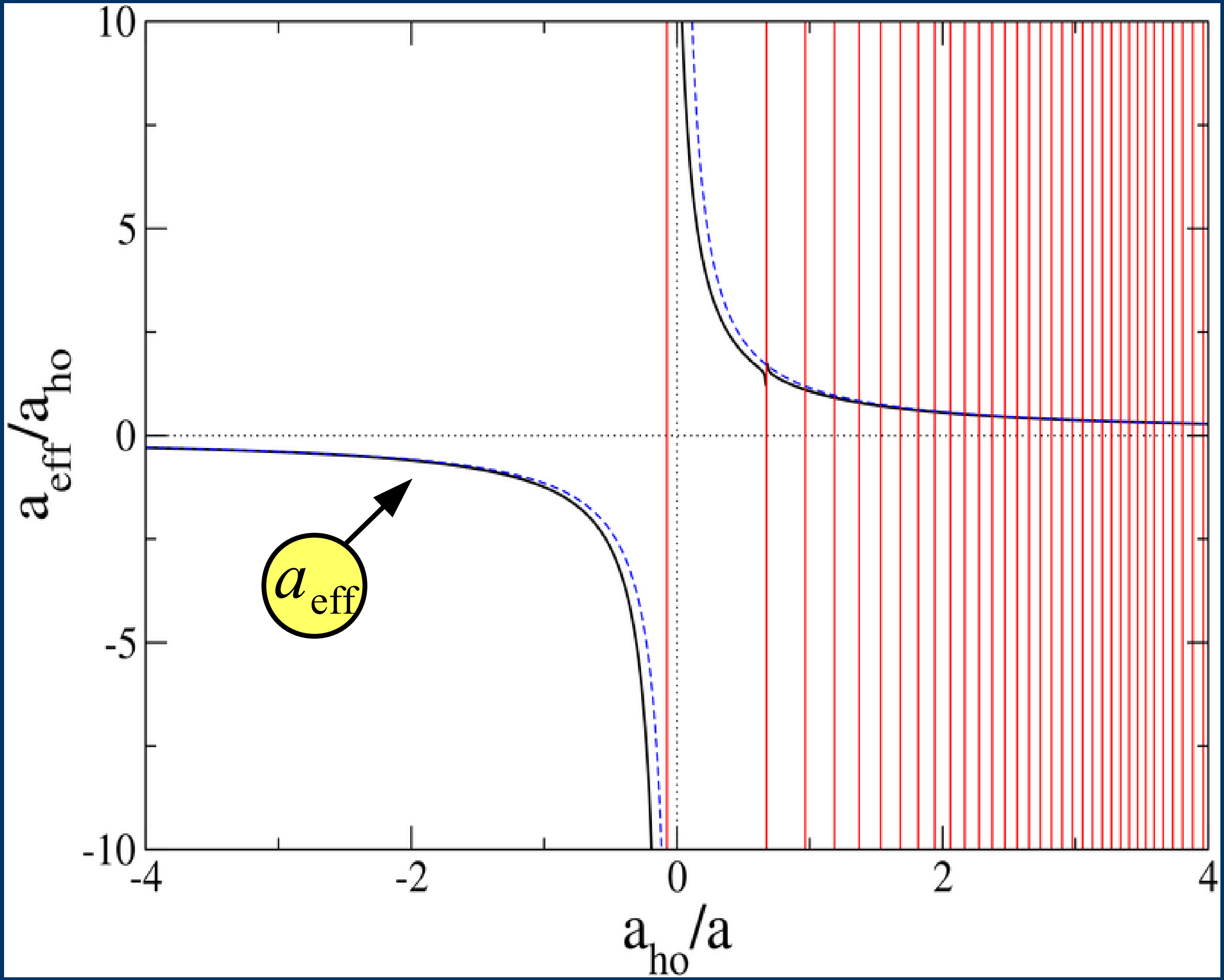
(single open channel: $\vec{n}=0$)

$$\psi(\vec{r}_A,\vec{r}_B)\underset{r_A\rightarrow\infty}{\simeq}\left[\frac{\sin(k r_A)}{k r_A}+f_k\frac{e^{ikr_A}}{r_A}\right]\phi_0(r_B)$$

$$a_{eff}=-\lim_{k\rightarrow 0} f_k=a\frac{m_A}{\mu}\int d\vec{\rho}\,\phi_0(\rho)\psi_{reg}(\rho)$$

$$g_{eff}=\frac{2\pi\hbar^2 a_{eff}}{m_A}$$

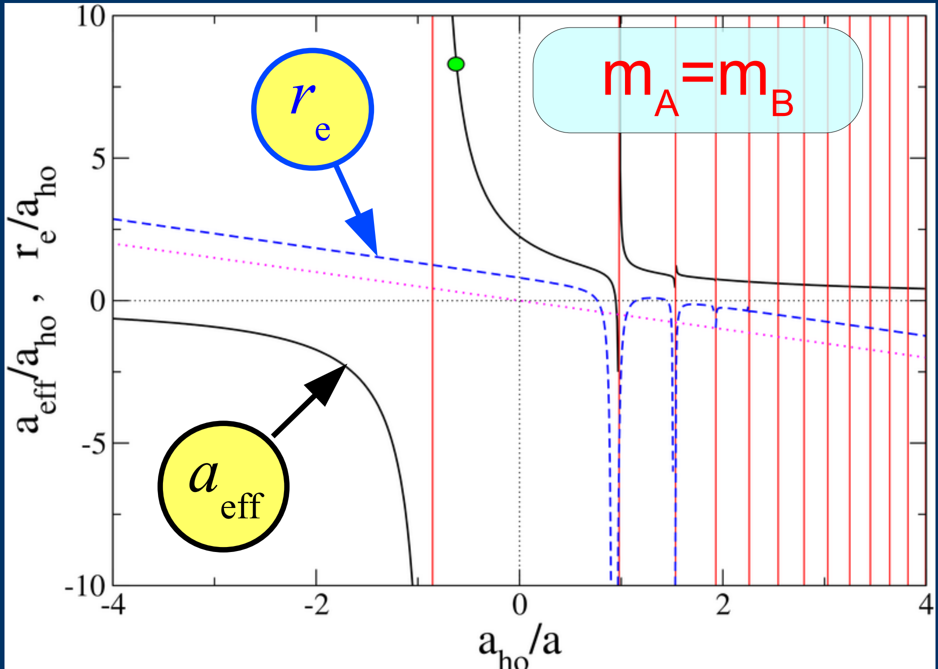
Trapped ^{40}K and free ^{6}Li

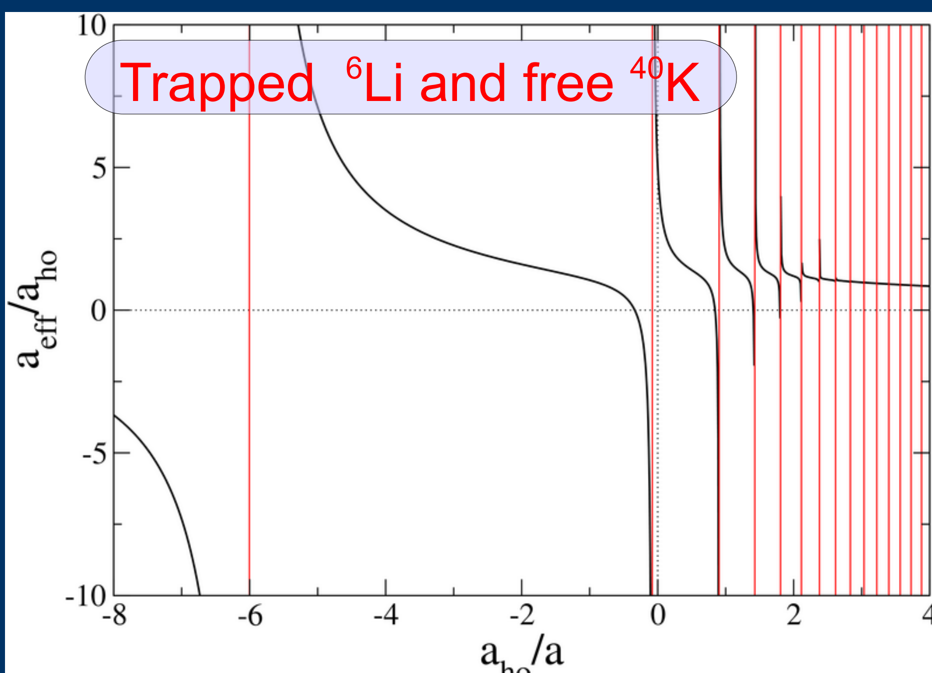


confinement induced resonances!

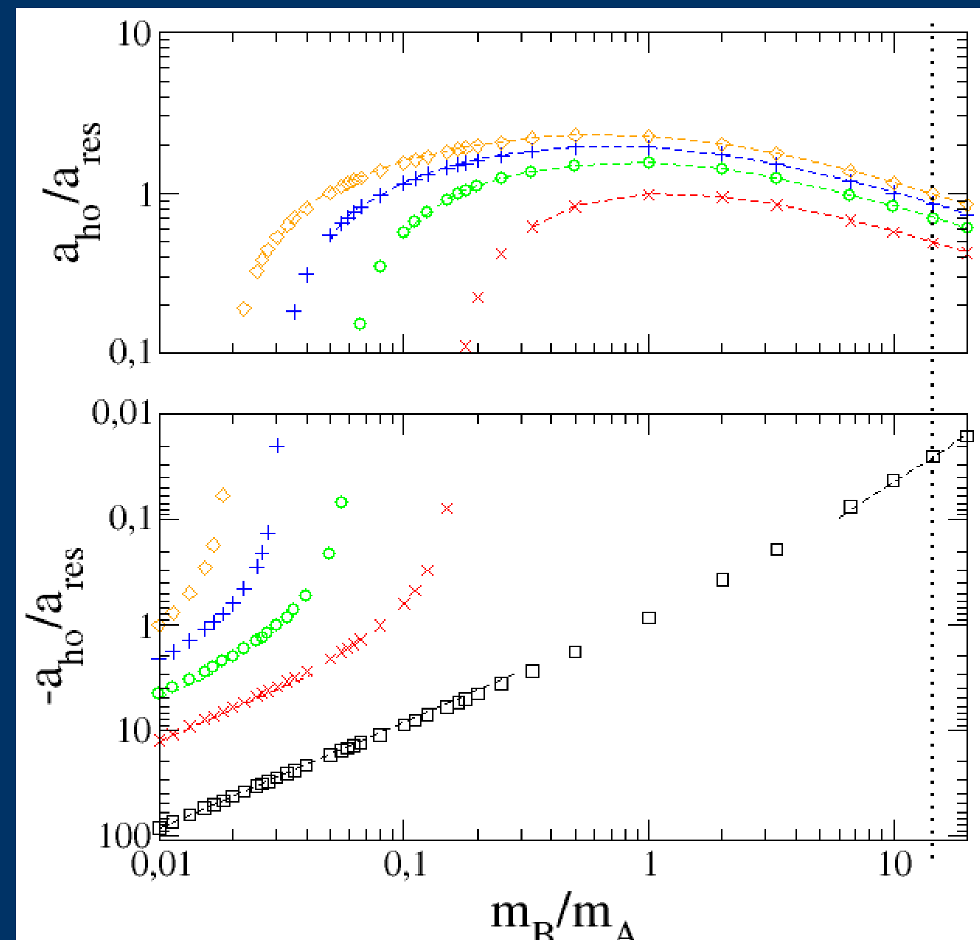
by tuning either scattering length or a_{ho}

(Massignan & Castin, PRA 2006)


$m_A = m_B$
r_e
a_{eff}
a_{eff}/a_{ho} , r_e/a_{ho}
a_{ho}/a


Trapped ^{6}Li and free ^{40}K
a_{eff}/a_{ho}
a_{ho}/a

CIRs!


a_{ho}/a_{res}
$-a_{ho}/a_{res}$
m_B/m_A

Position of the CIRs

$$H=-\frac{\hbar^2\Delta_R}{2(m_A+m_B)}+\frac{1}{2}m_B\omega^2R^2-\frac{\hbar^2\Delta_r}{2\mu}+g\,\delta(\vec{r})\frac{\partial}{\partial r}(r\cdot)+\left[\frac{1}{2}\frac{m_A\mu}{m_A+m_B}\omega^2r^2-\mu\,\omega^2\vec{R}\cdot\vec{r}\right]$$

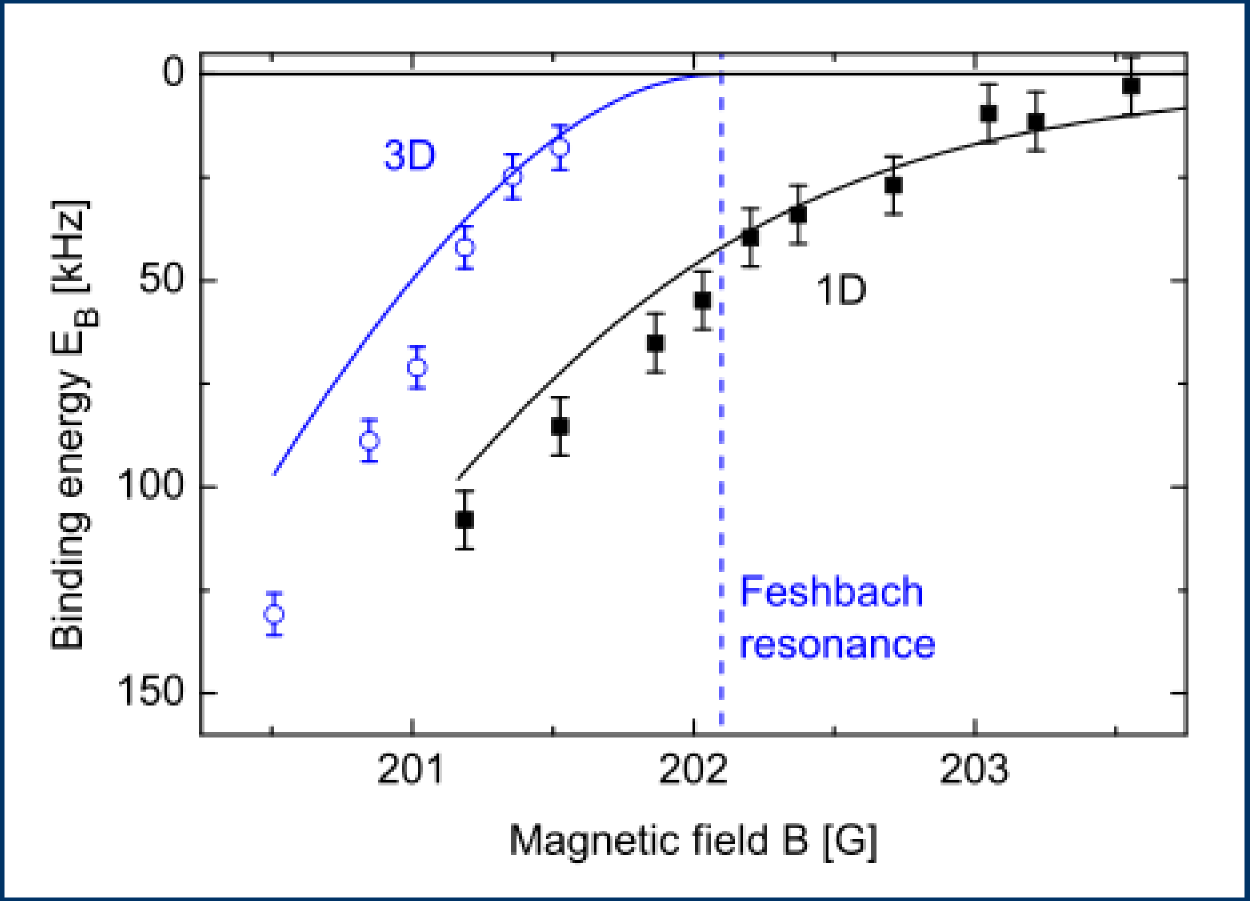
$$a>0:\quad \frac{3}{2}\hbar\omega=\left(2\mathrm{n}+\frac{3}{2}\right)\hbar\omega\sqrt{\frac{m_B}{m_A+m_B}}-\frac{\hbar^2}{2\mu\,a_{\mathrm{res}}^2}$$

$m_B/m_A\ll 1\rightarrow$ Born-Oppenheimer,

$$a<0:\quad H_{\mathrm{eff}}=-\frac{\hbar^2}{2m_A}\Delta_{\vec{r}_A}-\frac{2\pi\hbar|a|}{m_B}\frac{\exp\left(-r_A^2/a_{\mathrm{ho}}^2\right)}{\left(\sqrt{\pi}\,a_{\mathrm{ho}}\right)^3}$$

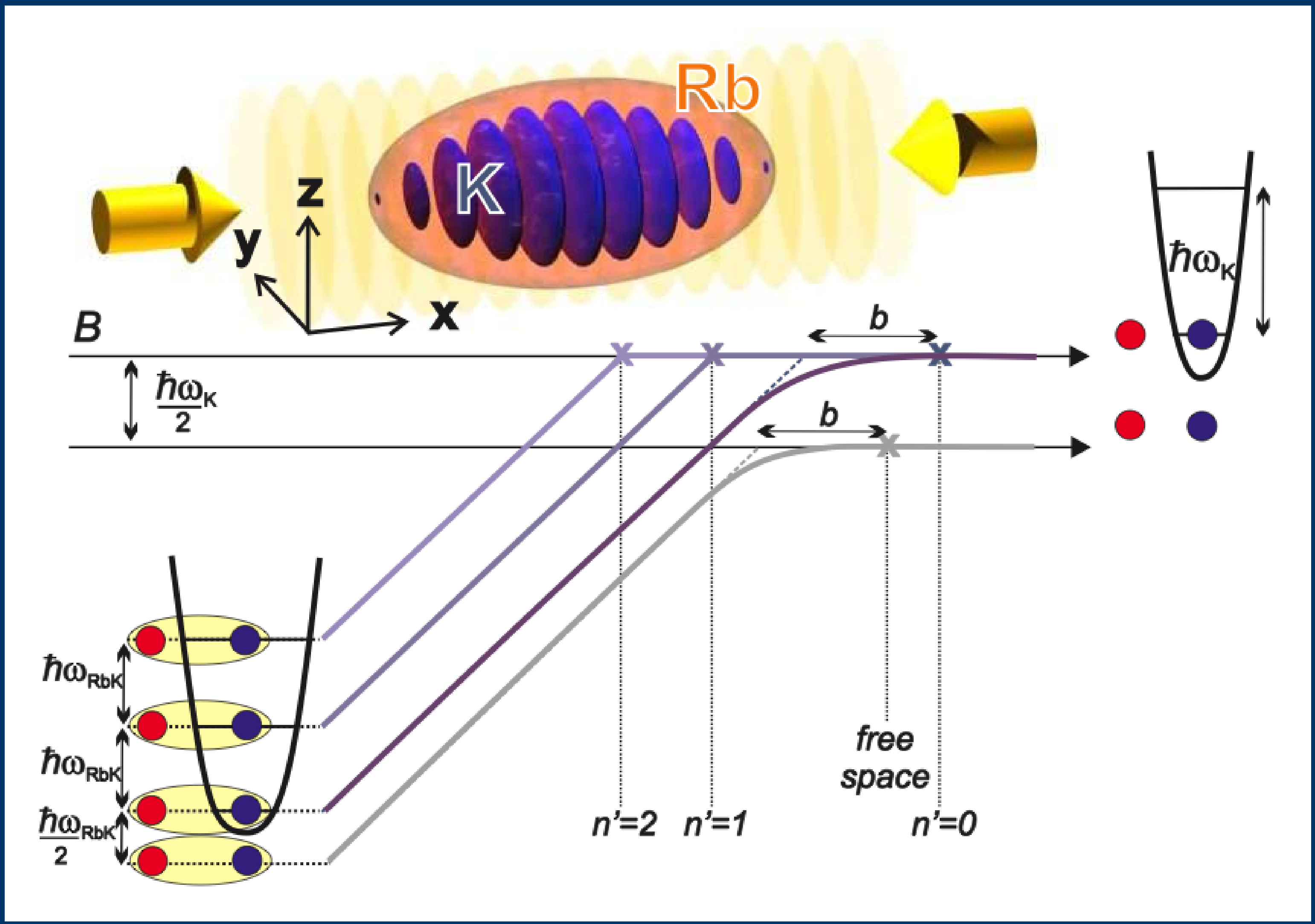
1D Confinement Induced Resonance

Moritz et al., PRL 2005



Experiment @ LENS

Lamporesi et al., PRL 2010



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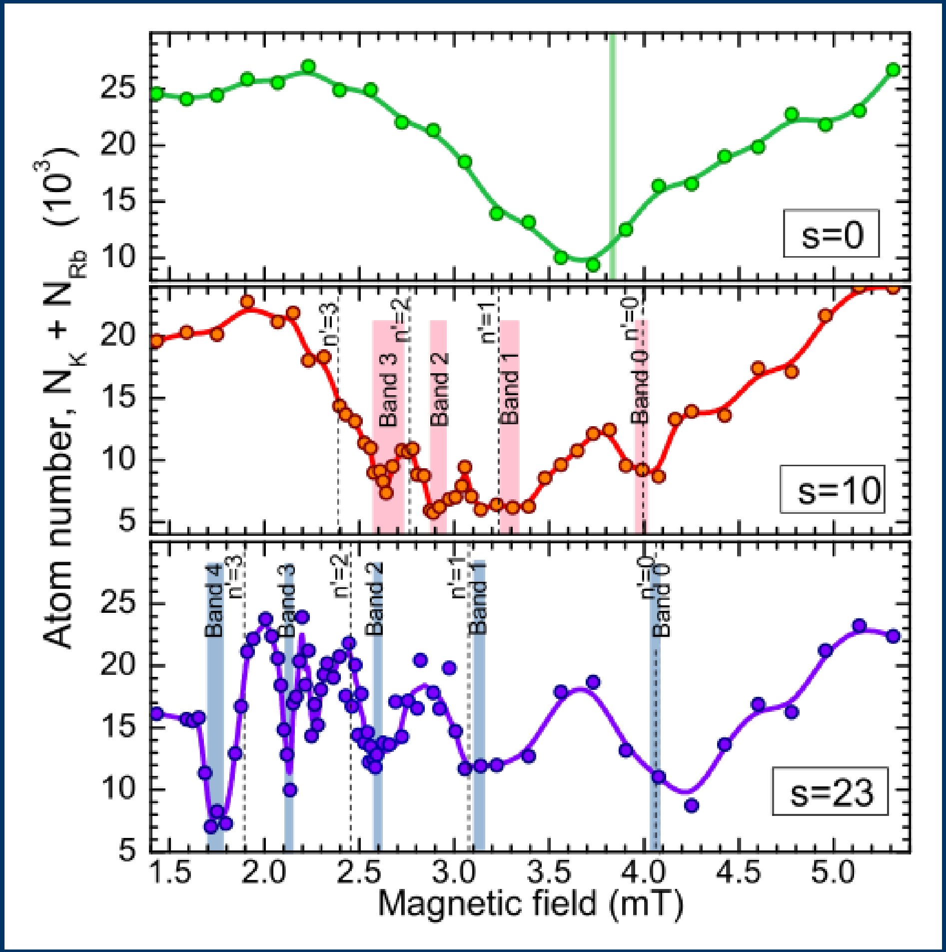

Eigenstates of the model

$$(E+i0^+-H)G_E=1$$

$\phi(\vec{r};\vec{r}_0)=Im\langle\vec{r}|G_E|\vec{r}_0\rangle$ *is an eigenstate of* H *for any* $\vec{r}_{0,}E$

$$\text{since}(E+i0^+-H)\langle\vec{r}|G_E|\vec{r}_0\rangle=\delta(\vec{r}-\vec{r}_0)$$

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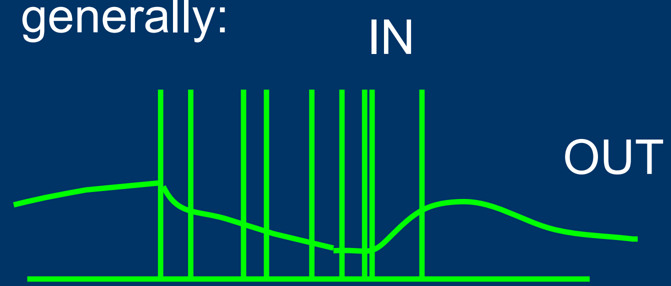
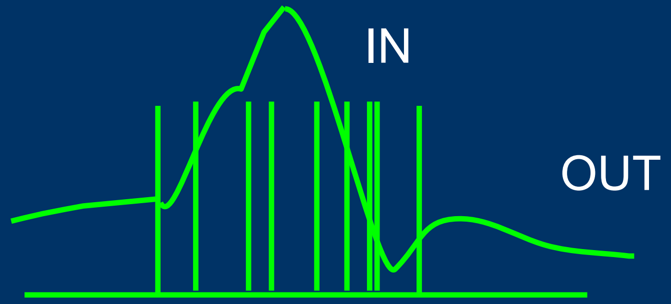
$$V(\vec{r}_A)=g_{eff}\sum_{j=1}^{N}\delta(\vec{r}_A-\vec{r}_j)\partial_{|\vec{r}_A-\vec{r}_j|}|\vec{r}_A-\vec{r}_j|$$

ϕ is given in terms of (the inverse of) a complex NxN matrix generally:

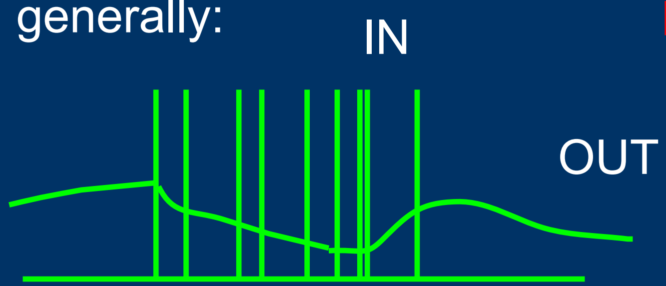


localized state:

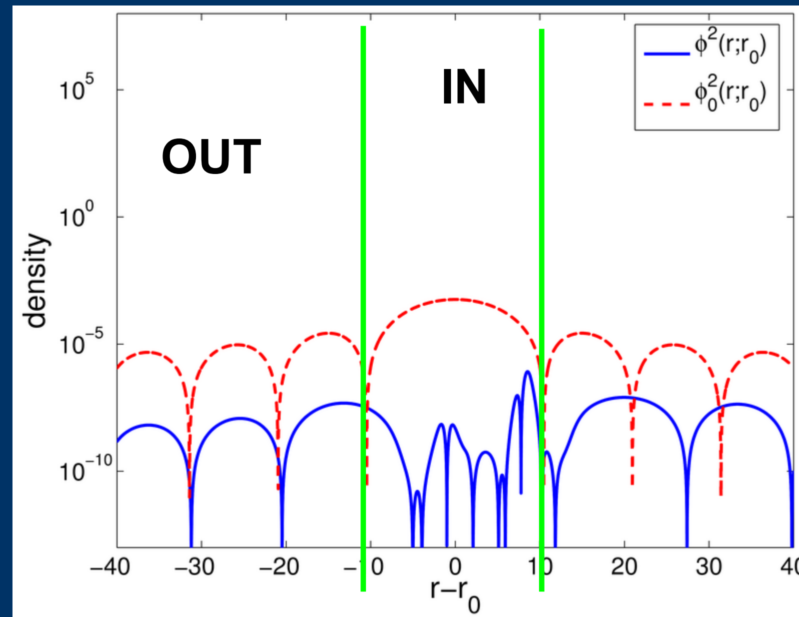


Two states

generally:

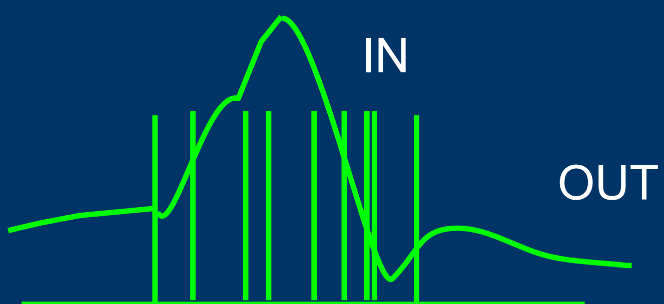


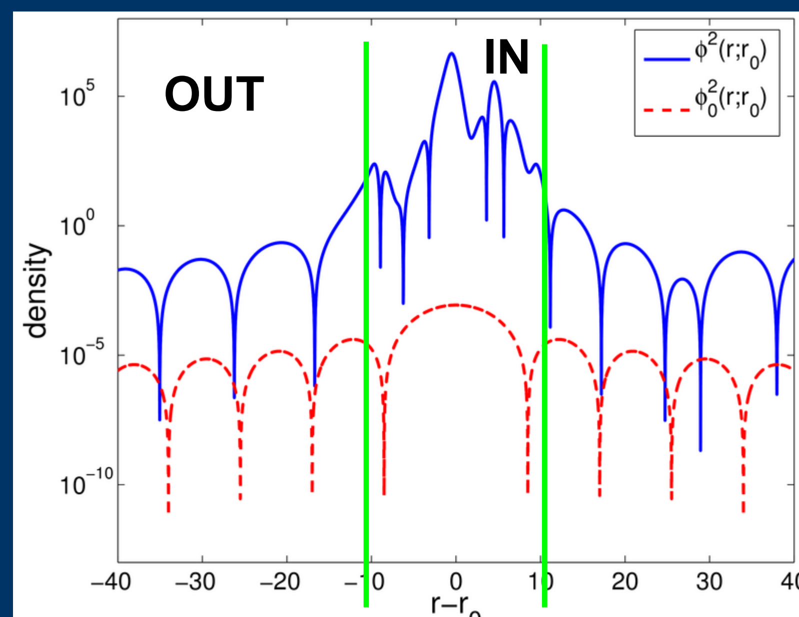
k=0.3 :



localized state:

k=0.3695215... :





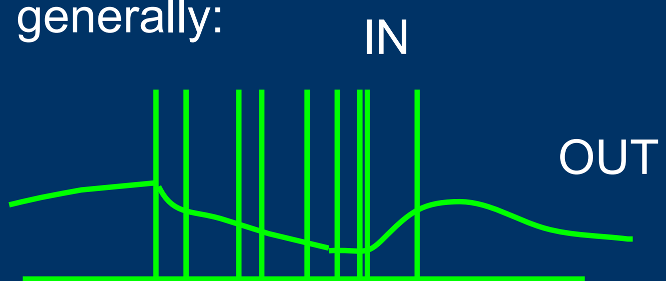
$$(\phi_0 : \text{non-interacting}, a_{\text{eff}} = 0)$$

Many localized states!

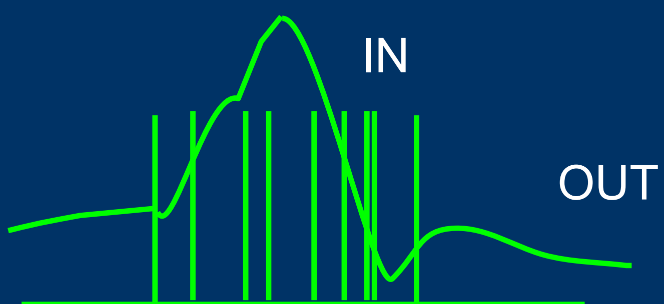
$$\text{for } k\tilde{d} < 1 \text{ and } a_{eff} \sim \tilde{d}$$

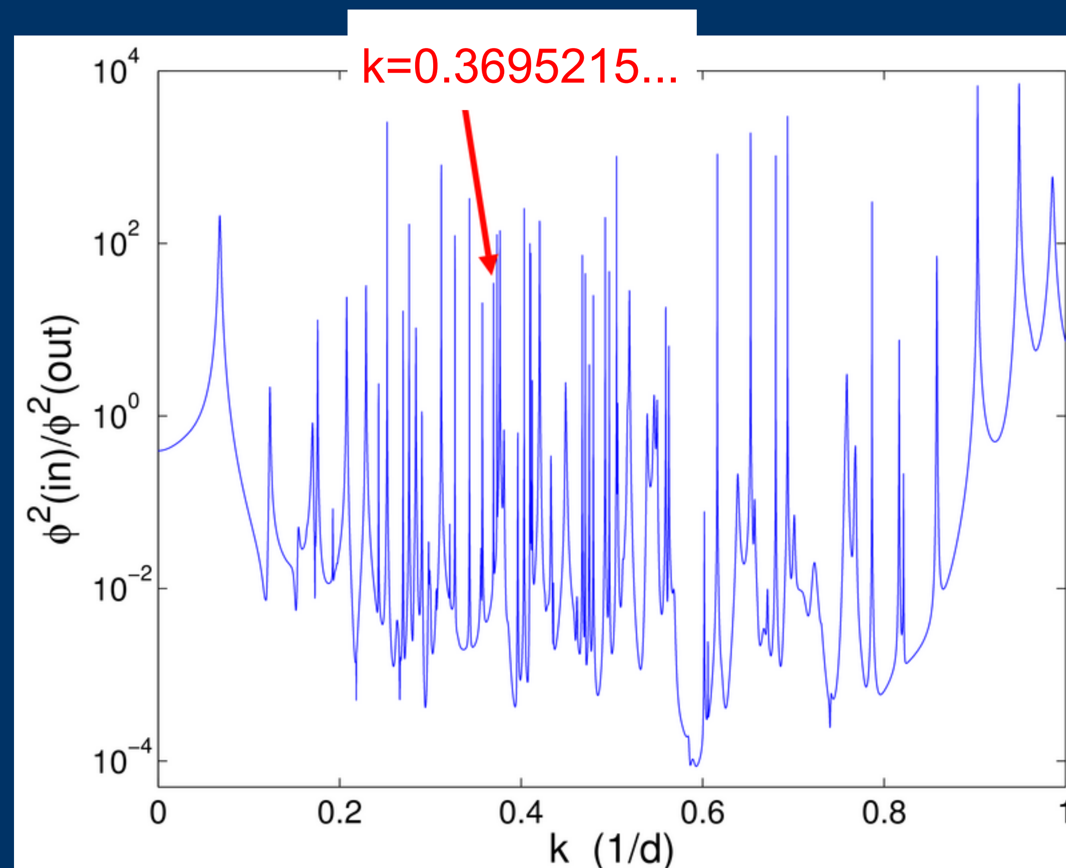
$$\tilde{d} = d\, p^{-1/3}$$

generally:



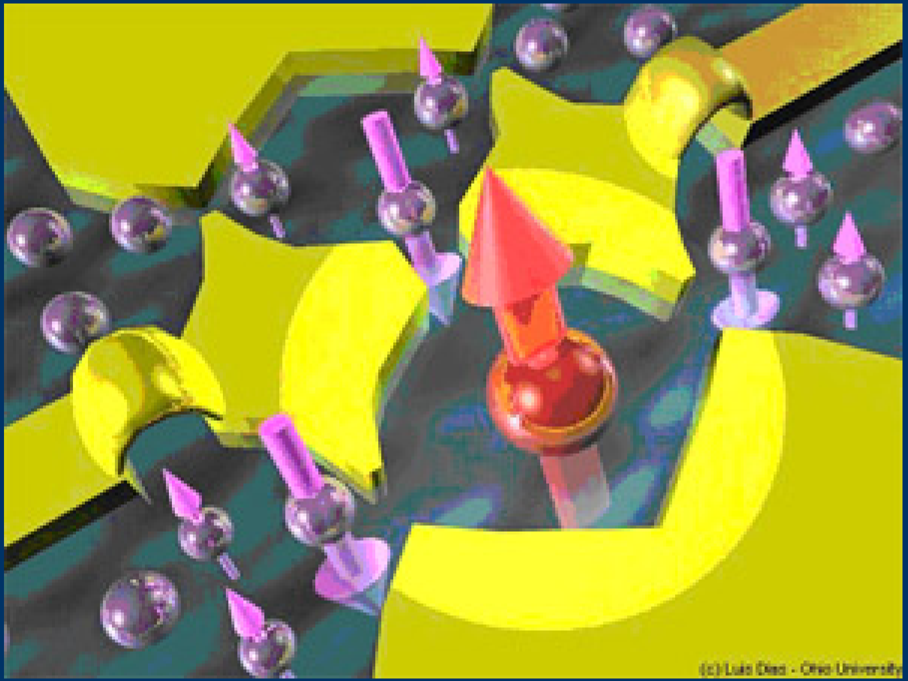
localized state:





9000 sites available, p=0.1, a_{eff}=mean particle distance

Kondo physics



- Spin ½ massive impurities
- Spin ½ gas
- Spin-exchange interactions lead to screening of the spin's impurity by the surrounding cloud
- Alkali-earth atoms ideal for studying single and many Kondo impurities problem
  (e.g., regular / disordered Kondo lattice,
  long range RKKY interactions, ...)

Conclusions and perspectives

- CIRs → unitarity limited interactions by tuning a_{ho}
- Many long-lived localized states at low energy
- Dynamics, dimensionality, interactions in the matter wave...
- Kondo physics!

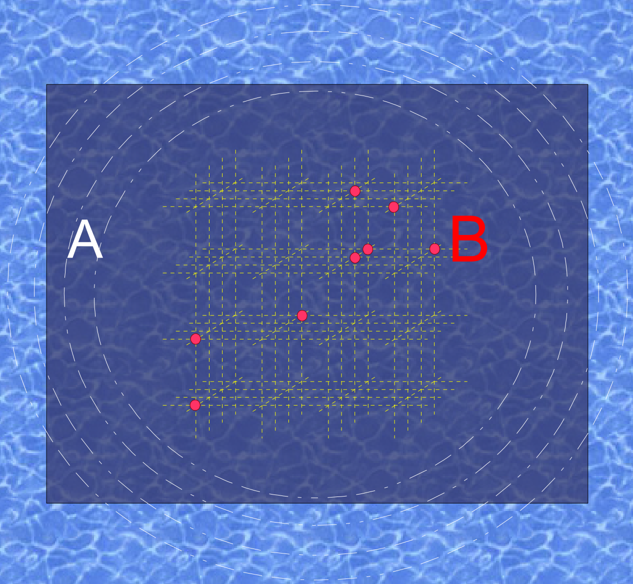
Conclusions and perspectives

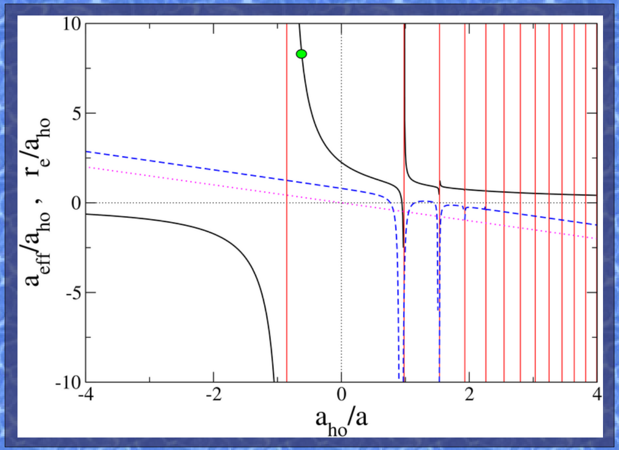
Many long-lived localized states for $k\,\tilde{d} < 1$ and $a_{eff} \sim \tilde{d}$

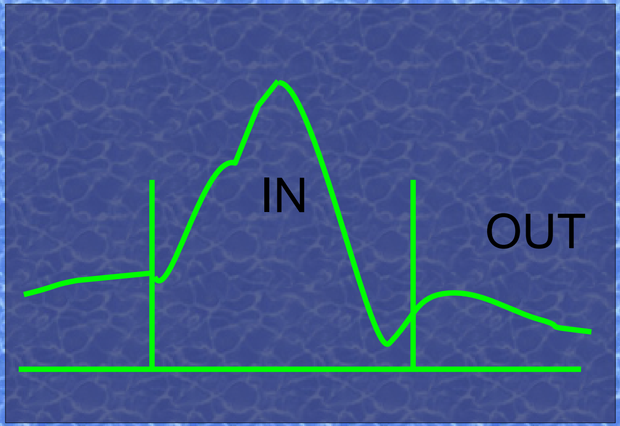
$$\tilde{d} = \frac{d}{p^{1/3}}$$

- Dynamics?
- Thermal bath?
- Dimensionality?
- Interactions in the matter wave?
- New experiment!

  Localization induced by impurities (S. Ospelkaus *et al.*, PRL 2006)







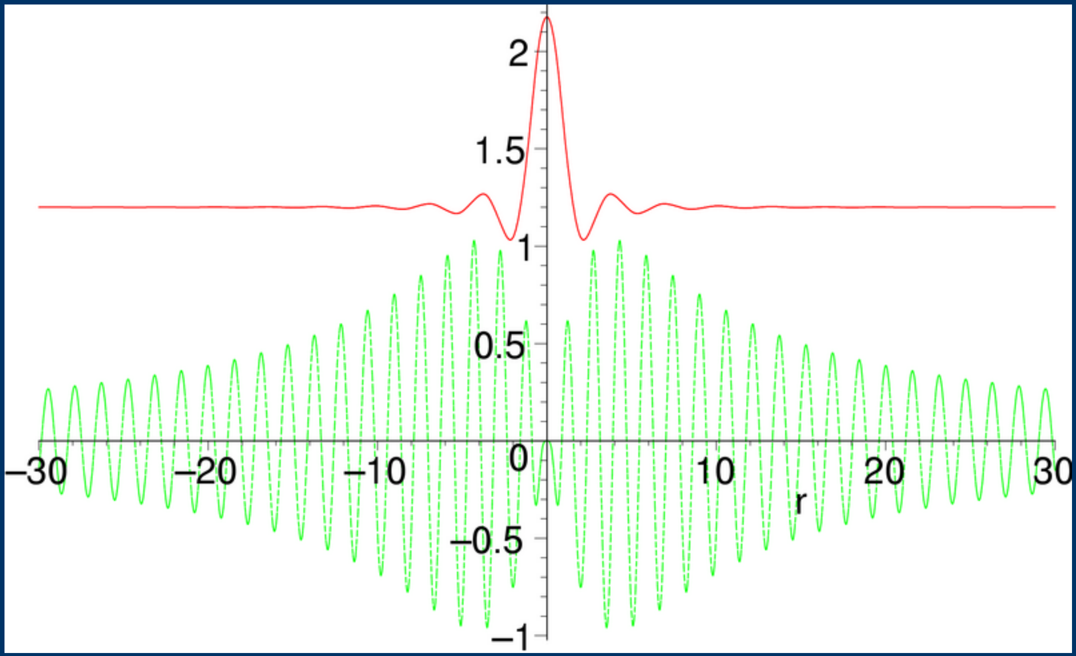
P. Massignan and Y. Castin, Phys. Rev. A (2006)

A localized state?

Eigenstate with:

★energy higher than the maximum of the potential

★square integrable wave function



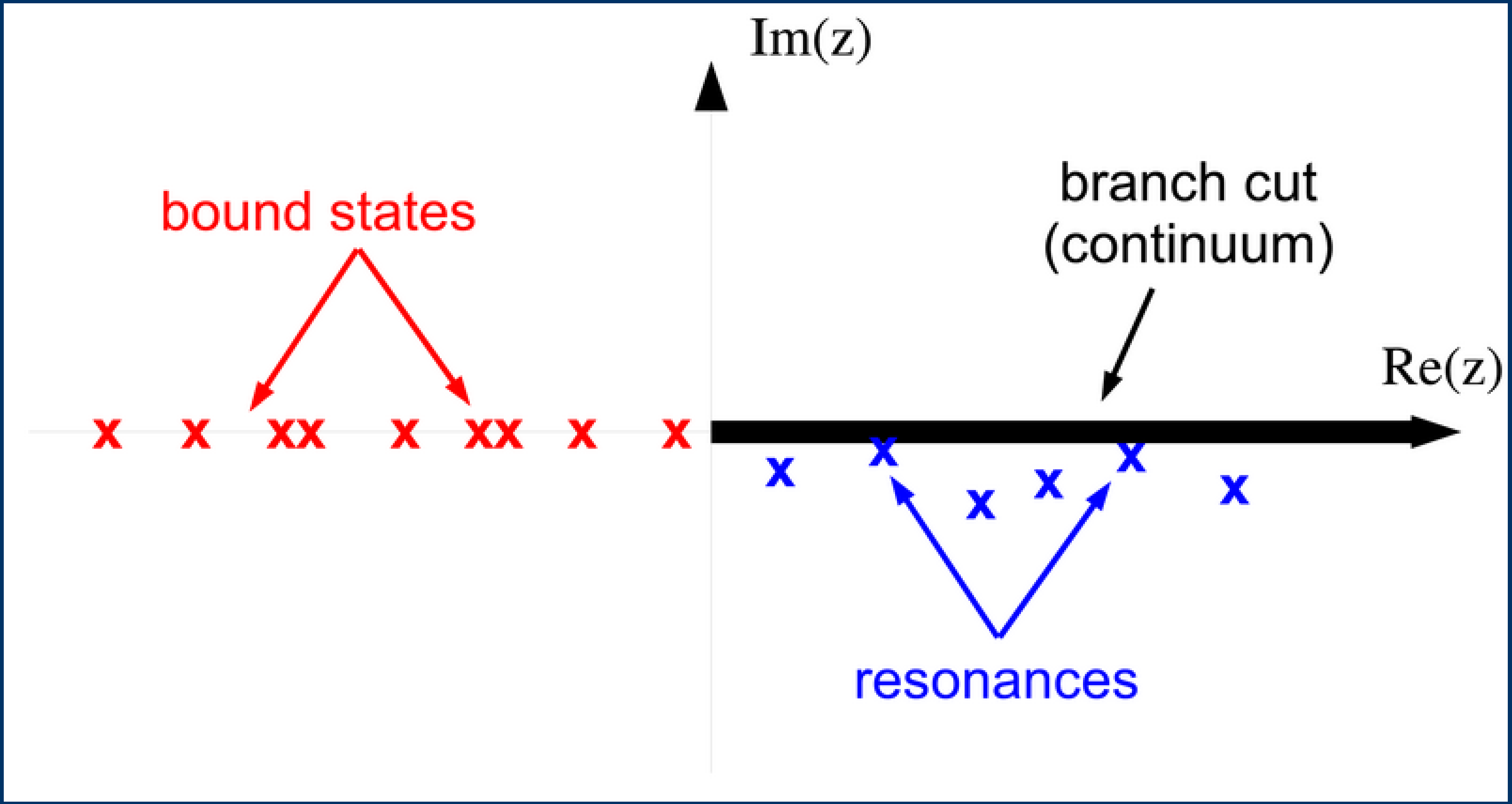
$$E = 2V_{max}$$

Why? quantum interference!

[von Neumann and Wigner, Phys. Z. **30**, 465 (1929)]

Poles of the Green's function

$$(z-H)G=1$$



Ioffe-Regel criterion

$$\lambda > \frac{1}{n\sigma}$$

With a unitarity-limited cross-section, $\sigma = \frac{4\pi}{k^2}$

one obtains $kd < (4\pi p)^{1/3}$

For p=0.1, this yields $k < \frac{1}{d} = \frac{k_L}{\pi}$

$$(T \sim 100\text{nK})$$

N static δ-scatterers

$$(E+i0^+-H)G=1$$

$$\Psi(\vec{r};\vec{r}_0)=\langle\vec{r}|G|\vec{r}_0\rangle=g_0(\vec{r}-\vec{r}_0)+\frac{2\pi\hbar^2}{m_A}\sum_{j,l}g_0(\vec{r}-\vec{r}_j)[M^{-1}]_{jl}\,g_0(\vec{r}_l-\vec{r}_0)$$

$$M=\frac{I}{a_{eff}}+M^{\infty}$$

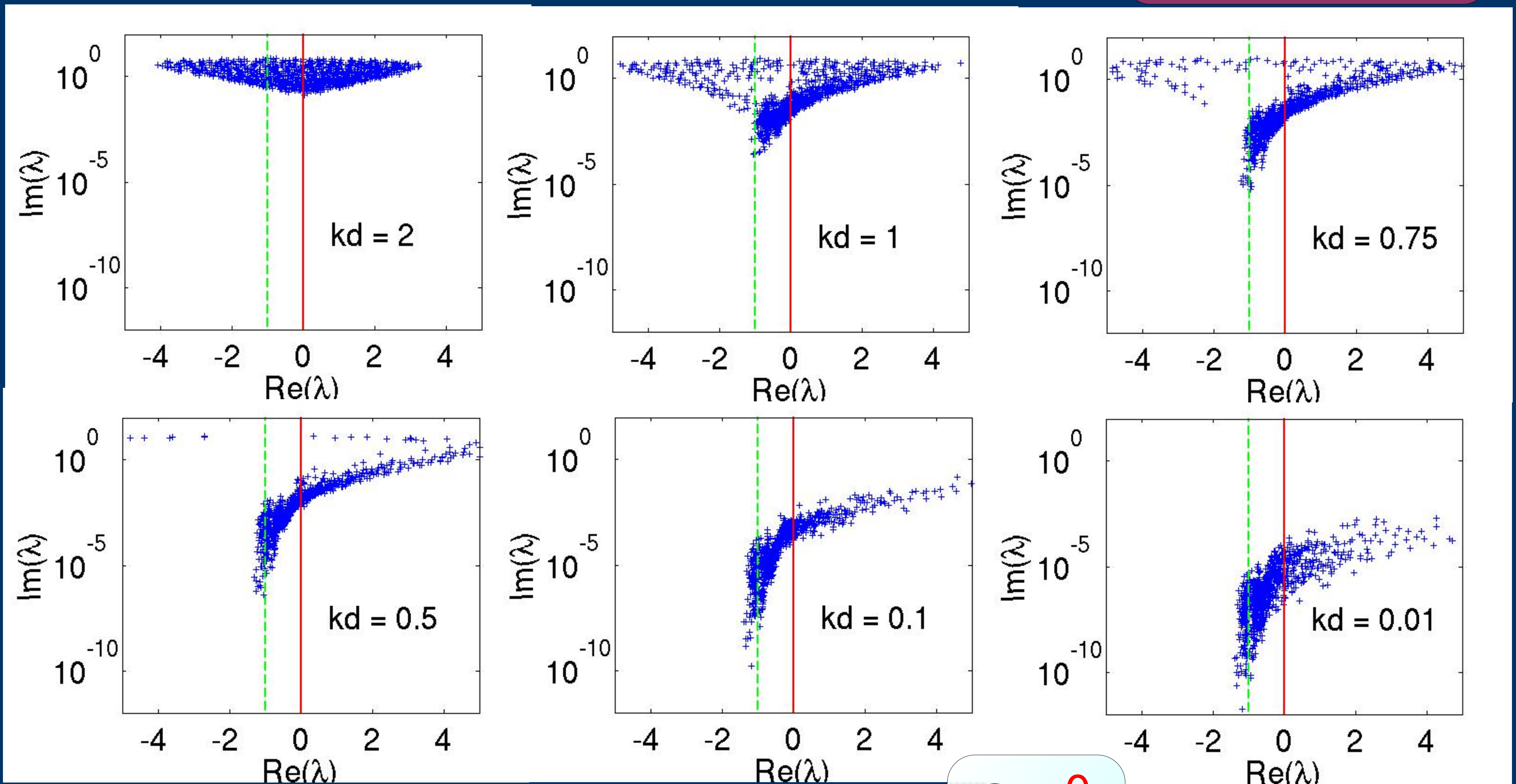
$$M_{jl}^{\infty}=\frac{e^{ik|\vec{r}_j-\vec{r}_l|}}{|\vec{r}_j-\vec{r}_l|}$$

$(E+i0^+-H)\Psi(\vec{r};\vec{r}_0)=\delta(\vec{r}-\vec{r}_0)$, then :

$\phi(\vec{r})=Im\,\Psi(\vec{r};\vec{r}_0)$ *is an eigenstate of* H *for any* $\vec{r}_0$

Eigenvalues of M

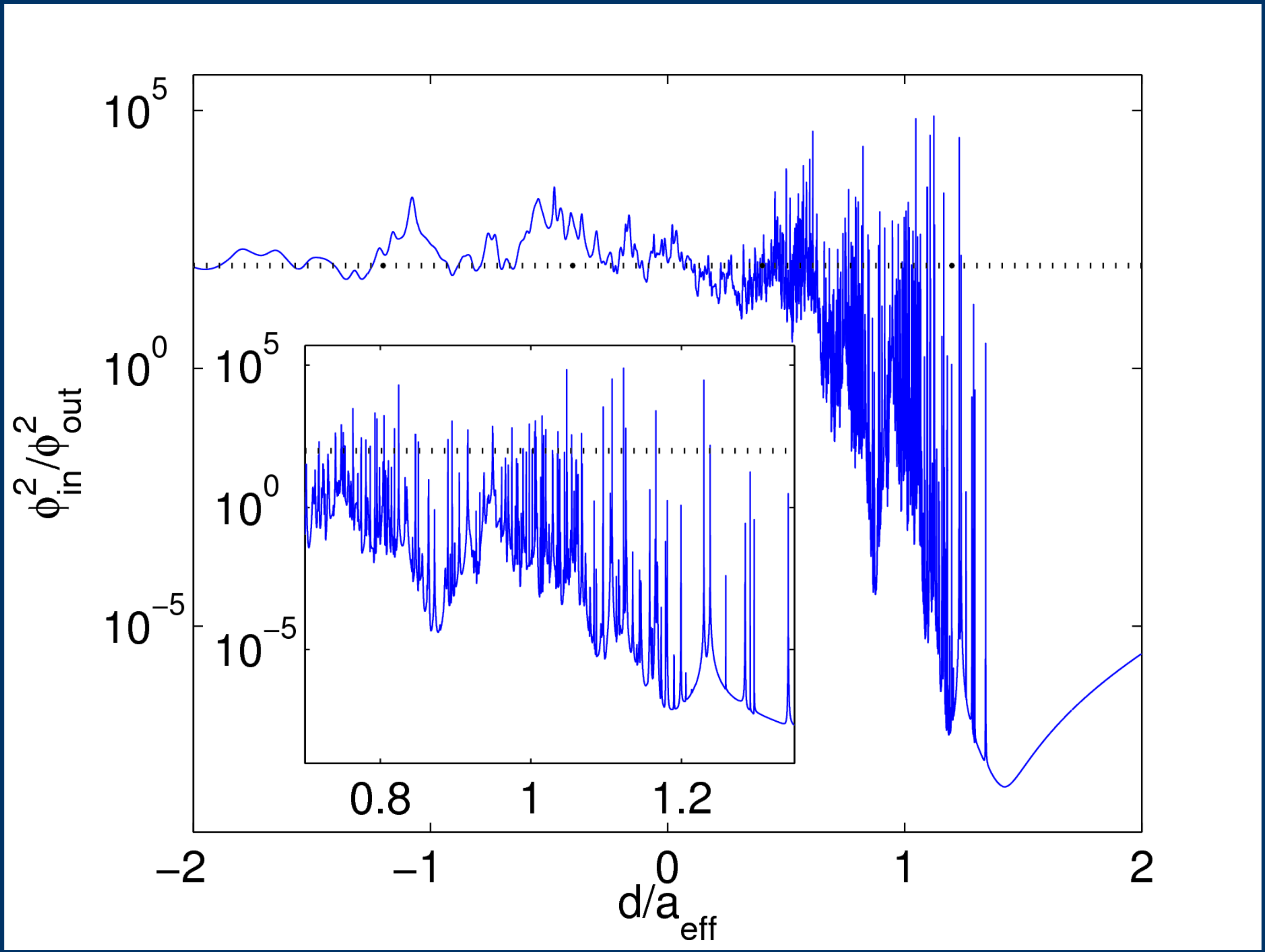
$$M = \frac{1}{a_{eff}} + M^{\infty}$$



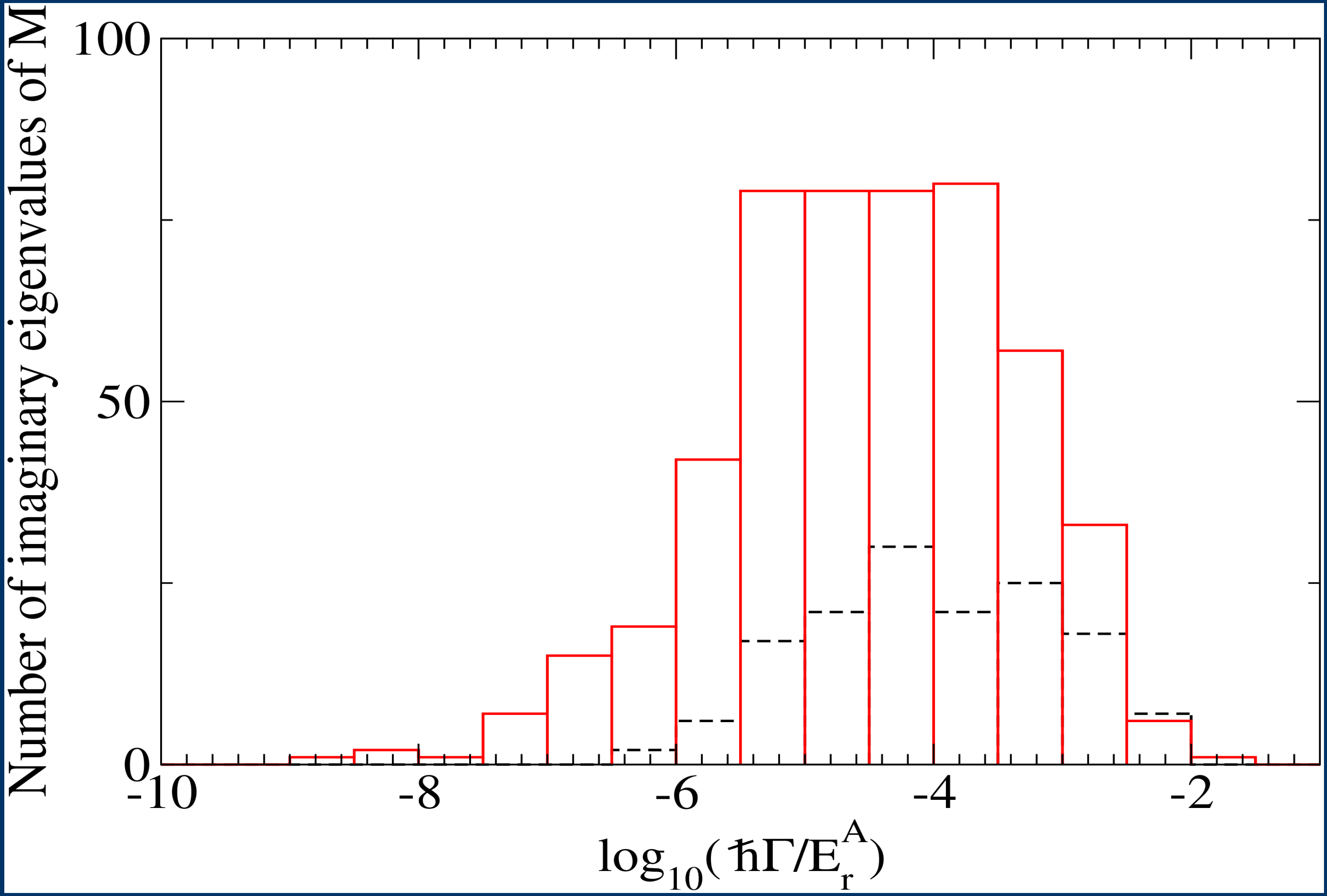
$a_{eff}=0$

$a_{eff}=d$

$d = \frac{\lambda}{2}$



10^{5}
10^{0}
10^{-5}
ϕ_{in}^2/ϕ_{out}^2
−2
−1
0
1
2
d/a_{eff}
0.8
1
1.2



Number of imaginary eigenvalues of M
100
50
0
-10
-8
-6
-4
-2
$\log_{10}(\hbar\Gamma/E_r^A)$