Efimov states close to a Feshbach resonance with large background scattering length

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October 30, 2009

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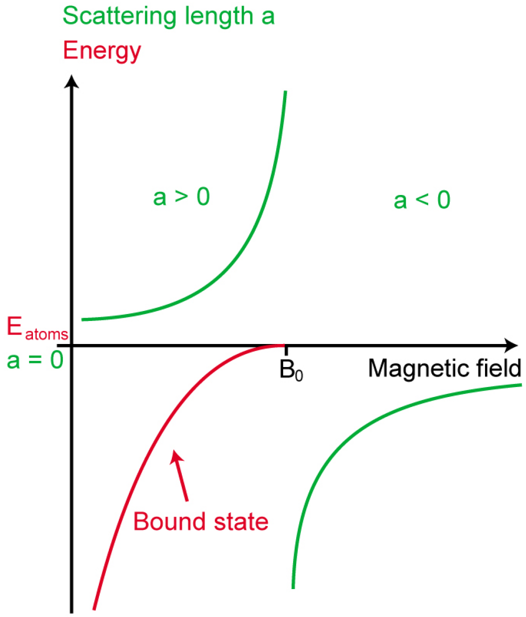
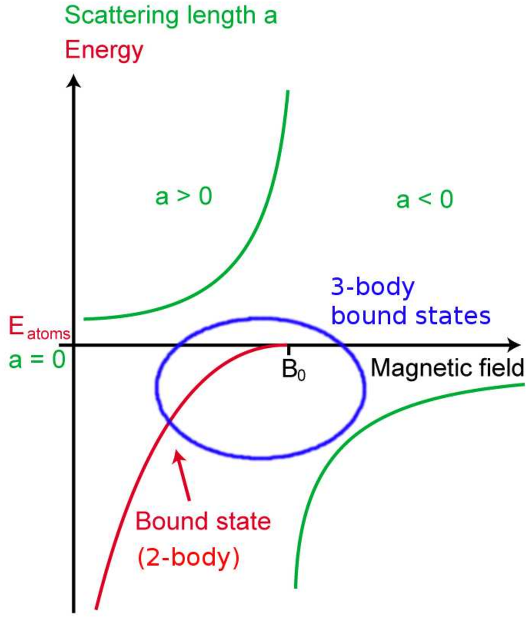
(work done in collaboration with Henk Stoof, ITF Utrecht)

Conference on "Efimov states in molecules and nuclei", Rome

Feshbach resonances in ultracold gases

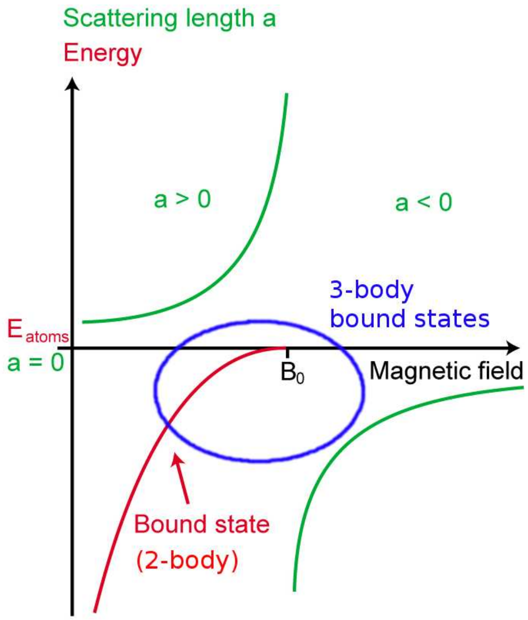


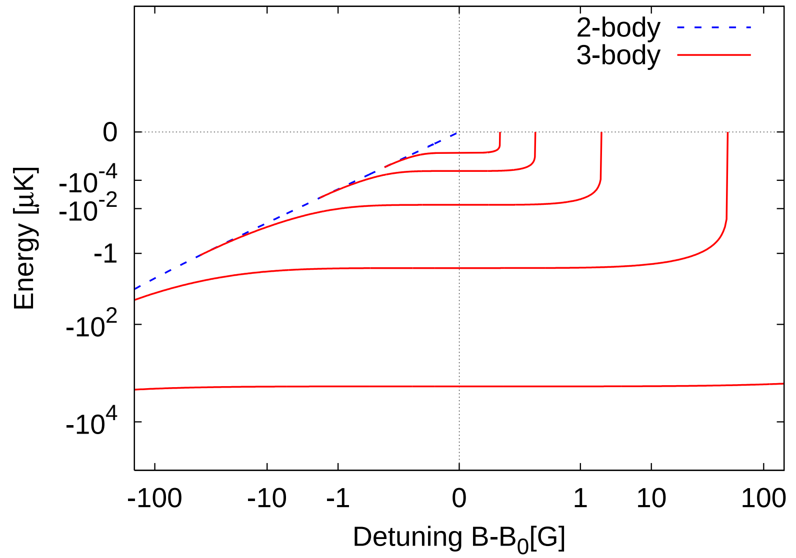
as $|a| \gg R$ there's more…(Efimov, 1970s)



as $|a| \gg R$ there's more...(Efimov, 1970s)

rescaled units: $E^{1/10}$ vs $(B - B_0)^{1/5}$





Discrete scaling: $\frac{a_{n+1}}{a_n} = 22.7, \qquad \frac{E_{n+1}}{E_n} = 22.7^{-2}$

Review: Braaten and Hammer (2006).

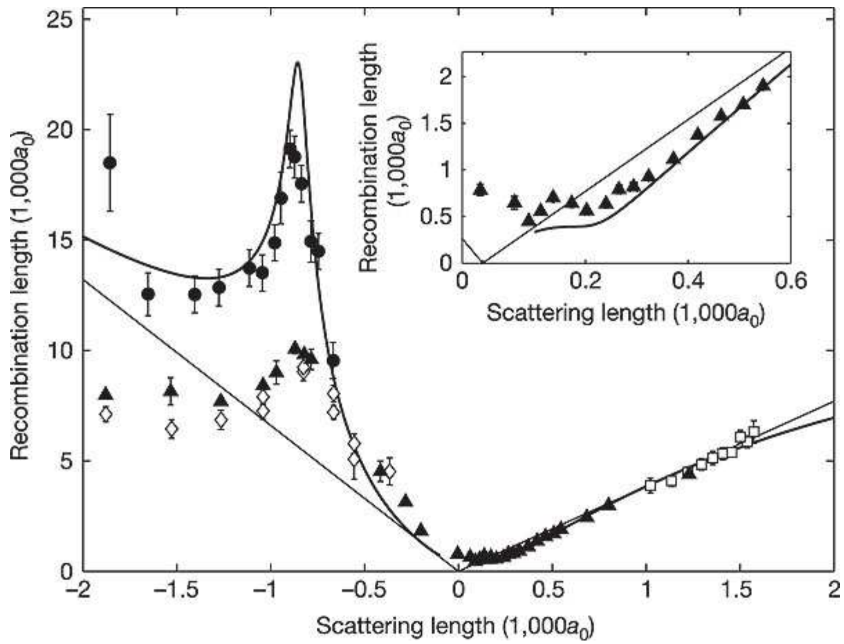
Universality: physics depends on a and an additional (3-body or low-energy) parameter.

Innsbruck experiment (Nature 2006)

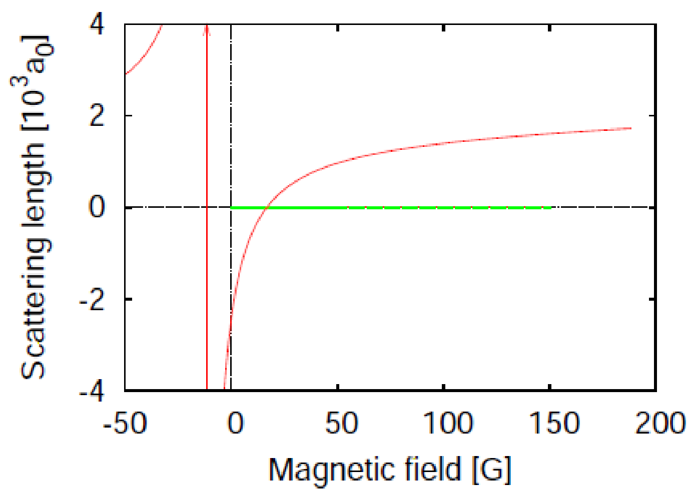
First experimental evidence for Efimov physics in cold gases.

Measurement of the recombination rate for A+A+A⇌A+D (A:atom, D:dimer) in a thermal gas of bosonic caesium.

$$\mathrm{d}n/\mathrm{d}t \propto -n^3 \rho_{\mathrm{rec}}^4$$

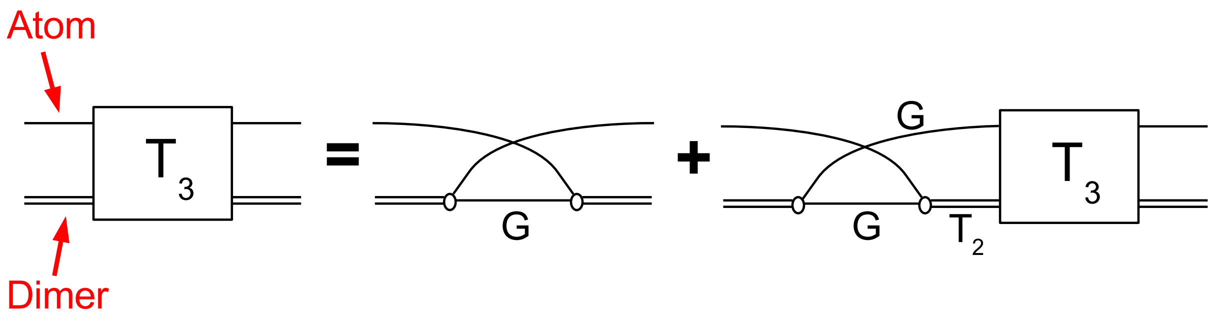


$$\rho_{\mathrm{rec}} \propto a \cdot C(a), \text{ with } C(a) \text{ log-periodic in } a$$



Universality? 1) $B_0 < 0$, 2)$|a| \sim R$, 3)$a_{\mathrm{bg}} \sim 1800 a_0 \gg R$

T_3: T-matrix for atom-dimer scattering



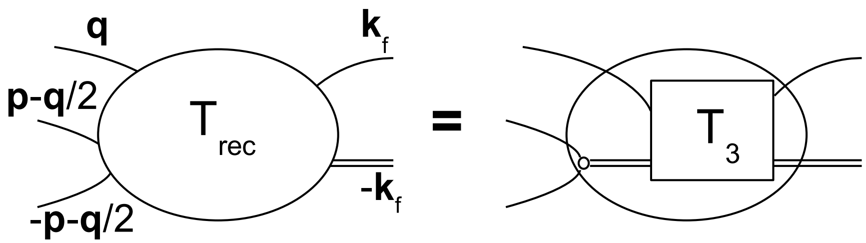
2-body s-wave interaction $\longrightarrow$ **STM** eq. (Skorniakov&Ter-Martirosian, 1956):

$$T_3(k, k'; E) = G(\ldots) + \int_0^\infty \mathrm{d}\mathbf{k}'' G(\ldots) G(\ldots) T_2(\ldots) T_3(\ldots).$$

G: atom propagator, T_2 : atom-atom scattering matrix (or dimer propagator), more to come. . .

Scattering length: $a_{\mathrm{AD}} \propto T_3(0, 0; E_b)$.

3-body recombination rate: AAA⇌AD

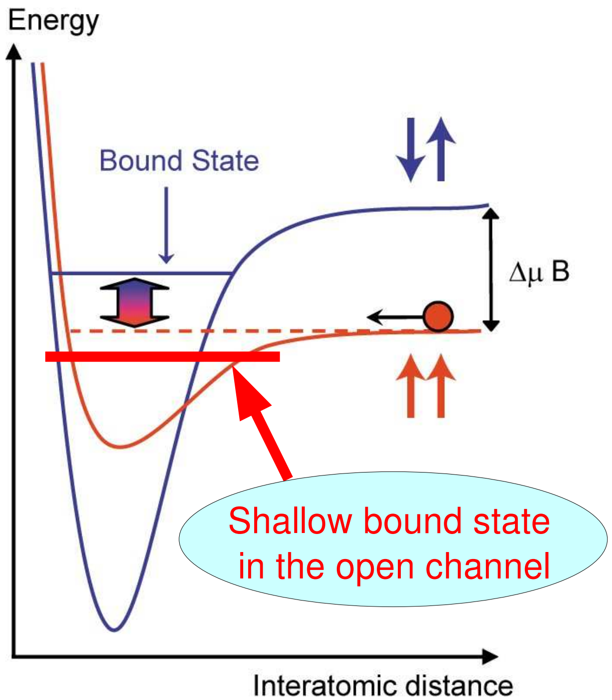


$$T_{\rm rec}(p,q;E_b) \propto T_2(\ldots)\;T_3(\ldots)$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\frac{n^3}{3!}\int_\Omega \frac{\mathrm{d}\Omega}{6\pi^2} k_f \left|T_{\rm rec}\right|^2 = -\left(\frac{\sqrt{3}\hbar}{2m}\right) n^3 \rho_{\rm rec}^4$$

($n^3/3!$ is the density of triples in the gas)

Dimer propagator for Feshbach resonances with large a_{bg}



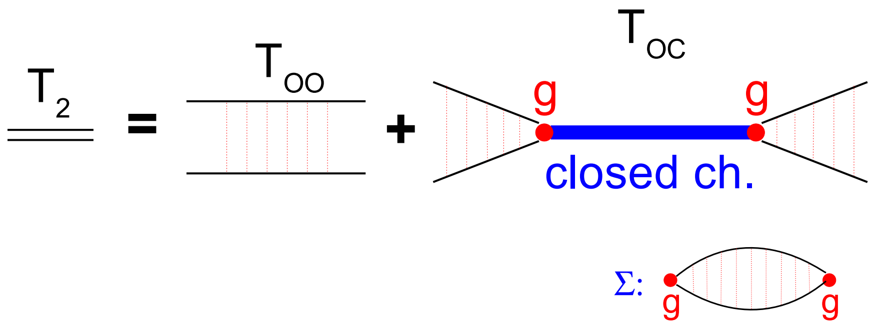
Scattering length: $a = a_{\mathrm{bg}} \left(1 - \frac{\Delta B}{B - B_0}\right)$

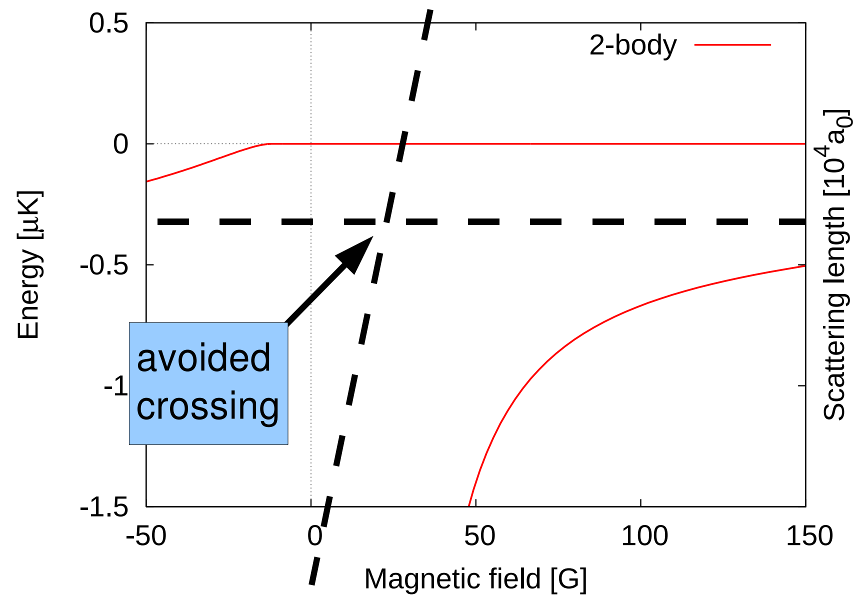
avoided crossing between the two bound states

$$T_2 = T_{\mathrm{oo}} + T_{\mathrm{oc}}$$

Bruun, Jackson, and Kolomeitsev (PRA 2005), Duine and Stoof (Phys. Rep. 2004).

Pole structure of T_2





$$T_2(E) = \frac{8\pi\hbar^2}{m} \frac{1}{\left[a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0 - E/\Delta\mu}\right)\right]^{-1} + ik}$$

$$k = \frac{\sqrt{mE}}{\hbar}$$

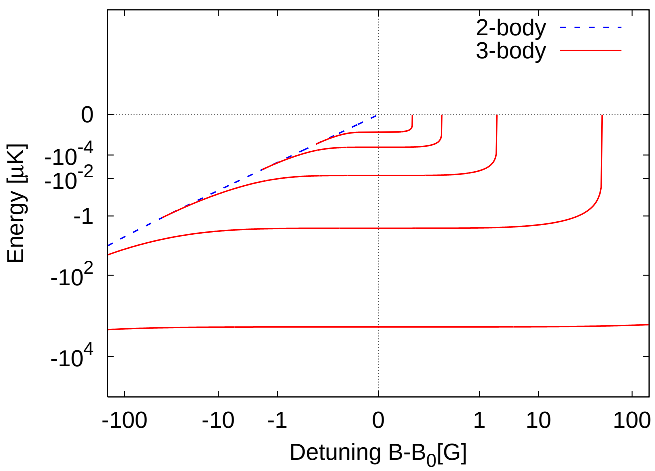
depending on the sign of a_{bg}, T_2 has 1 or 2 bound states: correct low-energy spectrum

Cut-off dependence

The STM eq. with simple $T_2 \sim \frac{1}{a^{-1}+ik}$

has a well-known UV cut-off dependence:

$$T_3(\ldots) = \ldots + \int_0^\Lambda \mathrm{d}q \ldots T_2(\ldots) T_3(\ldots).$$



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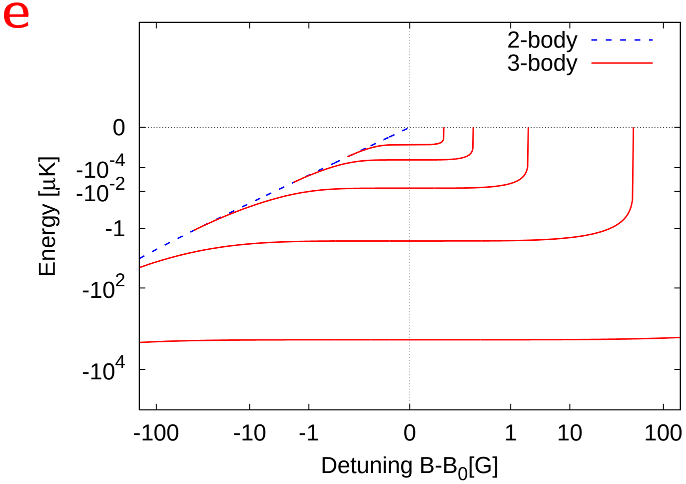

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Possible solutions

- finite momentum cut-off Λ or, equivalently, high energy correction to $T_2(E)$ fixed by an observable of the system
- for narrow resonances $(R^* \gg R)$, a natural cut-off is present:

$$f(E) \approx -\frac{1}{\frac{1}{a} + ik + R^* k^2} \qquad \text{with } R^* = \frac{\hbar^2}{m a_{\mathrm{bg}} \Delta B \Delta\mu}$$

Refs.: Danilov 1961, Fedorov and Jensen 2001, Petrov 2004

Same problem with our more refined $T_2(E)$

.
$$T_2(E) = \frac{8\pi\hbar^2}{m}\frac{1}{\left[a_{\rm bg}\left(1-\frac{\Delta B}{B-B_0-E/\Delta\mu}\right)\right]^{-1}+ik} \qquad k=\frac{\sqrt{mE}}{\hbar}$$

Ok, let's include a k^2 correction for T_2 (which introduces an effective cut-off at $k_{max} \sim 1/\tilde{R}$):

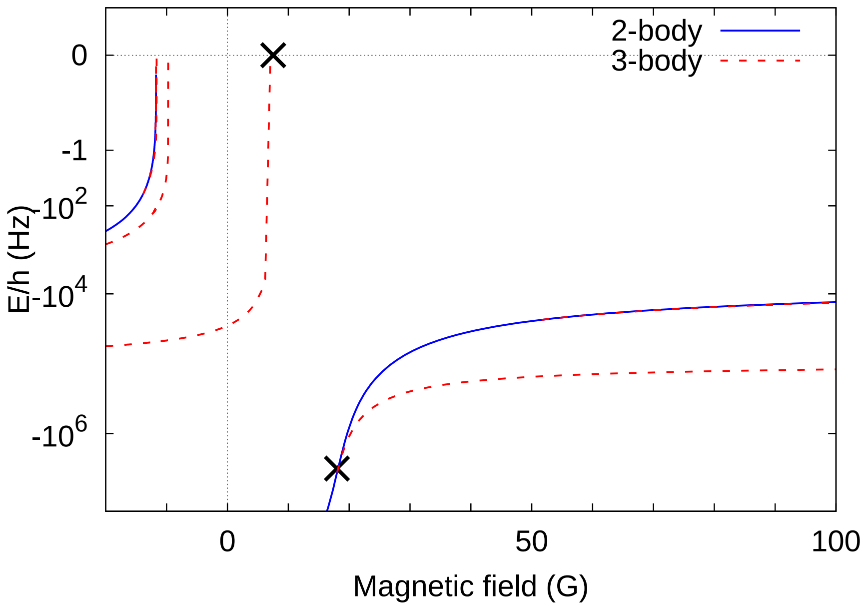
$$T_2(E) = \frac{8\pi\hbar^2}{m}\frac{1}{\left[a_{\rm bg}\left(1-\frac{\Delta B}{B-B_0-E/\Delta\mu}\right)\right]^{-1}+ik+\tilde{R}k^2}$$

$$\text{with} \begin{cases} \tilde{R} = \tilde{R}_{\rm fit} > 0 & \text{if } R^* \lesssim R \\ \tilde{R} = R^* \equiv \frac{\hbar^2}{m a_{\rm bg}\Delta B \Delta\mu} & \text{if } R^* \gtrsim R \end{cases}$$

(if $\tilde{R}<0$ the additional deep pole of T_2 has negative-norm)

^{133}Cs energy levels: comparison with Innsbruck experiments

$(R^*\!=\!0.15a_0, \qquad \tilde{R}_{\rm fit}\!=\!0.14a_0)$



rescaled y-axis: $E^{1/10}$ vs B

Theory (left): P. Massignan and H. Stoof, PRA **78**, 030701 (2008)

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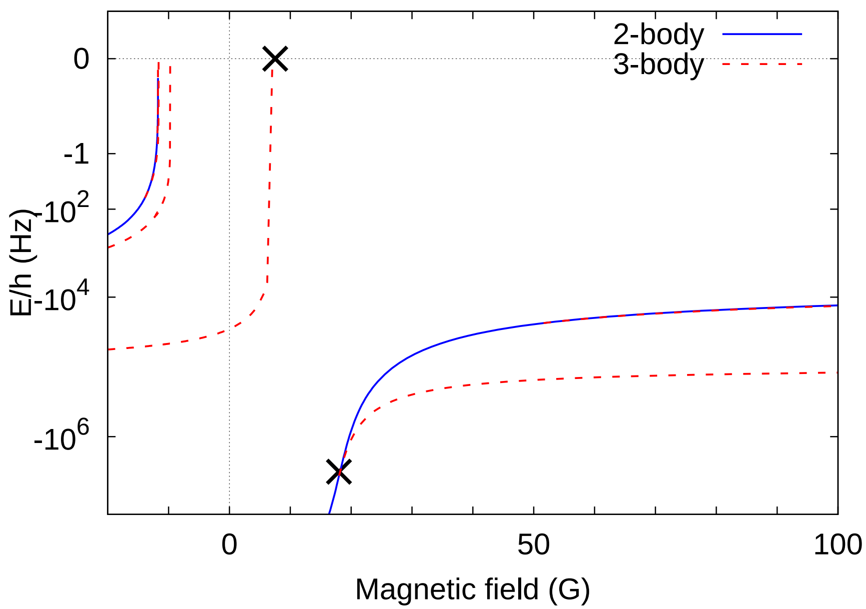
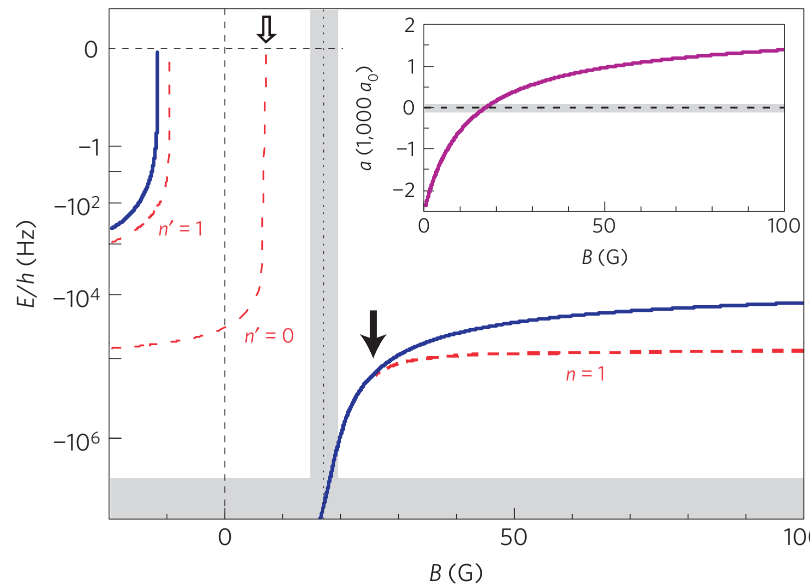


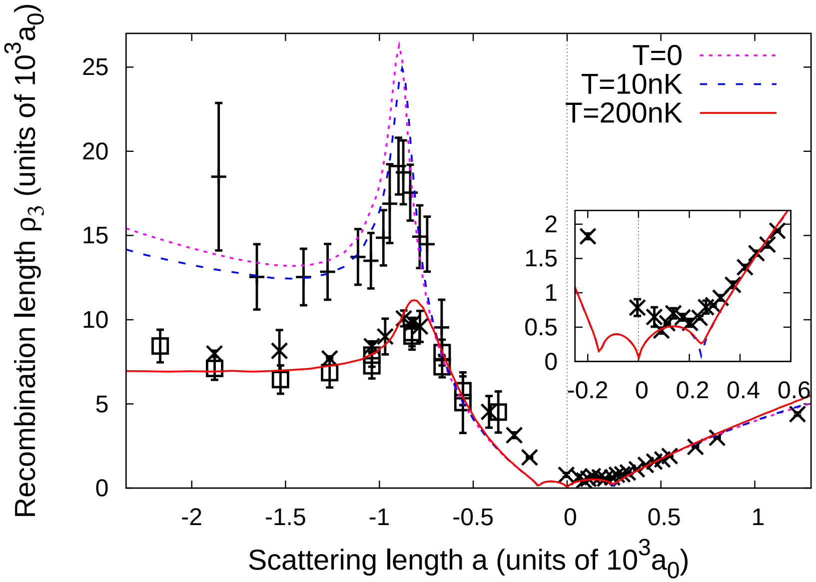


rescaled y-axis: $E^{1/10}$ vs B

Theory (left): P. Massignan and H. Stoof, PRA **78**, 030701 (2008)

Experiment (right): S. Knoop, F. Ferlaino, M. Mark et al., Nature Phys. **5**, 227 (2009)

3-body recombination: theory vs. experiment



✔ positions of min/max with a single free parameter (instead of the 4 needed in the original paper)

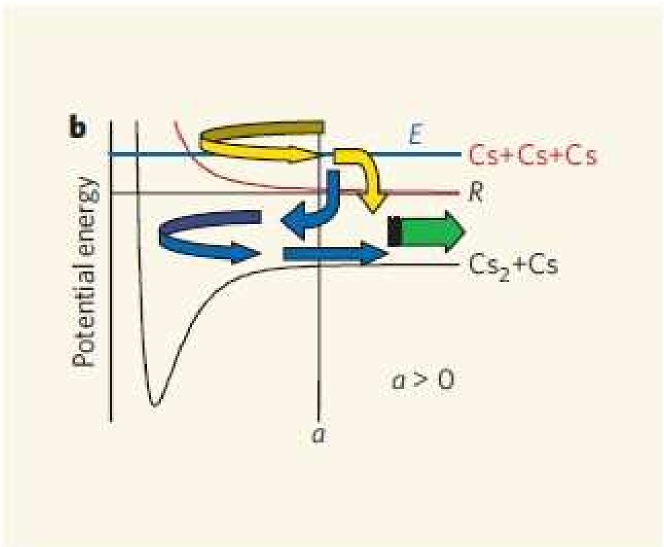
✔ good accord with the temperature dependence

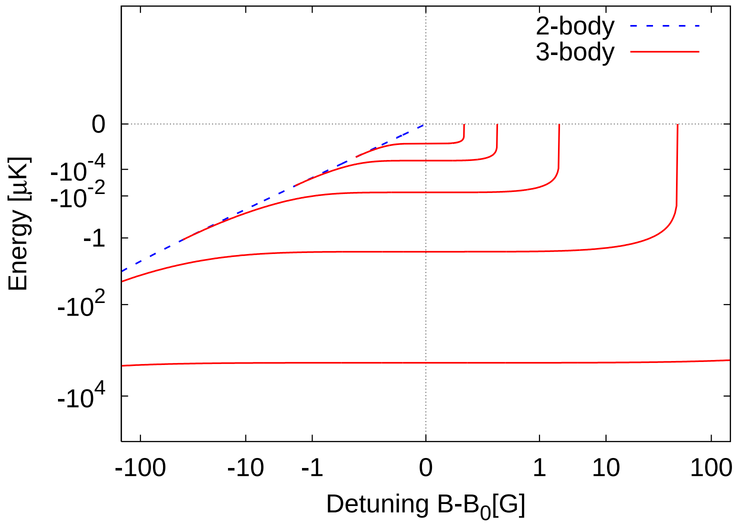
(see also: Jonsell; Yamashita, Frederico and Tomio; Braaten and Hammer; Lee, Köhler and Julienne)

- **Additional prediction:** an interference Stückelberg minimum at negative a $(a \simeq -130 a_0)$

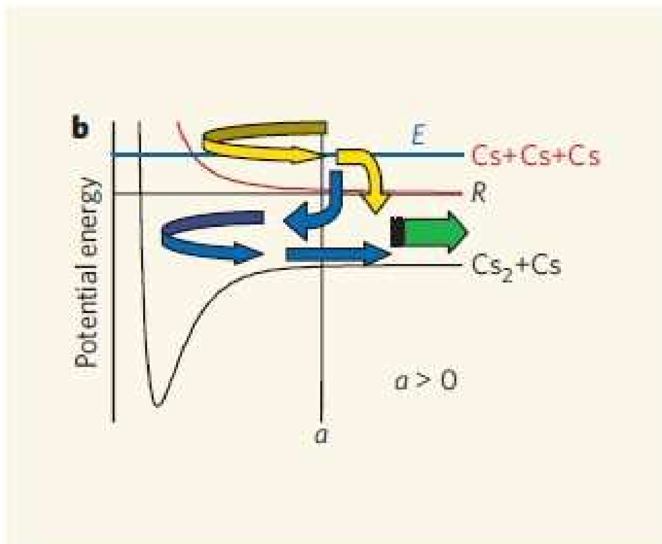
P. Massignan and H. Stoof, PRA **78**, 030701 (2008)

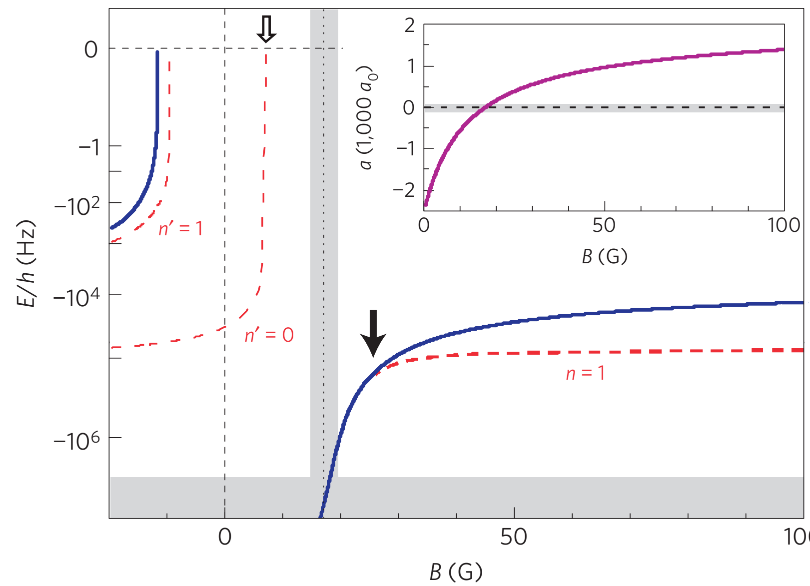
Stückelberg minima: destructive interference between two possible paths in the recombination process





Stückelberg minima: destructive interference between two possible paths in the recombination process



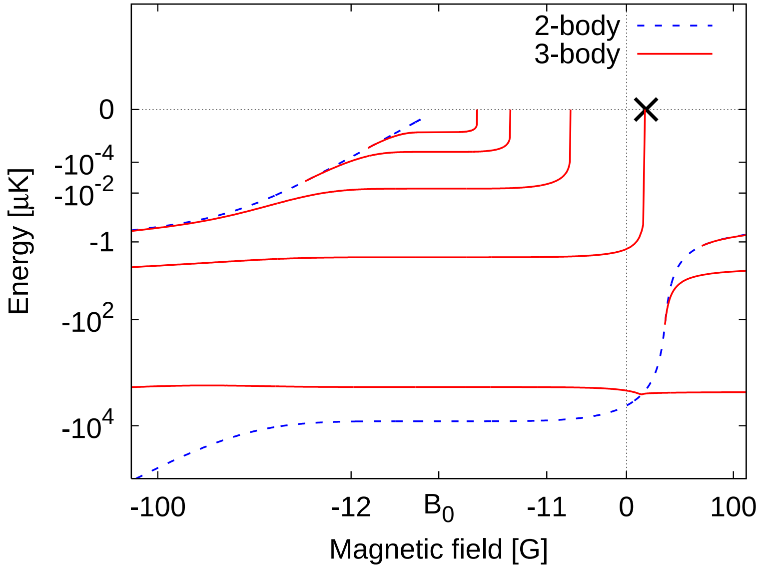


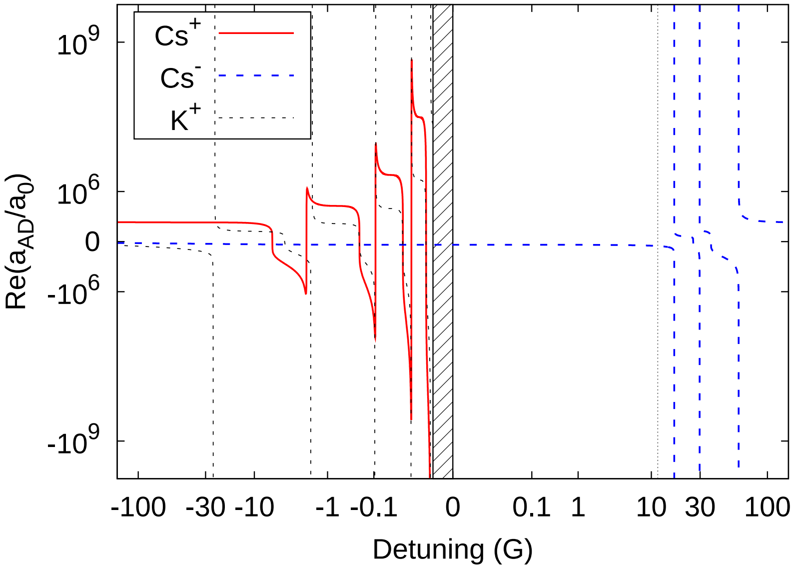
when $a_{\mathrm{bg}} \gg R$, even for negative a!

(in Cs $a_{bg} \sim 1800 a_0$)

Atom-dimer scattering lengths

$$\text{Scattering lengths: } \frac{3\pi\hbar^2 a_{\mathrm{AD}}^{\pm}}{m} = ZT_3(0,0;E_b^{\pm}).$$

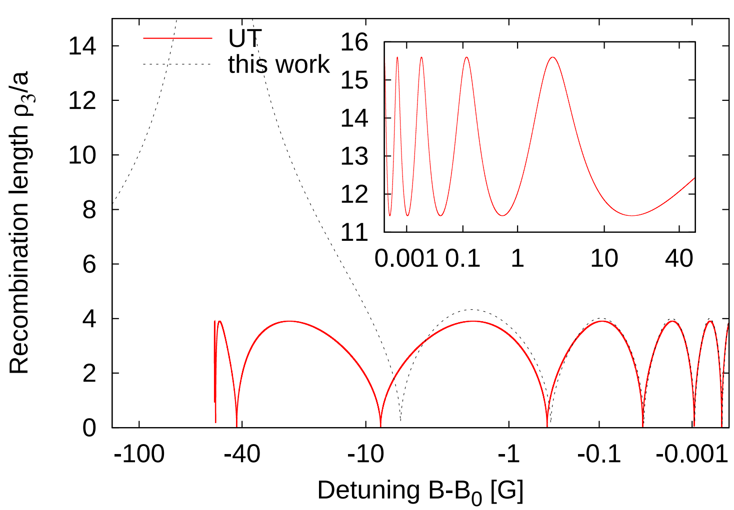


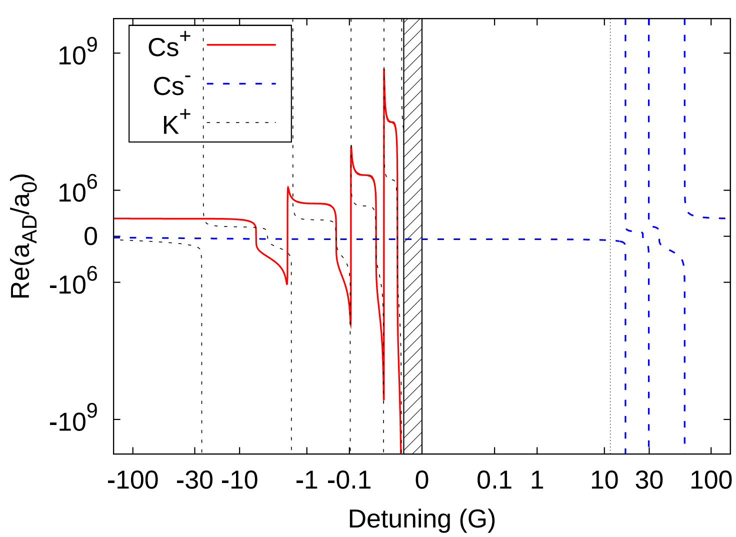


Results for ^{39}K: recombination length and a_{AD}

Predictions for ongoing experiments @ LENS with potassium 39

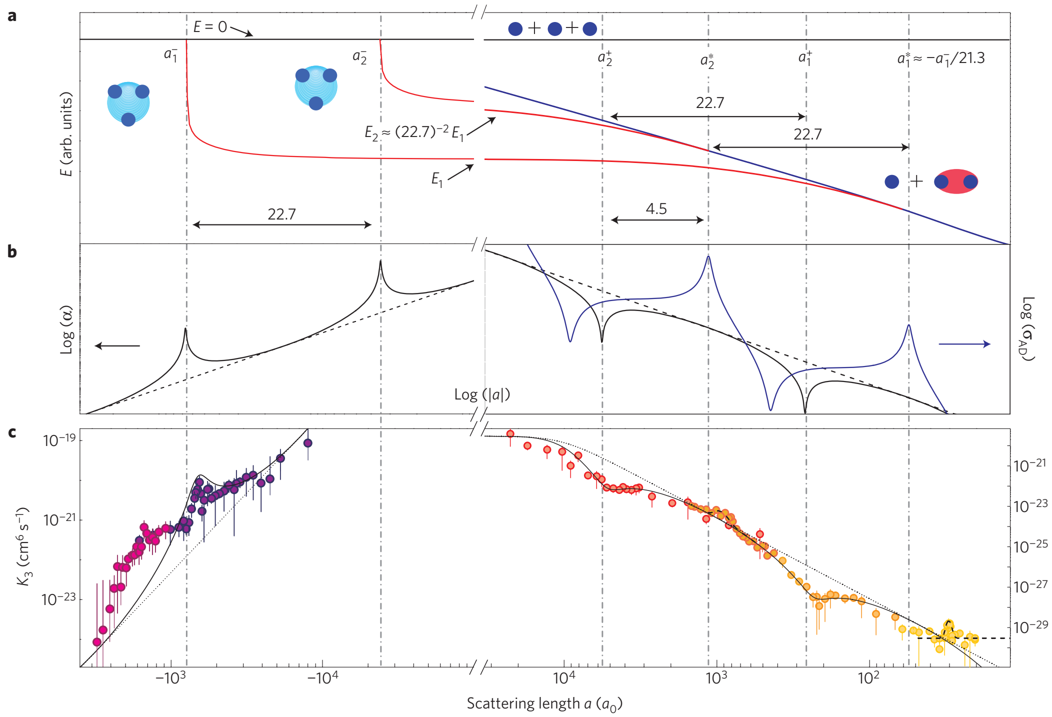
($B_0 = 403$G, $a_{\mathrm{bg}} = -29a_0$, $R^* = 29a_0 > R = 13a_0$) $\eta = 0.06$





P. Massignan and H. Stoof, PRA **78**, 030701 (2008)

^{39}K recombination length measured at LENS



$[a/a_0]$	Theory	Exp.
$\rho_{\rm rec}$ min.	212	224
	3850	5650
AD res.	30.7	30.4
	810	930
$\rho_{\rm rec}$ max.	-550	-1500
		-650

Experiment: M. Zaccanti, B. Deissler, C. D'Errico et al., Nature Phys. **5**, 586 (2009).

Note added*

The high-energy dependence discussed by Petrov is already in our T-matrix (without the k^2 correction),

as may be seen in the limit $|\frac{E}{\Delta\mu(B-B_0)}| \ll |\frac{B-B_0}{\Delta B}| \ll 1$, where:

$$\frac{1}{T_2(E)} \propto \frac{1}{a_{\rm bg}\left(1-\frac{\Delta B}{B-B_0-E/\Delta\mu}\right)} + ik \sim \frac{1}{a} + ik + R^* k^2$$

Consequence:

- for narrow resonances, our model (without $\tilde{R}$) should reduce to Petrov's
- it is not justified to assume $\tilde{R} = R^*$. The agreement between our a-priori predictions for ^{39}K resonances and the measured values seems therefore *accidental*.

*: thanks to Y. Castin, F. Werner, D. Petrov, E. Braaten, L. Pricoupenko

Can the problem be fixed?

Yes!

In the broad resonance limit, keep as a fit parameter either of the following:

- the coefficient of the quadratic correction in the momentum $\tilde{R}$
- a hard-core cut-off Λ in the STM integral eq. $T_3(\ldots) = \ldots + \int_0^\Lambda \mathrm{d}q \ldots T_2(\ldots)T_3(\ldots)$
- a "soft" high-energy cut-off (finite range) in $T_2(E)$ (see e.g. talk by M. Jona-Lasinio)

Is the model with $\tilde{R} = \tilde{R}_{fit}$ useful?

Yes!

In conclusion

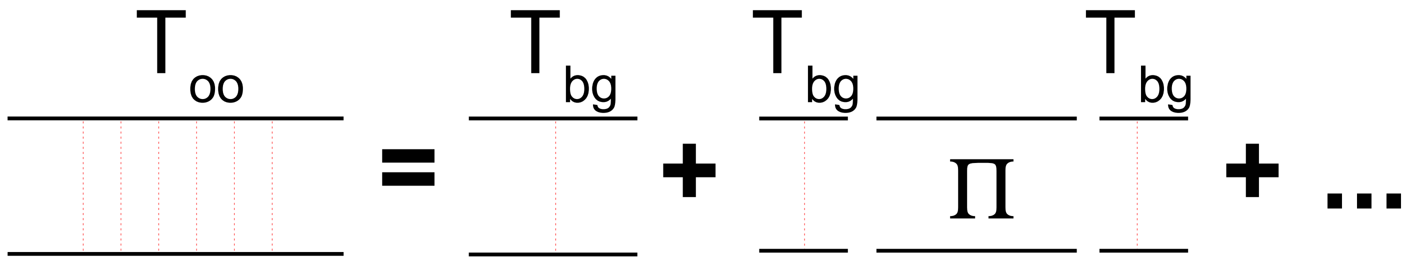
We have:

- introduced a model that carefully describes Feshbach resonance with large a_{bg} and reproduces the energy dependence of the associated 3-body bound states
- recovered the temperature dependence of the 3-body recombination for ^{133}Cs
- predicted the existence of interference Stückelberg minima at negative a
- calculated the atom-dimer scattering lengths for both ^{133}Cs and ^{39}K
- predicted atom-dimer crossings for ^{133}Cs at positive magnetic fields

Interesting developments:

- calculation of atom-dimer loss rates by inclusion of deeper bound states

Open-channel scattering



$$T_{\mathrm{OO}} = \frac{T_{\mathrm{bg}}}{1 - T_{\mathrm{bg}}\Pi(E)} \qquad \text{with } T_{\mathrm{bg}} = \frac{4\pi\hbar^2 a_{\mathrm{bg}}}{m}$$

$$\text{pair propagator: } \Pi(E) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\left(\frac{1}{E + i0^+ - k^2/m} + \frac{1}{k^2/m}\right) = \frac{m^{3/2}\sqrt{-E}}{4\pi\hbar^3}$$

T_{OC}

$$T_{OC}(E) = \left(\frac{g}{1 - T_{\rm bg}\Pi(E)} \right)^2 \frac{1}{E - \Delta\mu(B - B_0) - \hbar\Sigma(E)}$$

bare coupling between open and closed channels: g

$$g^2 = \frac{4\pi\hbar^2 a_{\rm bg} \Delta B \Delta\mu}{m}$$

self-energy of the closed ch. molecule: $\hbar\Sigma(E) = g \frac{\Pi(E)}{1 - T_{\rm bg}\Pi(E)} g$