# EFIMOV TRIMERS UNDER STRONG CONFINEMENT


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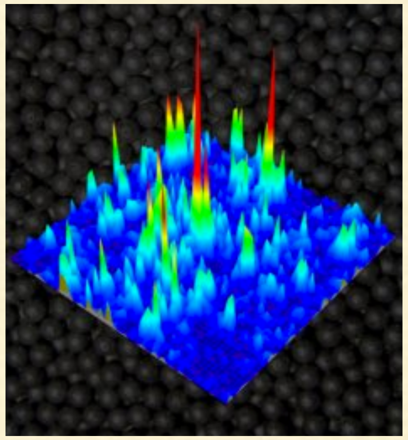
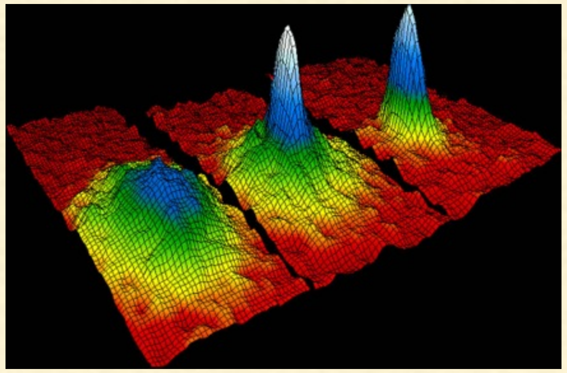
# STRONG CONFINEMENT EFFECTS

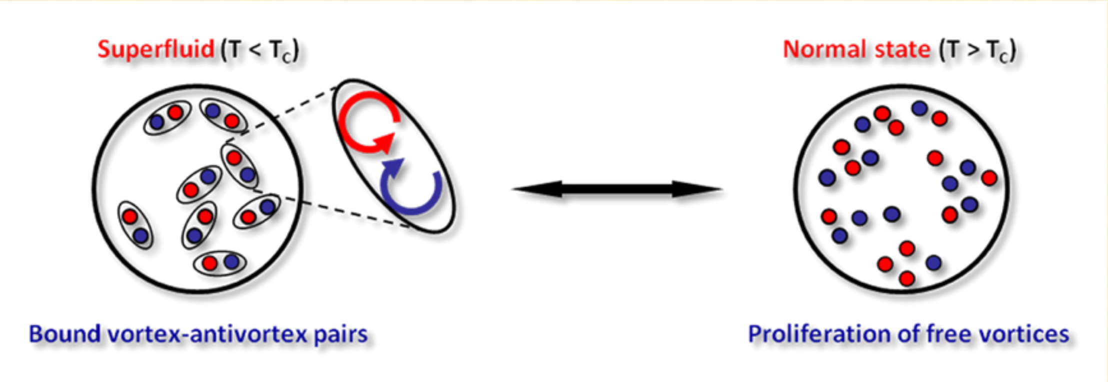
The dimensionality of the embedding space profoundly affects the system properties.

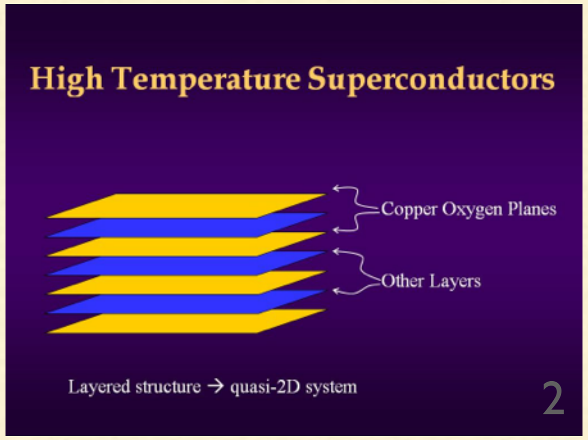
Examples:

- Anderson localization
- condensation & superfluidity

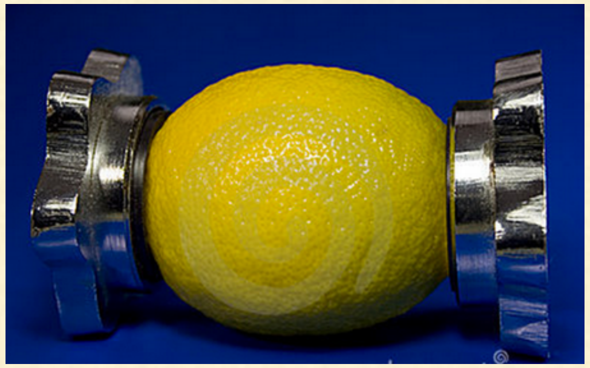






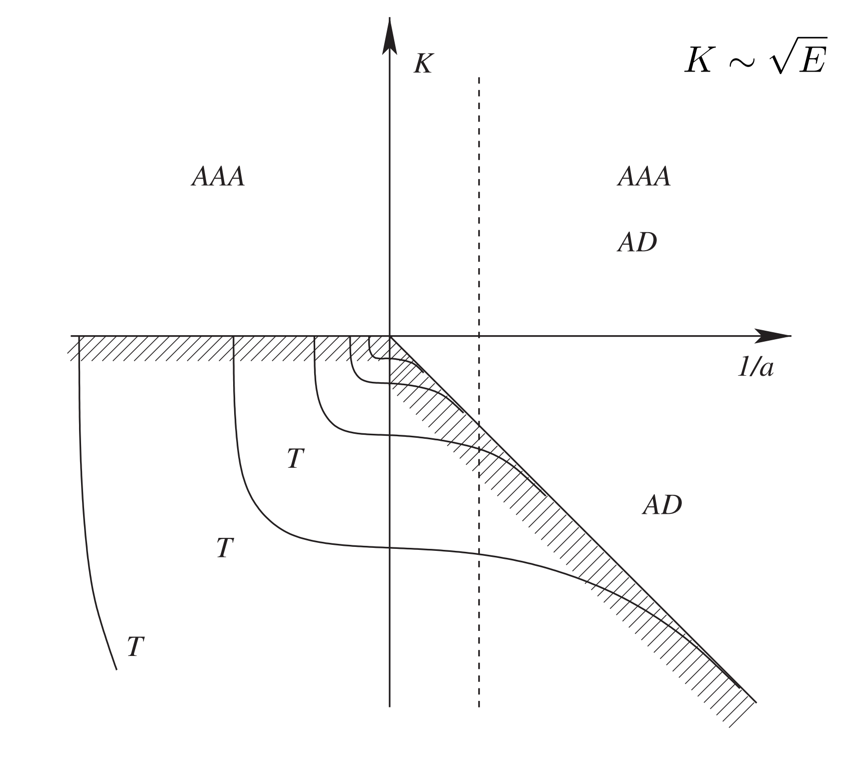
# OUTLINE



- Three identical bosons: 3D vs. 2D
- What happens in between? (quasi-2D)
  - trimer spectra and aspect-ratios
  - hyper-spherical potentials and wave functions
- Experimental consequences

# 2&3 IDENTICAL BOSONS IN **3D**

One universal dimer: $E_b = -\frac{\hbar^2}{ma^2} \qquad (a > 0)$

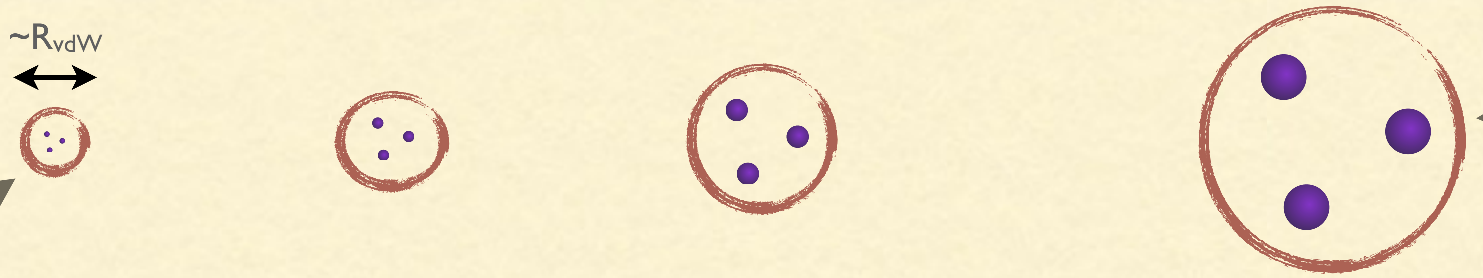



Braaten & Hammer, Phys. Rep. 2006

For resonant interactions (1/a=0), in principle ∃ an infinite tower of Efimov trimers.

Trimers map onto each other via the scale transformations $a \to \lambda_0^n a$ and $E \to \lambda_0^{-2n} E$

$\lambda_0 = 22.7$




Smallest set by short-distance physics

Largest set by the temperature, or the dimension of the container

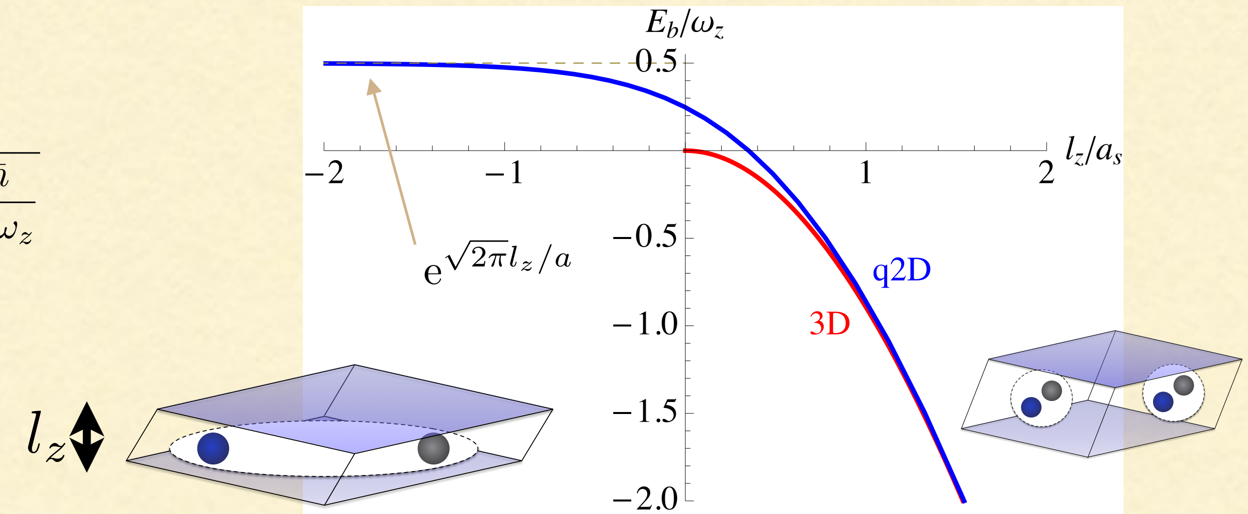
Scaling symmetry: continuous (two-body) vs. discrete (three-body)

# 2&3 IDENTICAL BOSONS IN **2D**

Apply harmonic confinement: $V(z) = \frac{1}{2} m\omega_z^2 z^2$

- CoM decouples
- continuum is shifted
- additional length scale appears

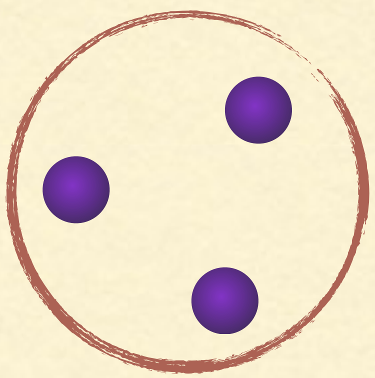
$l_z = \sqrt{\frac{\hbar}{m\omega_z}}$



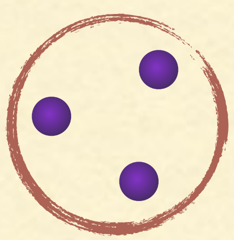

<u>One universal dimer</u>: $E_b = \frac{\hbar\omega}{2} - \frac{\hbar^2}{ma_{2D}^2}$

Petrov & Shlyapnikov PRA 2001
Bloch, Dalibard, Zwerger RMP 2008
Levinsen & Parish, arXiv 2014 (review)

<u>Two universal trimers:</u> $-1.27 \frac{\hbar^2}{ma_{\mathrm{2D}}^2}$



Bruch & Tjon, PRA 1979 $-16.5 \frac{\hbar^2}{ma_{\mathrm{2D}}^2}$



*<u>Both</u> two- and three-body problems display a <u>continuous scaling symmetry</u>*



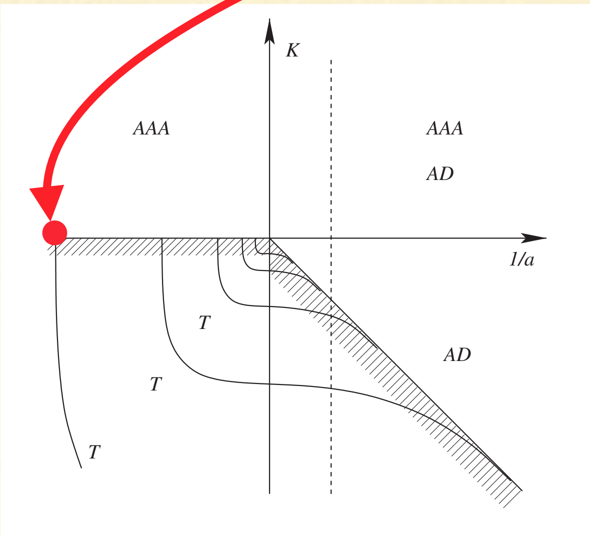
2D Bose gas experiments: Paris, Innsbruck, Chicago, Monash, …

# THREE BOSONS IN **QUASI-2D**

$$H = \sum_{\mathbf{k},n} (\epsilon_{\mathbf{k}} + n\hbar\omega_z) a^\dagger_{\mathbf{k},n} a_{\mathbf{k},n} + \sum_{\substack{\mathbf{k},\mathbf{k}',\mathbf{q} \\ n_1,n_2,n_3,n_4}} \mathrm{e}^{-(\mathbf{k}^2+\mathbf{k}'^2)/\Lambda^2} \langle n_1 n_2 | \hat{g} | n_3 n_4 \rangle a^\dagger_{\mathbf{q}/2+\mathbf{k},n_1} a^\dagger_{\mathbf{q}/2-\mathbf{k},n_2} a_{\mathbf{q}/2-\mathbf{k}',n_3} a_{\mathbf{q}/2+\mathbf{k}',n_4}$$

the UV cut-off Λ controls the three-body physics at short-distances,
and fixes the crossing of the deepest Efimov trimer with the 3-atom continuum ($a_-$)

Trimer wave function: $\sum_{\substack{\mathbf{k}_1,\mathbf{k}_2 \\ n_1,n_2,n_3}} \psi^{n_1,n_2,n_3}_{\mathbf{k}_1,\mathbf{k}_2} a^\dagger_{\mathbf{k}_1,n_1} a^\dagger_{\mathbf{k}_2,n_2} a^\dagger_{-\mathbf{k}_1-\mathbf{k}_2,n_3}$



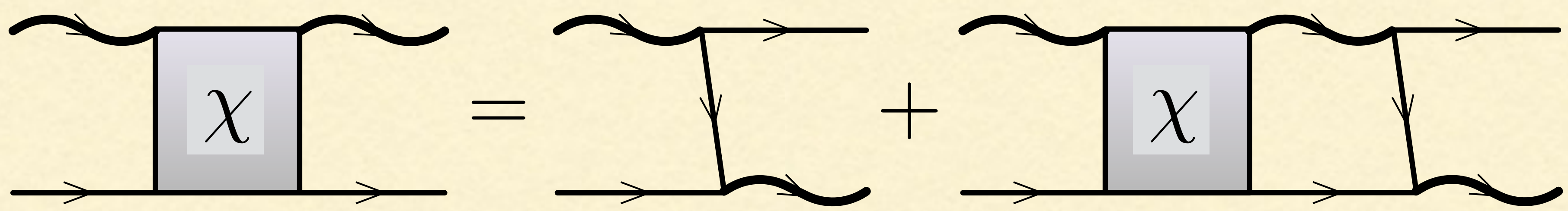

J. Levinsen, P. Massignan, and M. Parish, Phys. Rev X (2014)



see also the recent work on trimers in a box by M. Yamashita et al., arXiv:1404.7002

# SKORNIAKOV—TER-MARTIROSIAN EQ.

atom-dimer vertex:




relative z-motion wave function at $z_r$=0

Clebsch-Gordan coefficient

$$\mathcal{T}^{-1}\left(\mathbf{k}_1, E_3 - \epsilon_{\mathbf{k}_1} - N_1\omega_z\right)\chi^{N_1}_{\mathbf{k}_1} = 2\sum_{\mathbf{k}_2, N_2 n_{23} n_{31}} \frac{f_{n_{23}} f_{n_{31}} \langle N_1 n_{23} | N_2 n_{31}\rangle e^{-(k_1^2+k_2^2)/\Lambda^2}\chi^{N_2}_{\mathbf{k}_2}}{E_3 - \epsilon_{\mathbf{k}_1} - \epsilon_{\mathbf{k}_2} - \epsilon_{\mathbf{k}_1+\mathbf{k}_2} - (N_1 + n_{23})\omega_z}$$

(the CoM q.number N does not appear in the final formula!)

where:

- $\mathcal{T}(\mathbf{k}, E) = \frac{2\sqrt{2\pi}}{m}\left\{\frac{l_z}{a} - \mathcal{F}\left(\frac{-E + k^2/4m}{\omega_\perp}\right)\right\}^{-1}$
- $E_3$ is the energy measured from the 3-atom continuum
- $\mathbf{k}_i$ is the relative momentum of atom $i$ w.r.t. the pair $(j,k)$
- $n_{ij}$ and $N_i$ are the h.o. quantum numbers for motion along $z$ of a pair, and of an atom and a pair

Wave function for the atom-pair relative motion: $\psi(\boldsymbol{\rho}, Z) = R^{3/2}\sum_{\mathbf{k},N} e^{i\mathbf{k}\cdot\boldsymbol{\rho}}\phi_N(Z)\chi^N_{\mathbf{k}}$

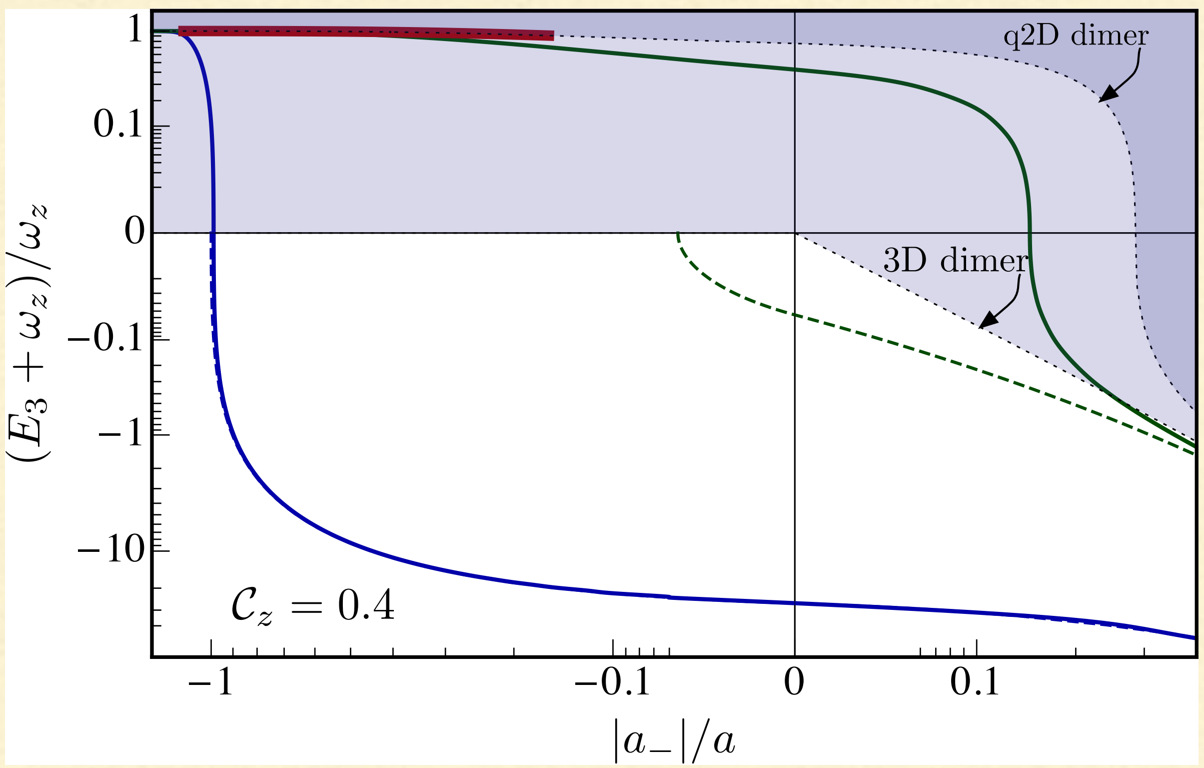
J. Levinsen, P. Massignan, and M. Parish, Phys. Rev X (2014)

# SPECTRUM

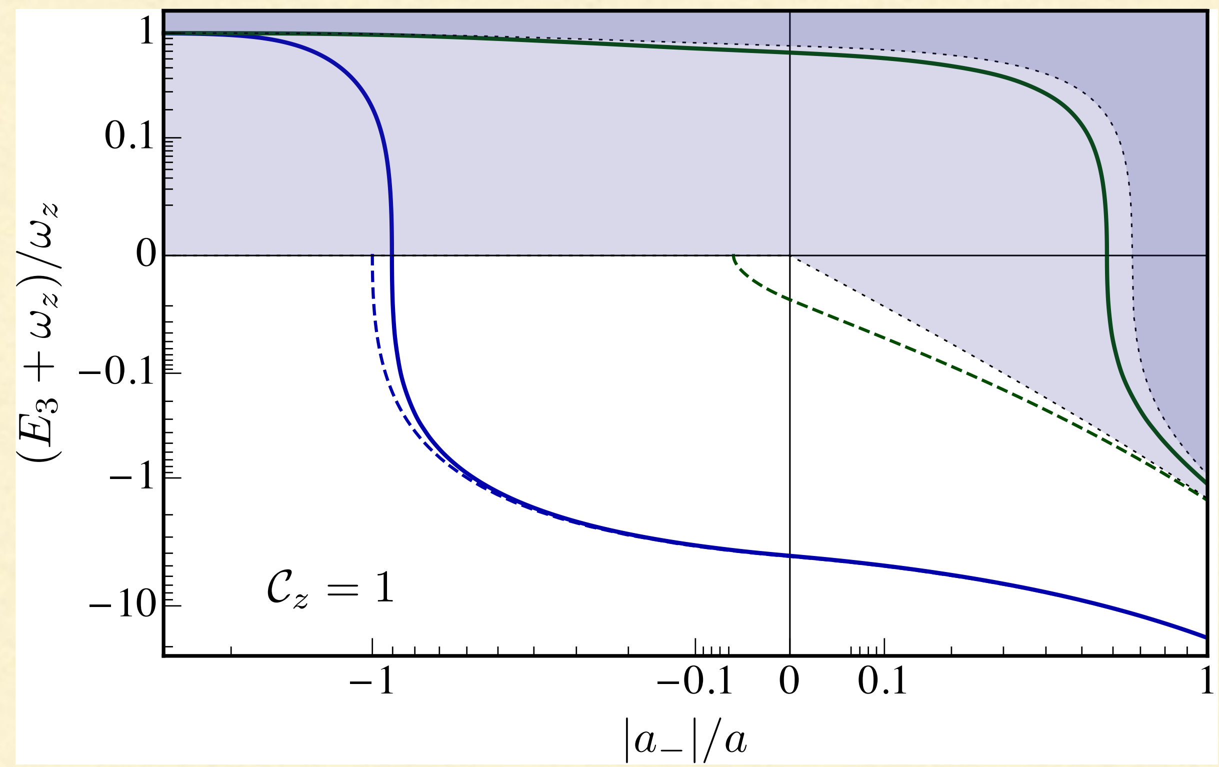
interaction strength: $|a_-|/a$

confinement strength: $\mathcal{C}_z \equiv |a_-|/l_z$

"weak" confinement




strong confinement

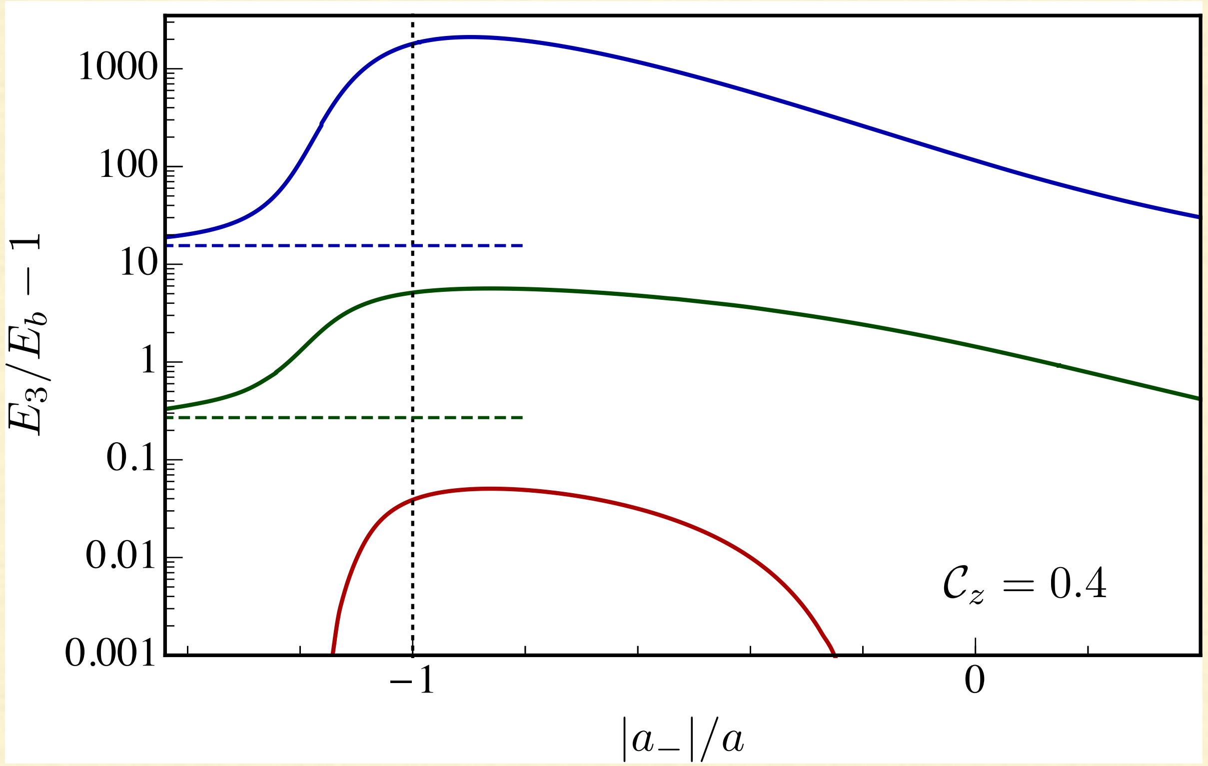


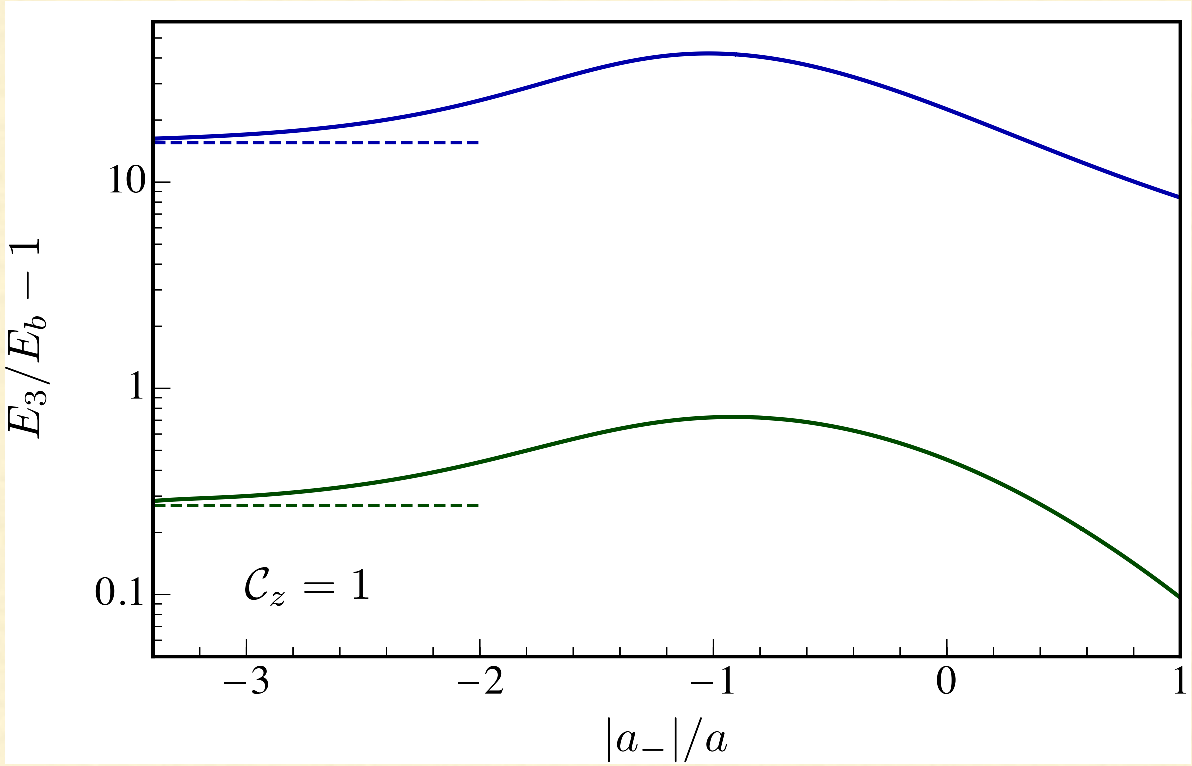

$^{133}$Cs: $\omega_z \approx 2\pi \times 5\text{kHz}$ $\omega_z \approx 2\pi \times 30\text{kHz}$

- deepest trimer closely resembles the 3D-one, even for strong confinement
- spectrum of trimers is strongly modified above the 3D continuum
- energy of trimer (measured from the q2D dimer) can be a significant fraction of $\omega_z$ even when $|a_-|/a<-1$, so trimers can be quite *resistant to thermal dissociation* when $T<<\omega_z$

# SPECTRUM (2D STYLE)

$\mathcal{C}_z \equiv |a_-|/l_z$

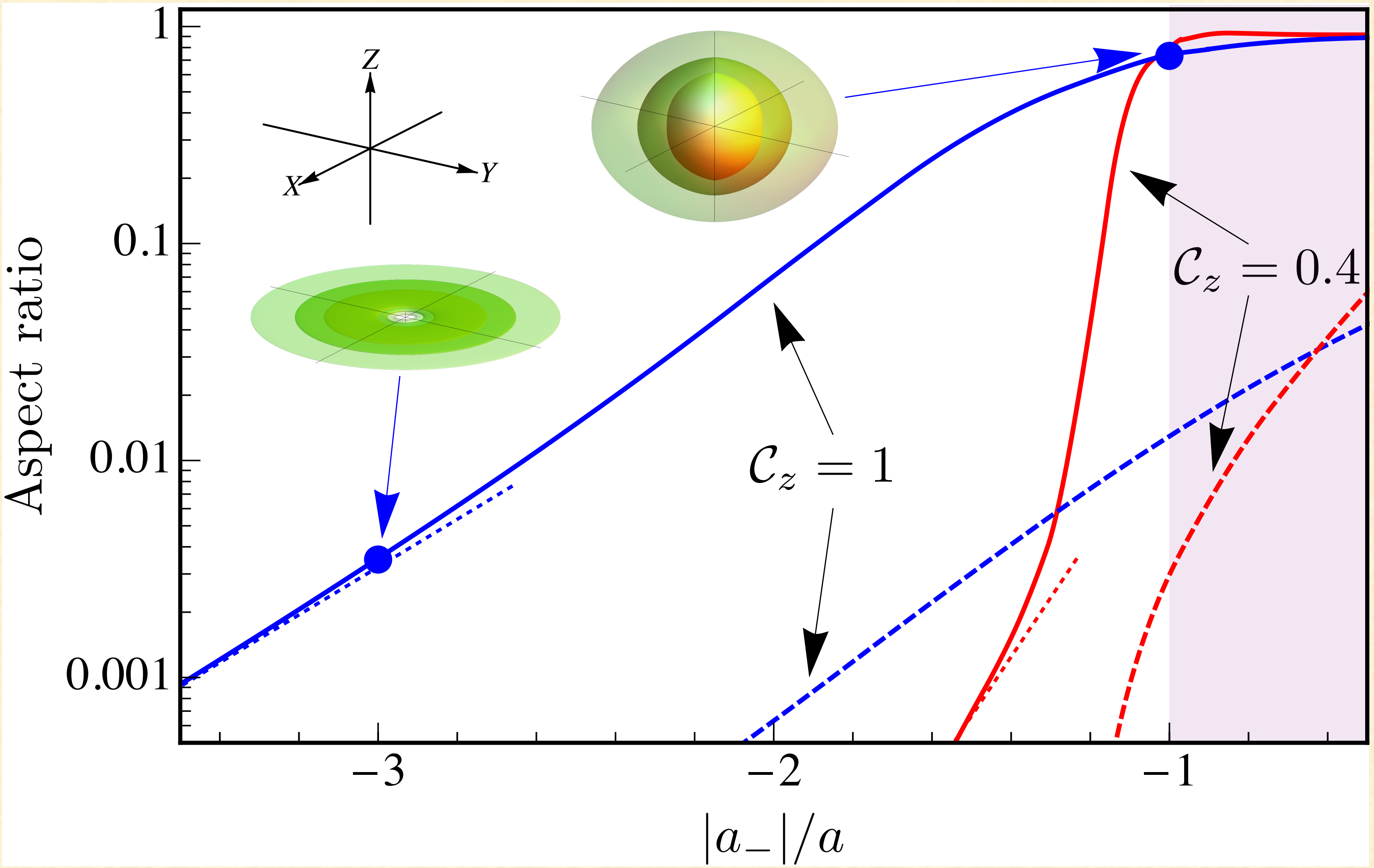



$^{133}$Cs: $\omega_z \approx 2\pi \times 5\text{kHz}$ $\qquad$ $\omega_z \approx 2\pi \times 30\text{kHz}$

- the 2D limit is recovered for small and negative scattering lengths (“BCS side” of the resonance)
- the two deepest trimers are stabilized for every negative scattering length
- avoided crossings: superposition of trimers with Efimovian + 2D-like character

# SHAPE OF THE TRIMERS




3D regime

2D limit

# HYPERSPHERICAL POTENTIALS

$$R^2 = r_1^2 + r_2^2 + r_3^2$$

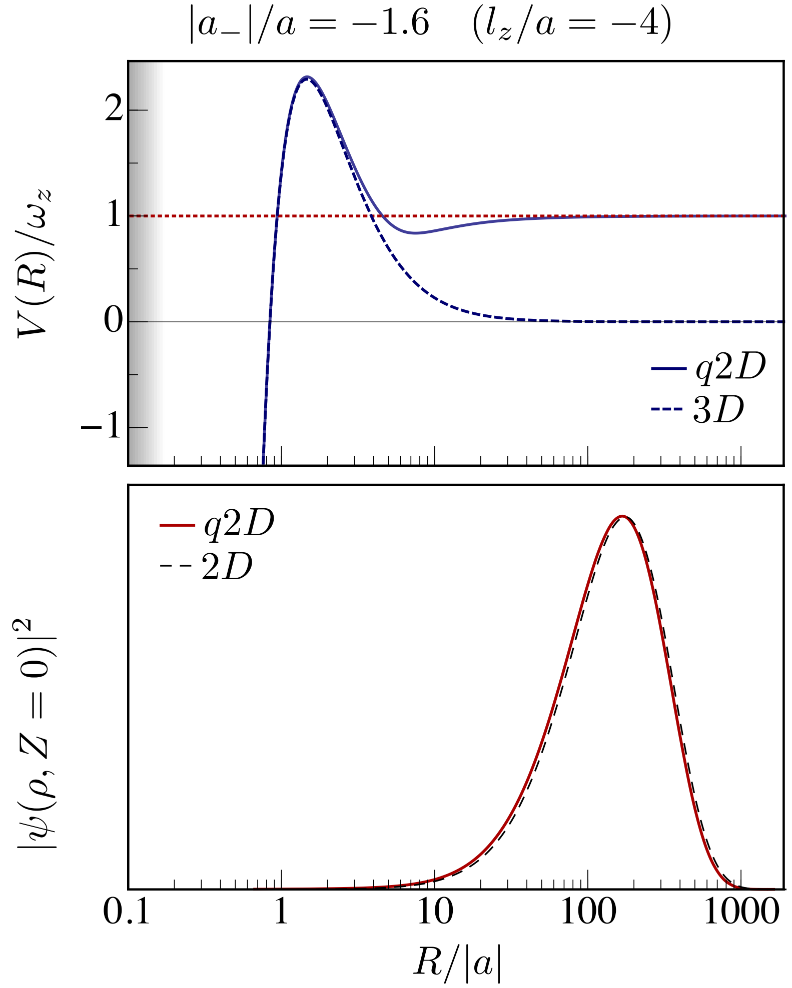
Hyper-spherical expansion: $$\Psi(R,\Omega) = \frac{1}{R^{5/2}\sin(2\alpha_k)} \sum_{n=0}^{\infty} f_n(R)\Phi_n(R,\Omega)$$

Hyper-radial Schrödinger equation:

$$\left[-\frac{1}{2m}\frac{\partial^2}{\partial R^2} + V(R)\right] f_0(R) = (E_3 + \omega_z)\, f_0(R)$$

*V(R)* depends on $l_z/a$, but not on the 3-body parameter.

# HYPERSPHERICAL POTENTIALS




- $V(R)$ approaches the 3D potential for $R \ll |a|$
  and the 2D potential for $R \gg l_z$
- When $l_z/a \lesssim -2.5$ the potential displays a repulsive barrier with height $\sim 0.15/ma^2$
- Small weight of trimers in the short distance region enhances lifetime

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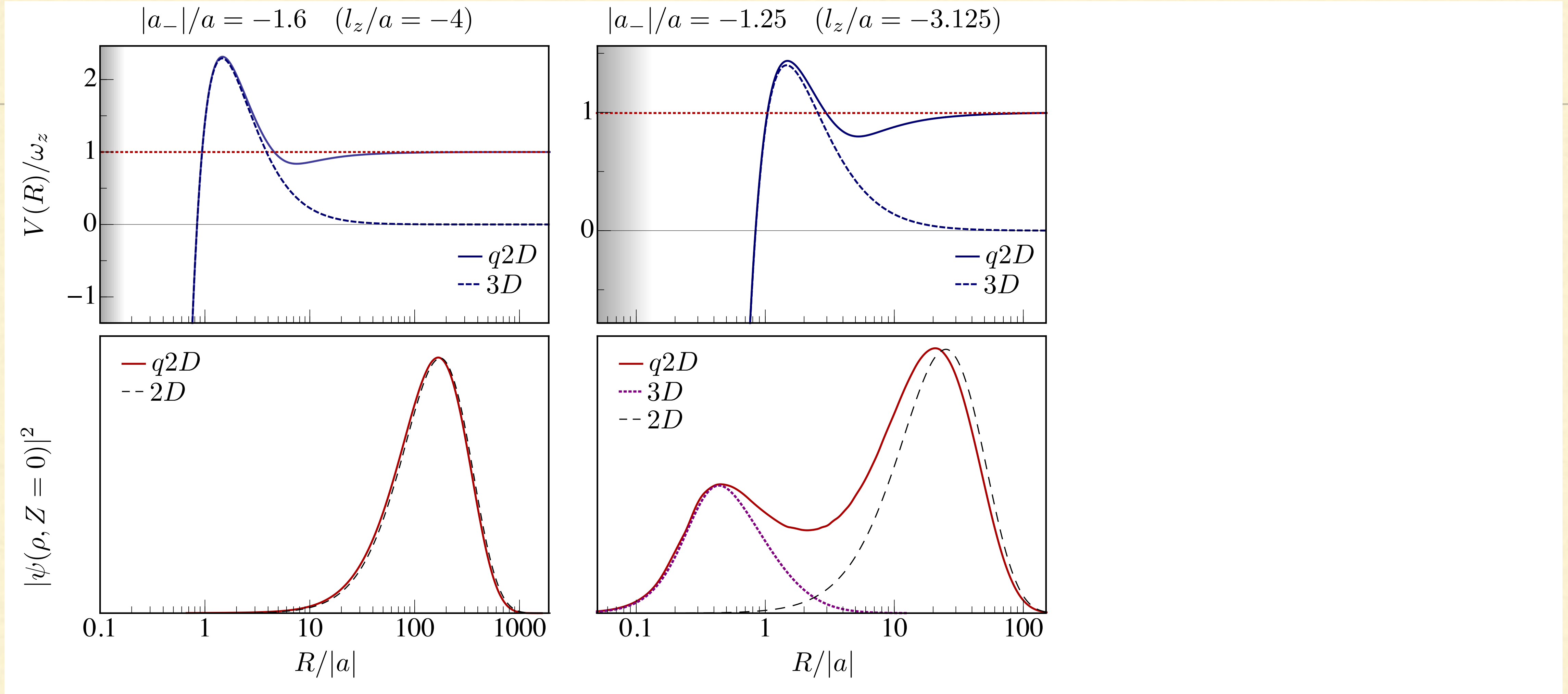


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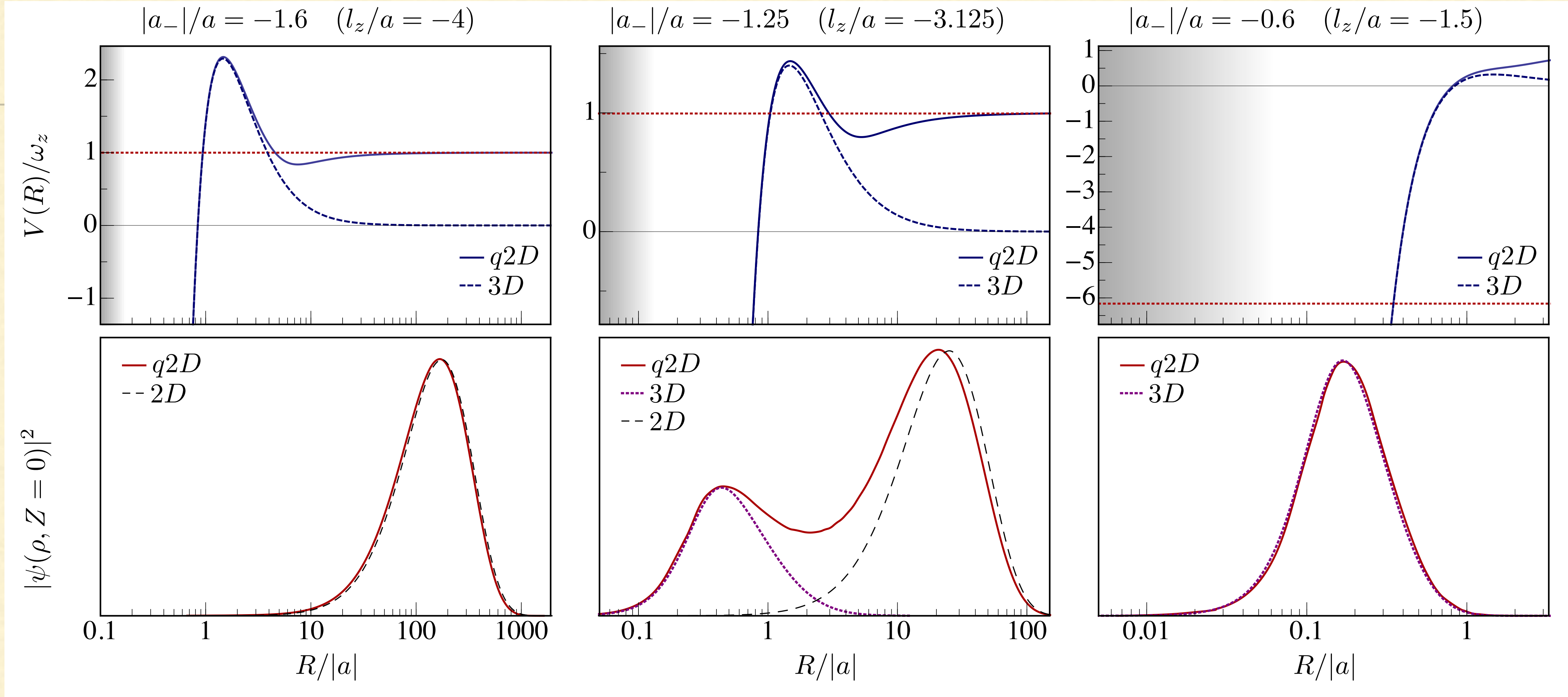


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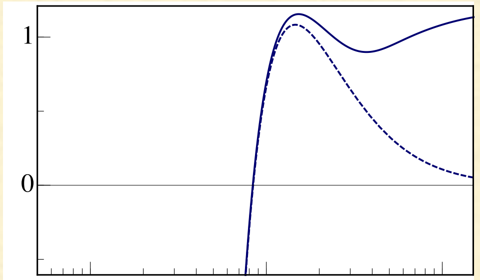
# EXPERIMENTAL CONSEQUENCES

- As "2D" experiments are performed at confinements often weaker than 5kHz, we expect this crossover physics to impact three-body correlations in realistic 2D studies on the *attractive* side of the Feshbach resonance

- Confinement raises continuum by $\hbar\omega_z$, so trimer resonance and loss peak disappear for $l_z/|a_-| \lesssim 2.5$, i.e., $C_z \gtrsim 0.4$

- When aiming at observing the discrete scaling symmetry: the 2nd trimer signature disappears once $C_z \gtrsim 0.4/22.7$ which for $^{133}$Cs corresponds to $\omega_z \approx 2\pi \times 10\text{Hz}$

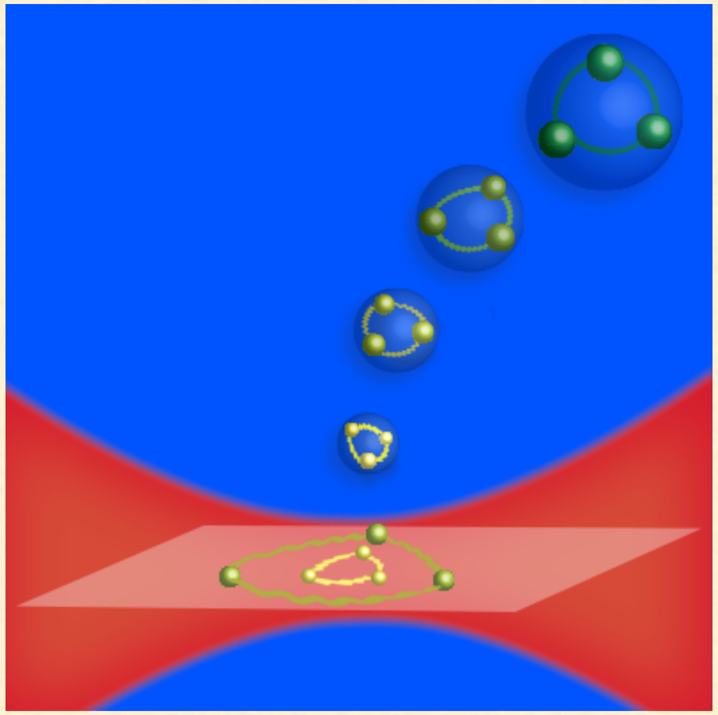



- Similar effects expected for 4-body states (as two tetramers exist in 2D), or in quasi-1D



observation of 2nd resonance in Cs recently reported by Bo Huang *et al*, PRL (2014)

# CONCLUSIONS



Efimov trimers under strong confinement

Discrete scaling survives only for $|a_-| \ll |a| \ll l_z$

Deepest trimer remains 3D-like even under strong confinement

Mixing with 2D trimers stabilizes two deepest trimers for ∀ a<0

Small weight at short distance will enhance lifetime (long-lived Efimov trimers?)

Consequences for correlations, quest to observe discrete scaling symmetry

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J. Levinsen,
P. Massignan and
M. M. Parish,
Phys. Rev. X 4, 031020 (2014)