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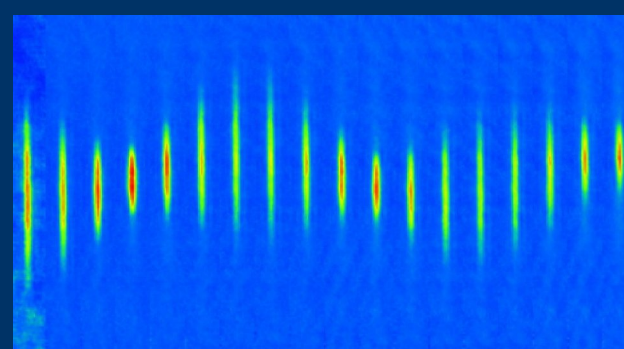
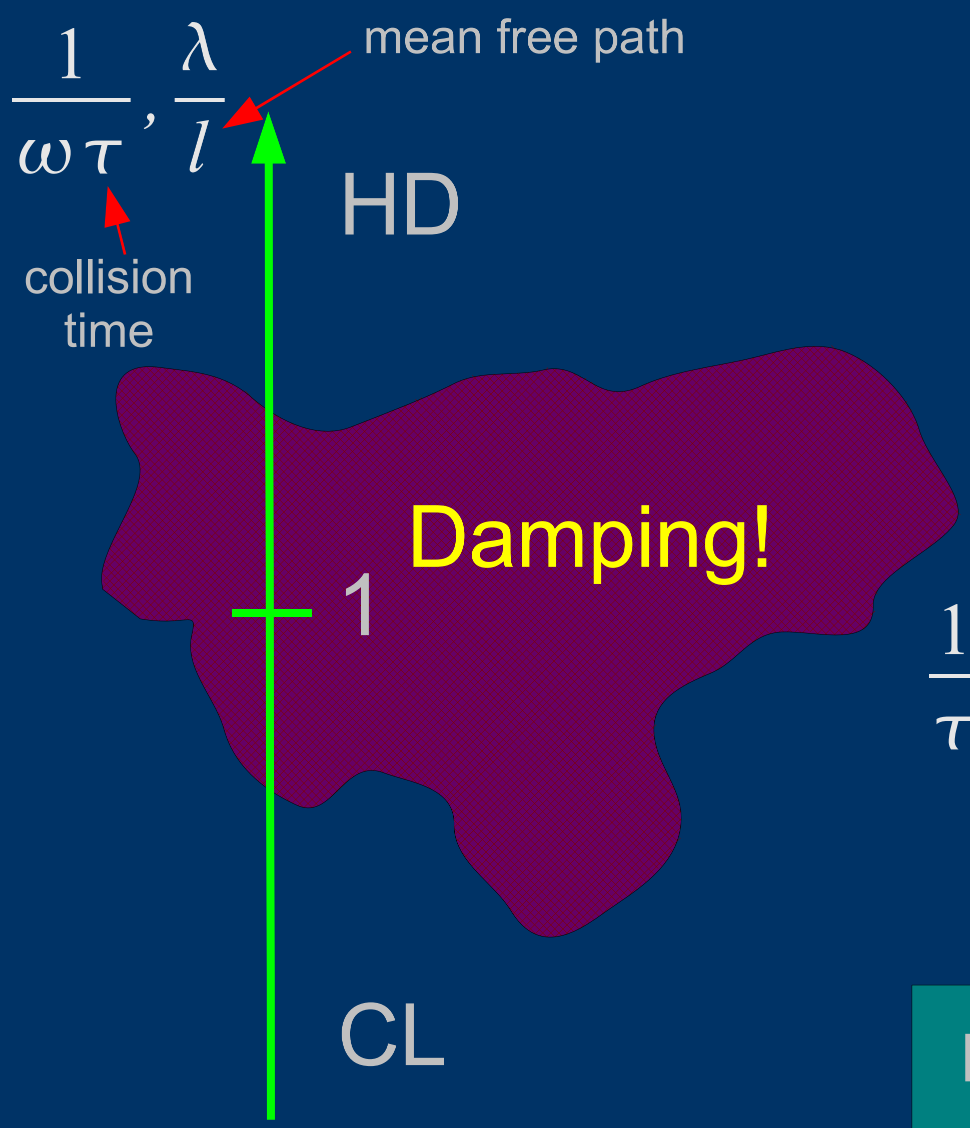
BEC and cold atomic gases group (NBI and NORDITA)

Viscous relaxation and collective oscillations in a trapped Fermi gas near the unitarity limit



Collective oscillations as a probe



CL: free particles, oscillations at $\omega = \sum_{i=x,y,z} N_i \omega_i$

HD: local thermodynamic equilibrium, oscillations at hydrodynamic freqs.

$$\frac{1}{\tau} \sim \frac{\bar{v}}{l} \sim \bar{v} n \sigma$$

[for fermions at $T \ll T_F$, add additional factor of $(T/T_F)^2$]

HD freqs. measured for any T?

What are we looking at?

- Two-component Fermi gas in its normal phase

  $(f = f_{\uparrow} = f_{\downarrow})$

- Viscosity of a uniform system

- Study the frequency and the damping of the low-energy collective modes

  $\omega(T, k_F a)$

- Starting point: Boltzmann equation

$$\frac{\partial f}{\partial t} + \dot{\vec{r}} \frac{\partial f}{\partial \vec{r}} + \dot{\vec{p}} \frac{\partial f}{\partial \vec{p}} = -I[f]$$

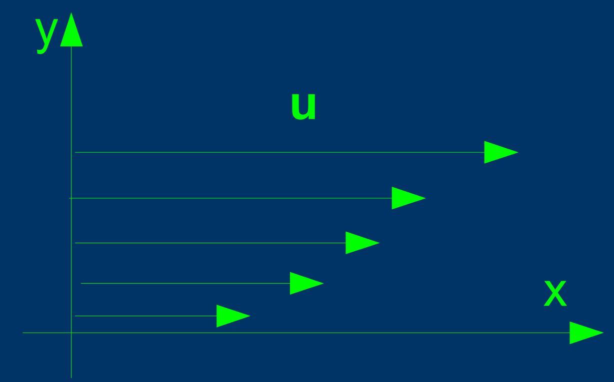
$$\dot{\vec{p}} = -\frac{\partial (V + g n)}{\partial \vec{r}}$$

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{1 + k_r^2 a^2}$$

Viscous flow

$$f^{\vec{u}}(\vec{r},\vec{p}) = \left\{ \exp\left[\frac{1}{kT} \left(\frac{p^2}{2m} - \mu - \vec{p}\cdot\vec{u} \right) \right] + 1 \right\}^{-1}$$

small $\vec{u}$: $\delta f = -\vec{p}\cdot\vec{u}\,\frac{\partial f^0}{\partial \epsilon}$



Flow velocity: $\vec{u}(\vec{r}) = (u_x(y), 0, 0)$

viscosity

$$\eta \stackrel{\text{def}}{=} -\frac{\Pi_{xy}}{\partial_y u_x}$$

momentum current density

$$\Pi_{xy} = 2\int \frac{d^3\vec{p}}{(2\pi\hbar)^3} \frac{p_x p_y}{m} f$$

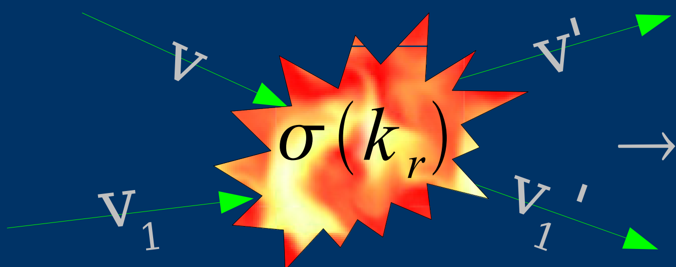
$$\partial_t f = 0$$
$$\delta\mu = 0$$
$$\delta T = 0$$

linearize the Boltzmann eq. in $\frac{\partial u_x}{\partial y}$

$$I[f] \approx \frac{\delta f}{\tau_\eta} \rightarrow \eta_{\text{relax}} \approx \frac{2\tau_\eta}{kT} \langle X^2 \rangle_{\vec{p}}$$

Viscous relaxation time

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{1 + k_r^2 a^2}$$



$$\rightarrow \eta_{\text{var}} \gtrsim \frac{2}{kT} \frac{(\langle X^2 \rangle_{\vec{p}})^2}{\langle X | H[X] \rangle_{\vec{p}}}$$

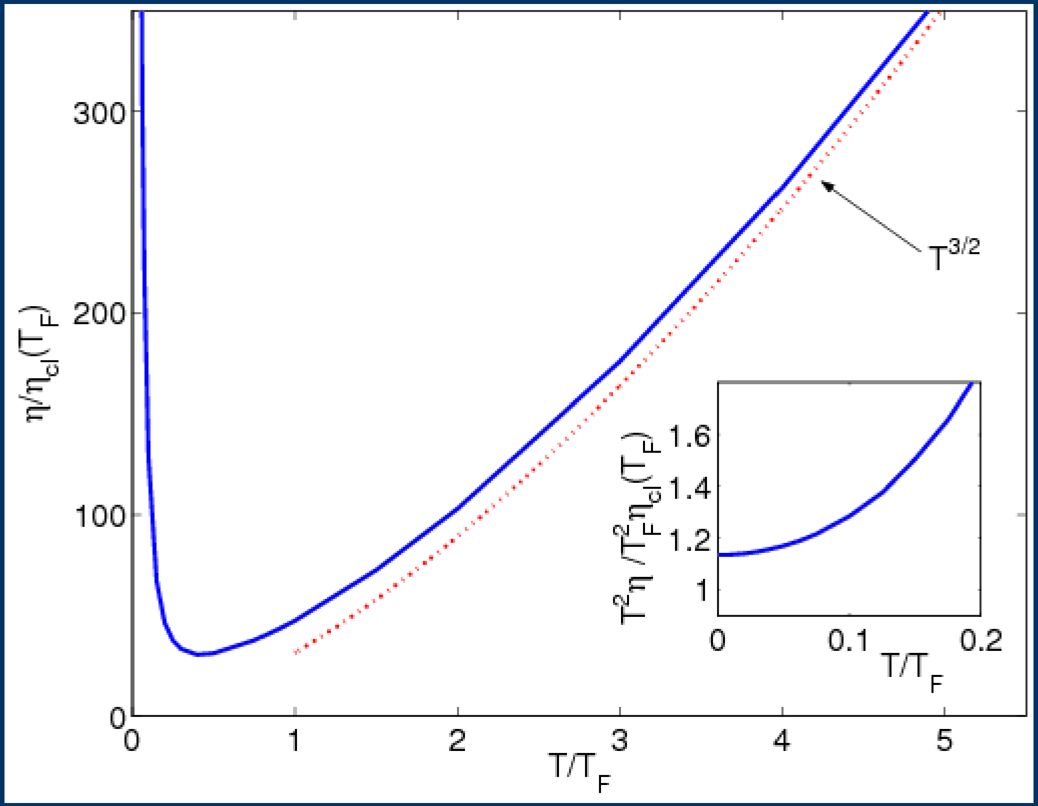
$$X \stackrel{\text{def}}{=} \frac{p_x p_y}{m}$$

$$H[\ldots] \stackrel{\text{def}}{=} \frac{I[\ldots]}{f^0(1 - f^0)}$$

$$\langle \ldots \rangle_{\vec{p}} \stackrel{\text{def}}{=} \int \frac{d^3 \vec{p}}{(2\pi\hbar)^3} f^0 (1 - f^0) \ldots$$

$$\eta_{\text{relax}} = \eta_{\text{var}} \rightarrow \frac{1}{\tau_\eta} \approx \frac{\langle X | H[X] \rangle_{\vec{p}}}{\langle X^2 \rangle_{\vec{p}}}$$

Viscosity of a uniform gas



$$k_F|a|=4.5$$

$$\eta_{cl}(T)=\frac{5\sqrt{\pi m k T}}{8\bar{\sigma}}$$

low T: $\frac{1}{\tau_\eta}\propto\frac{T^2}{T_F^2}\quad\left(\eta=\frac{2}{5}\tau_\eta n_{tot}E_F\right)$

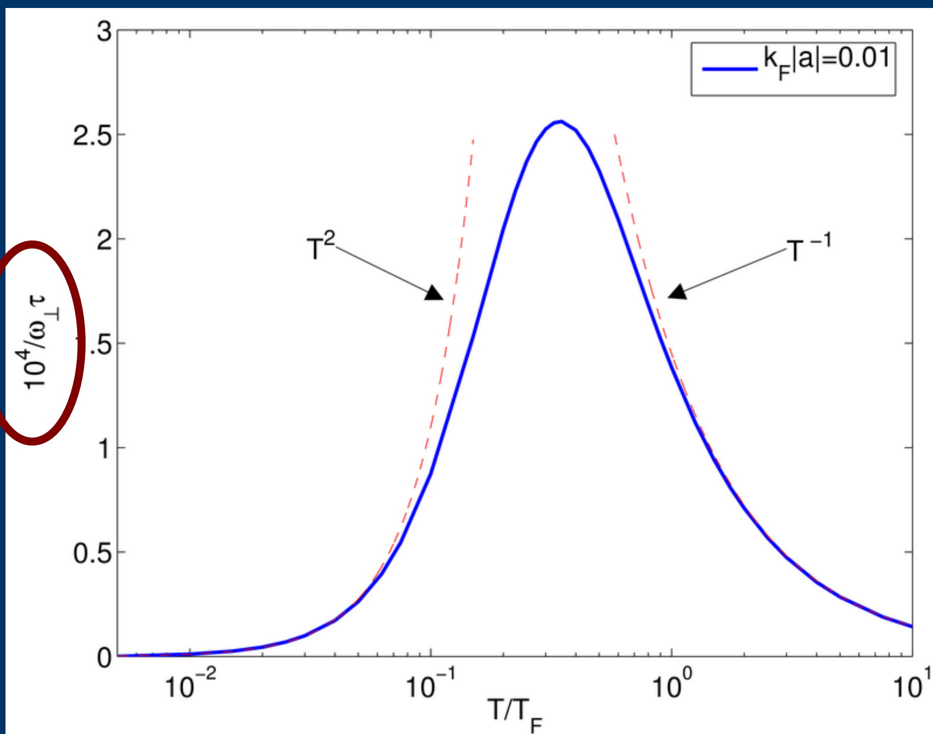
high T: $\tau_\eta\propto\frac{1}{n_{\rm tot}\bar{v}\bar{\sigma}}\quad\left(\eta=\tau_\eta n_{tot}kT\right)$

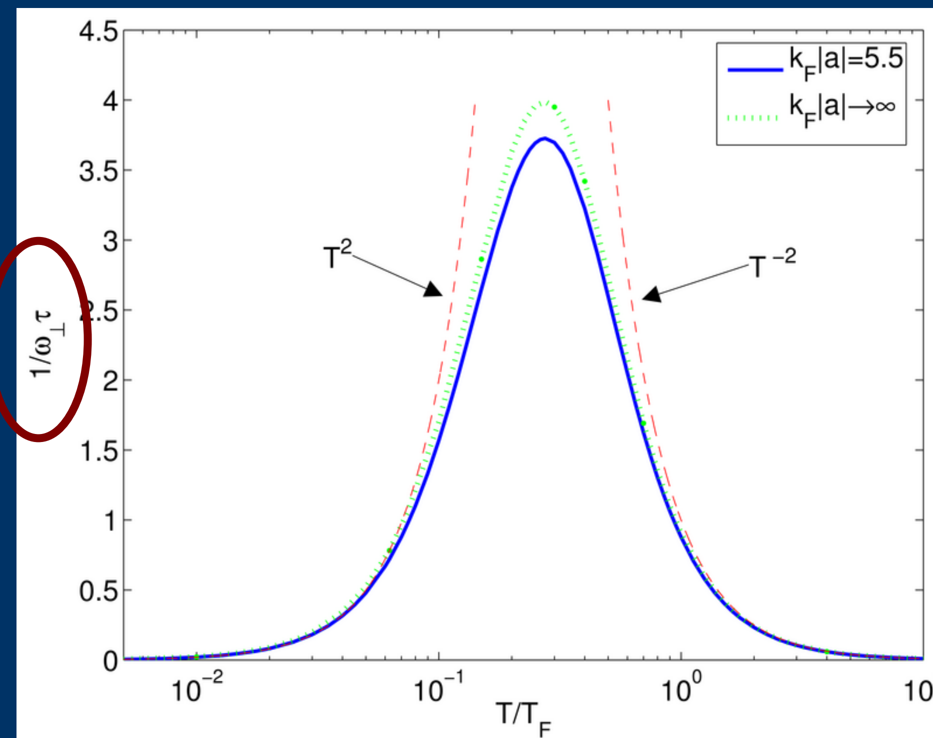
$\bar{\sigma}=$ → $4\pi a^2\quad(\text{if } a^2\ll\hbar^2/mkT)$

→ $2\lambda_{dB}^2/3\quad(\text{if } a^2\gg\hbar^2/mkT)$

Viscous relaxation rate, trapped gas:

$$\frac{1}{\tau_\eta} \stackrel{\text{def}}{=} \frac{\int d^3\vec{r} \langle X | H[X] \rangle_{\vec{p}}}{\int d^3\vec{r} \langle X^2 \rangle_{\vec{p}}}$$





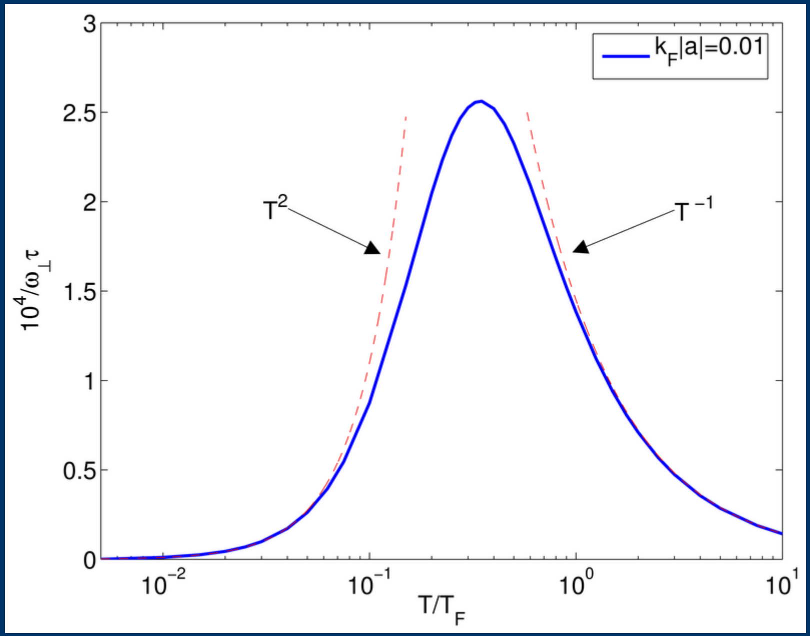
$\omega_\perp \sim 20\,\omega_z$

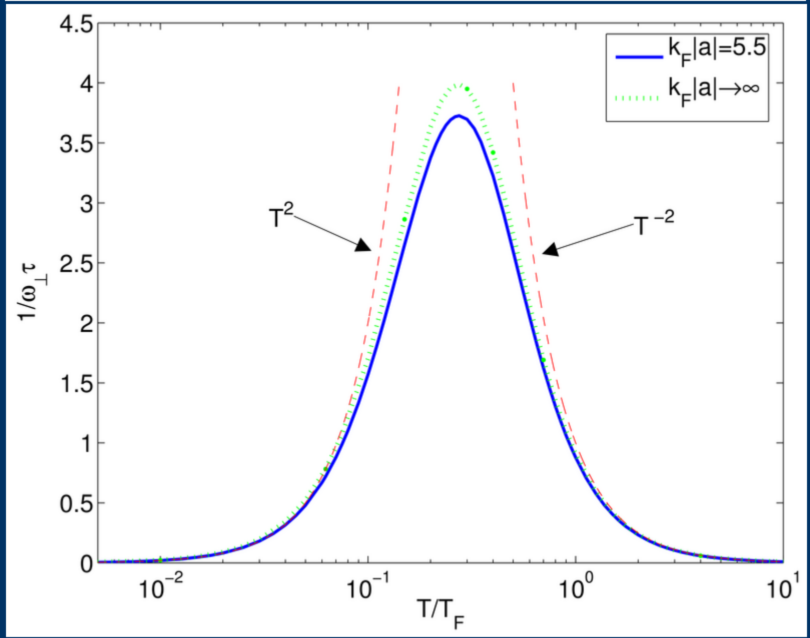
low T: Pauli blocking

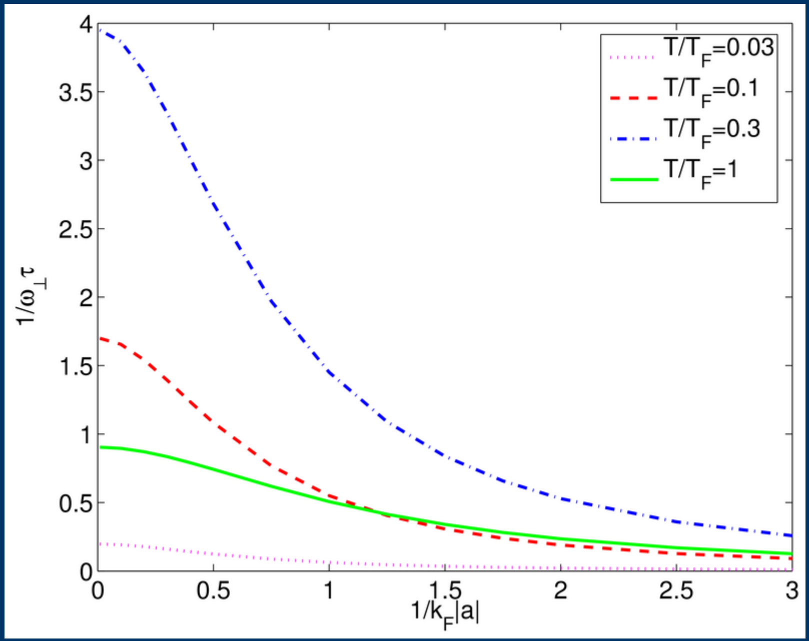
high T: $\frac{1}{\tau_\eta} \propto \bar{v}\, n_{\text{tot}}\, \bar{\sigma}$

$$\bar{\sigma} = \begin{cases} 4\pi a^2 & (\text{if } a^2 \ll \hbar^2/mkT) \\ 2\lambda_{dB}^2/3 & (\text{if } a^2 \gg \hbar^2/mkT) \end{cases}$$

$$\frac{1}{\tau_\eta} \stackrel{\text{def}}{=} \frac{\int d^3\vec{r} \left\langle X \middle| H[X] \right\rangle_{\vec{p}}}{\int d^3\vec{r} \left\langle X^2 \right\rangle_{\vec{p}}}$$







$N_{tot} \approx 3 * 10^5$ atoms

$\omega_z = 2\pi \times 70\,\text{Hz}$

$\omega_\perp = 2\pi \times 1550\,\text{Hz}$

+ interactions in the streaming terms

Linearized Boltzmann equation:

$$\frac{\partial \Phi}{\partial t}+\sum_{i=x,y,x}\omega_i\left[p_i\frac{\partial \Phi}{\partial \tilde{r}_i}-\left(\tilde{r}_i+mg\frac{\partial n^0}{\partial \tilde{r}_i}\right)\frac{\partial \Phi}{\partial p_i}-\frac{mg}{f^0(1-f^0)}\frac{\partial \langle \Phi \rangle_{\vec{p}}}{\partial \tilde{r}_i}\frac{\partial f^0}{\partial p_i}\right]=-\frac{I[\Phi]}{f^0(1-f^0)}$$

$$\Phi=\frac{\delta f}{f^0(1-f^0)}$$

$$\tilde{r}_i=m\omega_i r_i$$

Variational Ansatz: $u_i \propto r_i$

$$\Phi=\mathrm{e}^{-i\omega t}\sum_{i=x,y,z}\left(a_i\tilde{r}_i^2+b_i\tilde{r}_i p_i+c_i p_i^2\right)$$

Variational solution of the linearized Boltzmann equation

Taking moments with the 9 terms of the Ansatz,
9x9 linear system, ...

m=2 mode $\begin{pmatrix} \omega_{\mathrm{cl}}^2 = 4\,\omega_{\perp} \\ \omega_{\mathrm{hd}}^2 = 2\,\omega_{\perp} \end{pmatrix}$

$$\omega\left[(\omega^2-\omega_{\mathrm{hd}}^2)-i\,\omega\,\tau_\eta(\omega^2-\omega_{\mathrm{cl}}^2)\right]$$

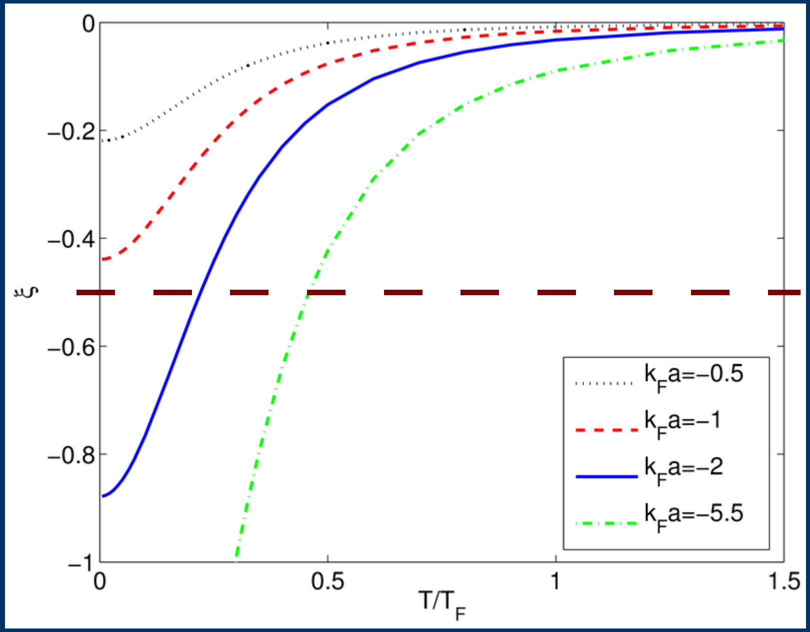
$$\left[(\omega^2-\omega_{\mathrm{hd+}}^2)(\omega^2-\omega_{\mathrm{hd-}}^2)-i\,\omega\,\tau_\eta(\omega^2-\omega_{\mathrm{cl+}}^2)(\omega^2-\omega_{\mathrm{cl-}}^2)\right]=0$$

2 m=0 modes

Interactions

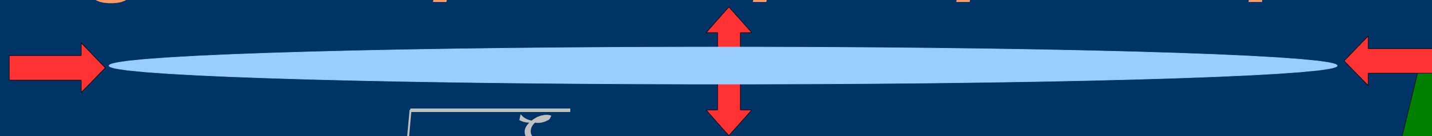
$$E_{kin} - E_{pot} + \frac{3}{2} E_{int} = 0$$

At equilibrium:



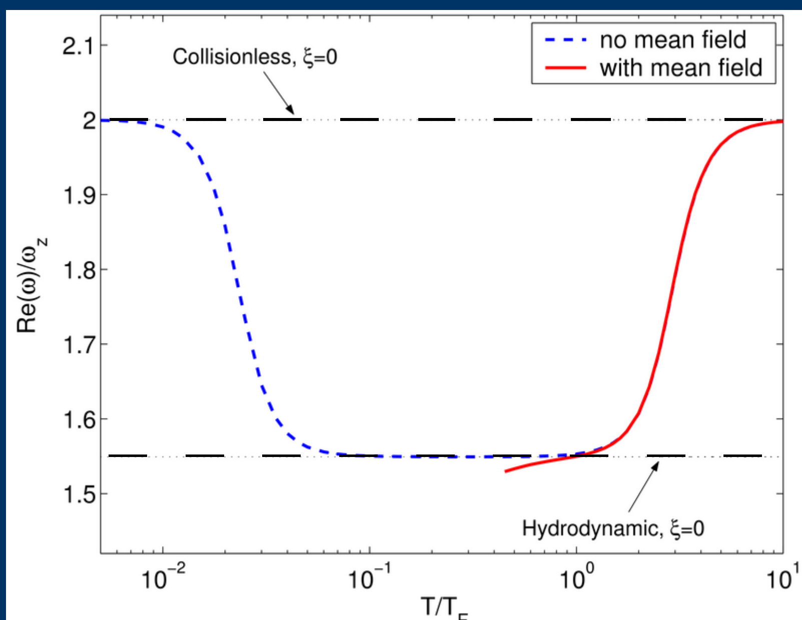
$$\xi \overset{\text{def}}{=} \frac{3 E_{int}}{2 E_{pot}}$$

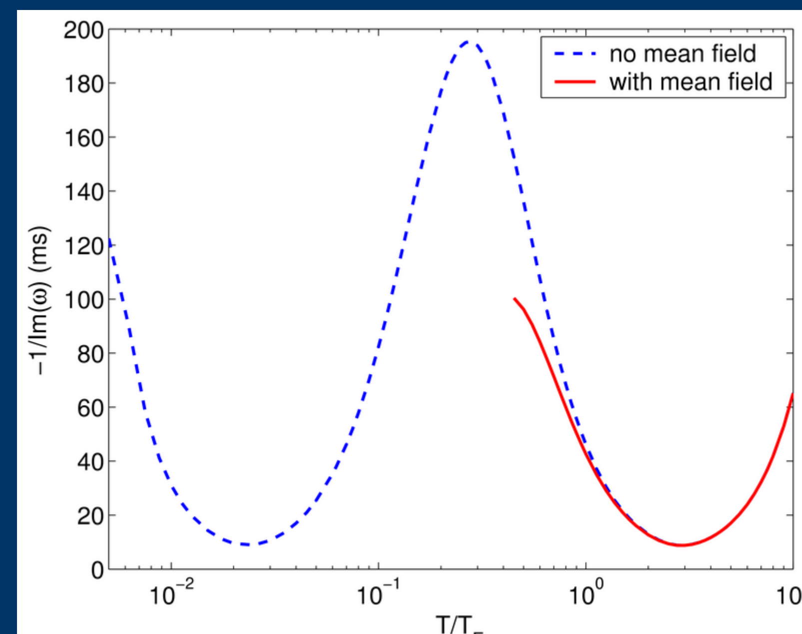
Cigar-shaped traps: quadrupole mode



$$\omega_{\text{cl-}} = 2\,\omega_z \sqrt{1 - \frac{\xi}{4}}$$

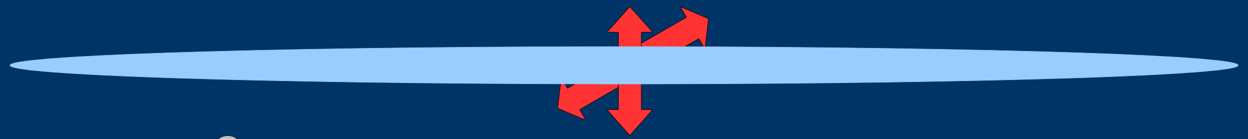
$$\xi \overset{\text{def}}{=} \frac{3\,E_{\text{int}}}{2\,E_{\text{pot}}}$$





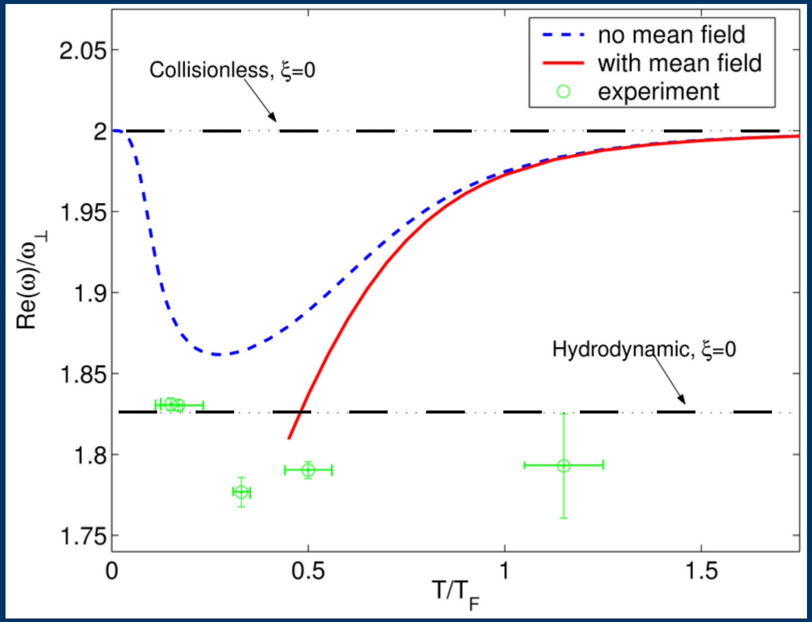
$$\omega_{\text{hd-}} = \omega_z \sqrt{\frac{12}{5}\left(1 + \frac{\xi}{20}\right)}$$

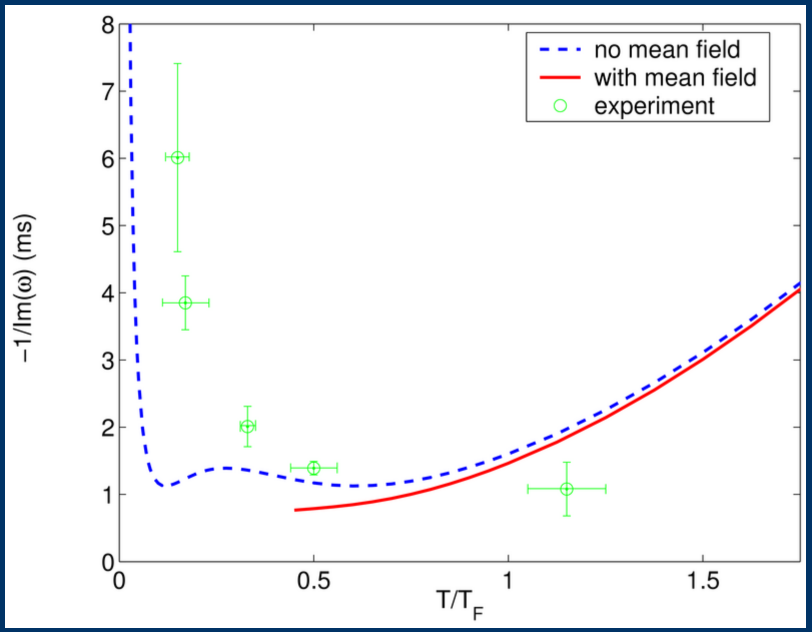
Cigar-shaped traps: breathing mode



$$\omega_{cl+} = 2\omega_{\perp}$$

$$\xi \stackrel{\text{def}}{=} \frac{3E_{\text{int}}}{2E_{\text{pot}}}$$





$$\omega_{hd+} = \omega_{\perp}\sqrt{\frac{10}{3}\left(1+\frac{\xi}{5}\right)}$$

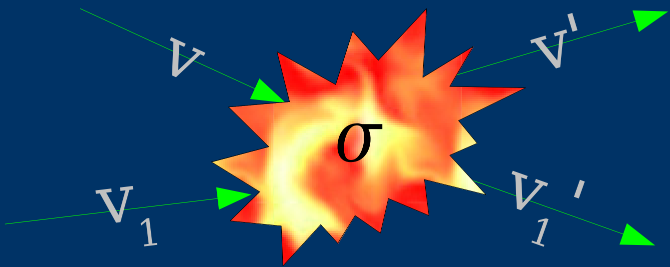
[Exp: J. Kinast *et al.*, PRL **92**, 150402 (2004)]

Perspectives

More accurate inclusion of interactions
(many-body effects, short-lived molecules, ...)

- P. Massignan, G. Bruun and H. Smith, PRA **71**, 033607 (2005)
- G. Bruun and H. Smith, PRA **72**, 043605 (2005)

$$I[f] \approx \frac{\delta f}{\tau_\eta} \quad \rightarrow \quad \eta_{\text{relax}} \approx \frac{2\tau_\eta}{kT} \langle X^2 \rangle_{\vec{p}}$$



Viscous relaxation time

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{1 + k_r^2 a^2}$$

$$I[\Phi] = \int \frac{d^3 \vec{p}_1}{(2\pi\hbar)^3} \int d\Omega \frac{d\sigma}{d\Omega} |\vec{v} - \vec{v}_1| [\Phi + \Phi_1 - \Phi' - \Phi_1'] f^0 f_1^0 (1 - f^0{}') (1 - f_1^0{}')$$

$$\eta_{\text{var}} \gtrsim \frac{2}{kT} \frac{(\langle X^2 \rangle_{\vec{p}})^2}{\langle X | H[X] \rangle_{\vec{p}}}$$

$$X \stackrel{\text{def}}{=} \frac{p_x p_y}{m}$$

$$\Phi \stackrel{\text{def}}{=} \frac{\delta f}{f^0 (1 - f^0)}$$

$$H[\ldots] \stackrel{\text{def}}{=} \frac{I[\ldots]}{f^0 (1 - f^0)}$$

$$\langle \ldots \rangle_{\vec{p}} \stackrel{\text{def}}{=} \int \frac{d^3 \vec{p}}{(2\pi\hbar)^3} f^0 (1 - f^0) \ldots$$

$$\eta_{\text{relax}} = \eta_{\text{var}} \rightarrow \frac{1}{\tau_\eta} \approx \frac{\langle X | H[X] \rangle_{\vec{p}}}{\langle X^2 \rangle_{\vec{p}}}$$