Synthetic gauge fields in synthetic dimensions

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Abstract

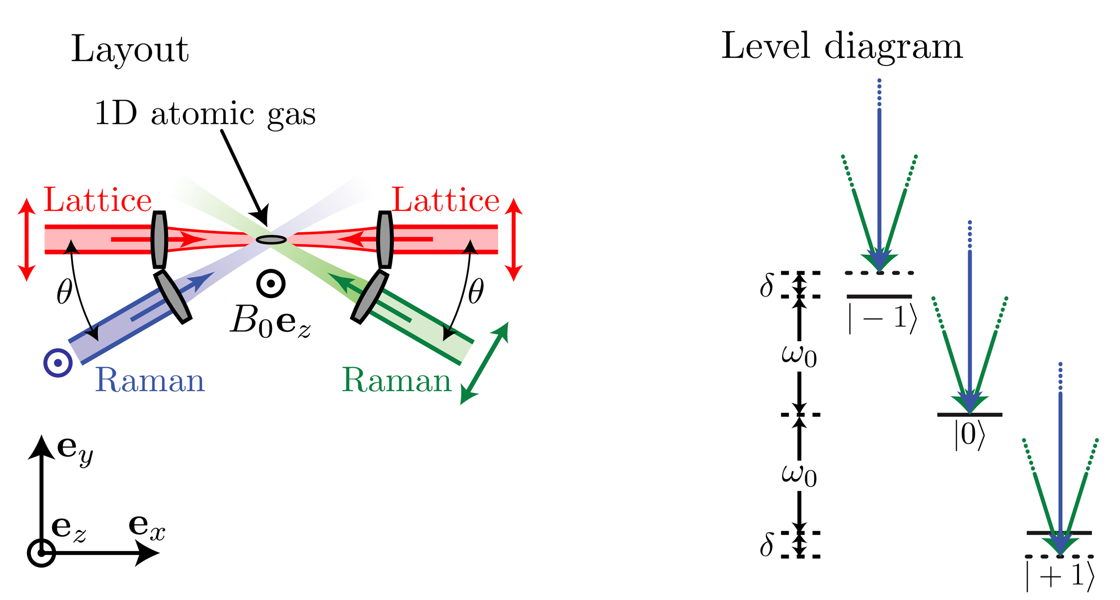
We describe a simple technique for generating a cold-atom lattice pierced by a uniform magnetic field. Our method is to extend a one-dimensional optical lattice into the "dimension" provided by the internal atomic degrees of freedom, yielding a synthetic 2D lattice.
Suitable laser-coupling between these internal states leads to a uniform magnetic flux within the 2D lattice. We show that this setup reproduces the main features of magnetic lattice systems, such as the fractal Hofstadter butterfly spectrum and the chiral edge states of the associated Chern insulating phases.

Goals

- **Realize synthetic *uniform* magnetic fields**
- **Engineer a lattice with *sharp* edges**
- **Study *infinite-ranged* interactions**

Raman-coupling of internal states

Proposal:
- F=1 ^{87}Rb atoms in a 1D optical lattice (deep, but not in the Mott regime)
- two linearly polarized λ_R Raman beams, providing recoil $k_R = 2\pi\cos\theta/\lambda_R$
- uniform magnetic field in the orthogonal direction



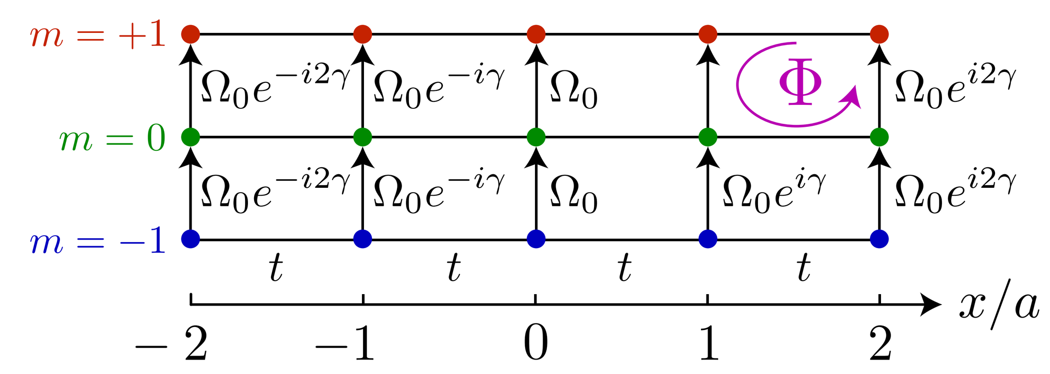
Effective magnetic field: $\mathbf{\Omega}_T = \delta\mathbf{e}_z + \Omega_R\left[\cos(2k_R x)\,\mathbf{e}_x - \sin(2k_R x)\,\mathbf{e}_y\right]$

Atom-light coupling: $H_{\rm al} = \mathbf{\Omega}_T\cdot\mathbf{F} = \delta F_z + (F_+ e^{ik_R x} + F_- e^{-ik_R x})\Omega_R/2$

Raising (or *spin-hopping*) operator: $F_+|m\rangle = g_{F,m}|m+1\rangle$

$$g_{F,m} = \sqrt{F(F+1) - m(m+1)}$$

2D synthetic lattice (*extra-dimension*)



$$H = \sum_{n,m}\left(-t a^\dagger_{n+1,m} + \Omega_{m-1}e^{-i\gamma n}a^\dagger_{n,m-1}\right)a_{n,m} + \text{H.c.}$$

synthetic tunneling strength: $\Omega_m = \Omega_R g_{F,m}/2$

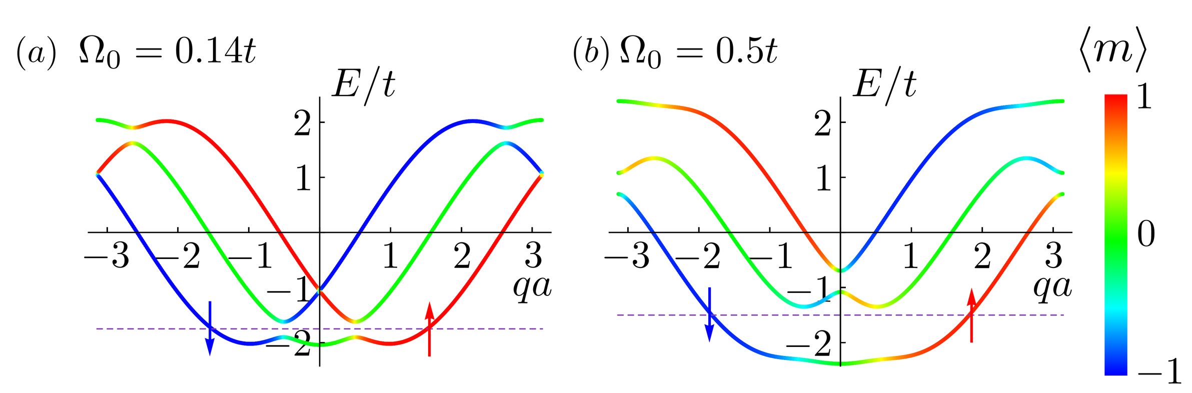
phase, and magnetic flux per plaquette: $\gamma = 2k_R a = 2\pi\Phi$

By a gauge- and Fourier-transform, one obtains:

$$H = \sum_q\sum_{m=-F}^{F}\varepsilon_{q+\gamma m}b^\dagger_{q,m}b_{q,m} + \left(\Omega_m b^\dagger_{q,m+1}b_{q,m} + \text{H.c.}\right)$$

with $\varepsilon_k = -2t\cos(k)$, and $q \equiv 2\pi l/N$

Chiral edge states

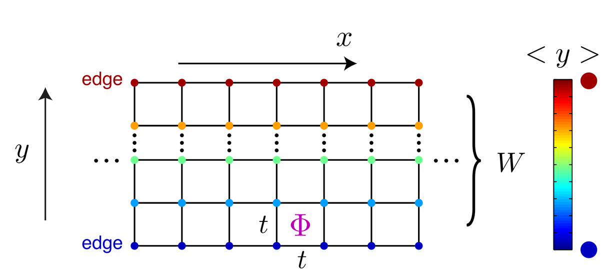


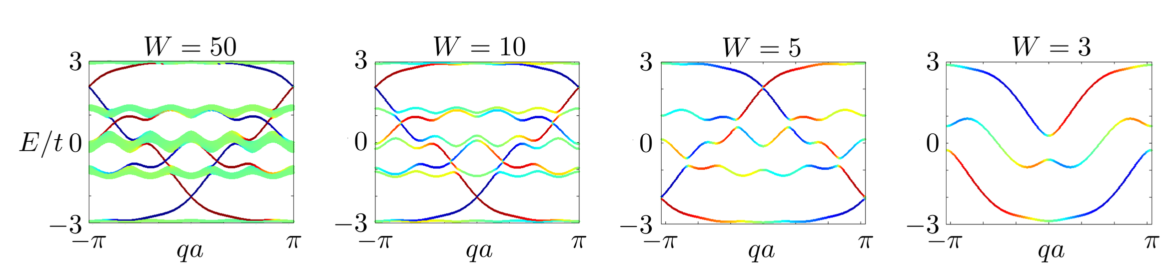
States inside the Raman-gap have $|\langle m\rangle| \approx F$,
i.e., are **chiral edge states on the synthetic lattice**

$\Phi = 1/2\pi$

Hofstadter strip (of width W=2F+1)

Regular 2D square N*W lattice, pierced by a uniform magnetic field:



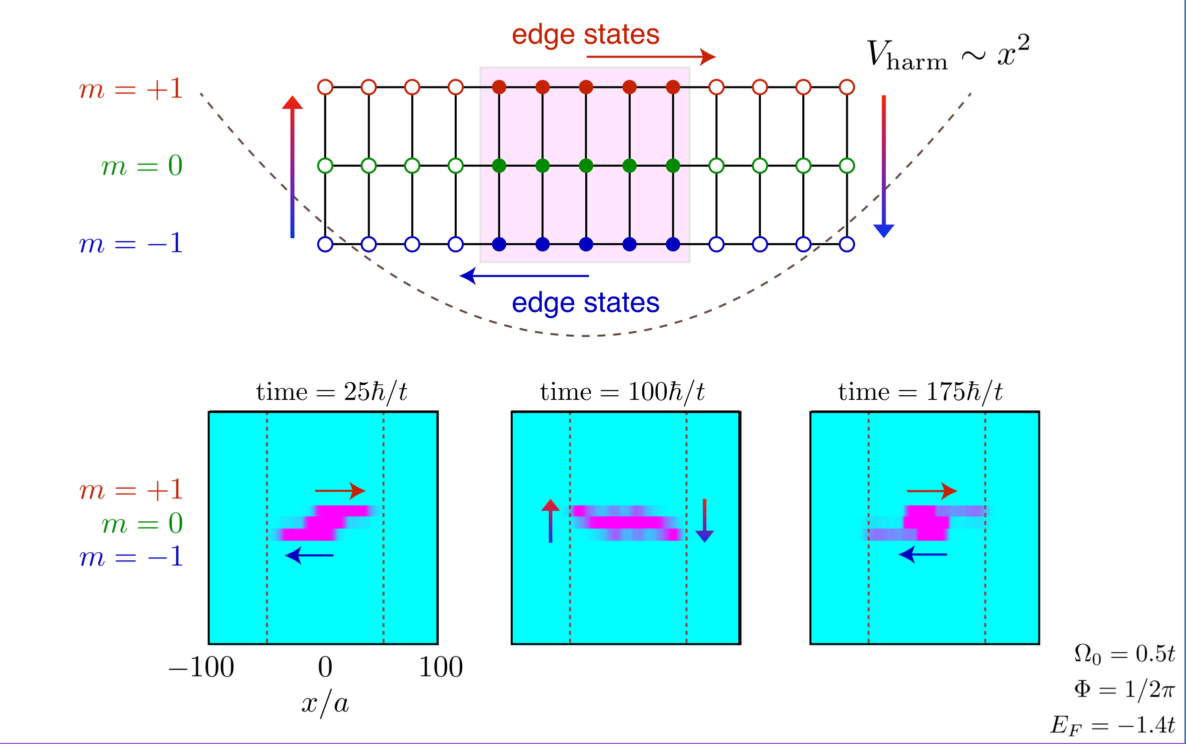


(see Ref. [5] for a detailed discussion of the case W=2)

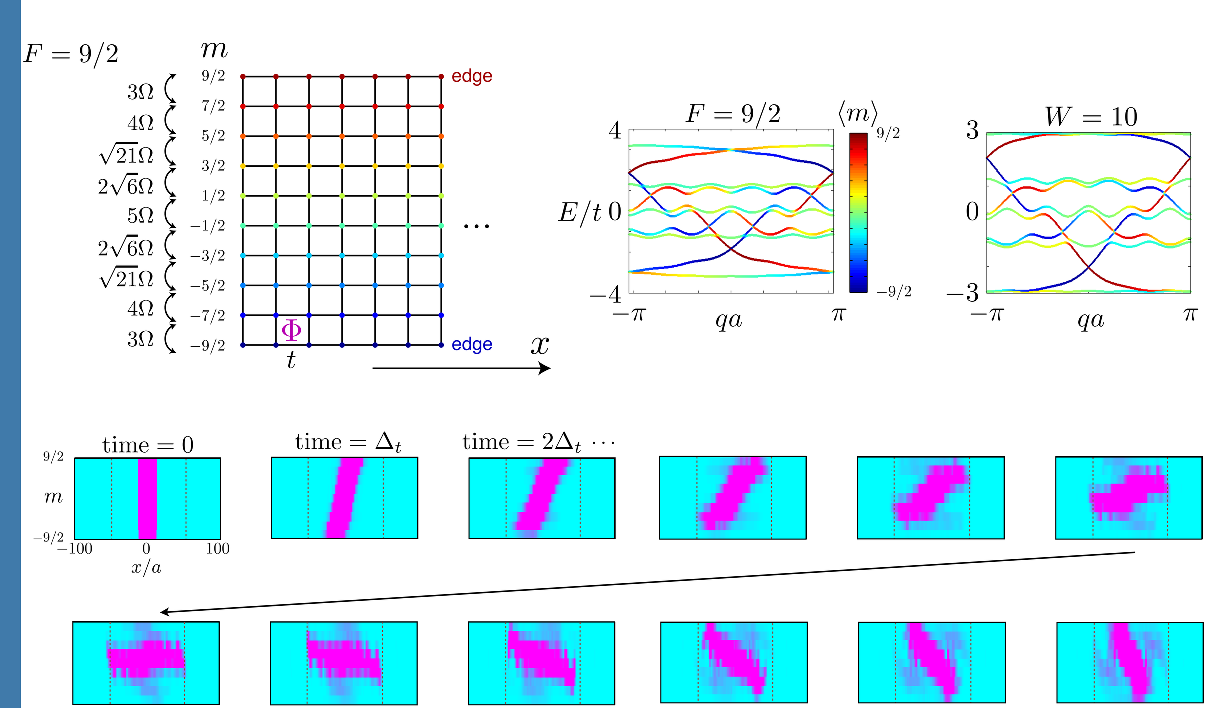
$\Phi = 1/5$

Edge state dynamics

Dynamics of an "F=1 Fermi gas" in a harmonic trap
For t<0, the gas is confined by hard walls in the shaded region

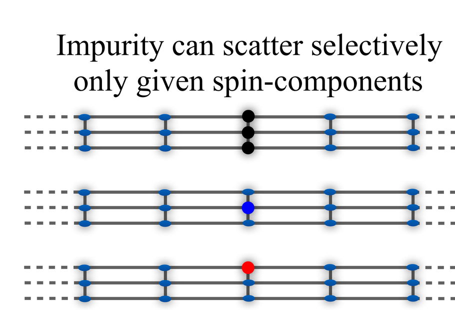


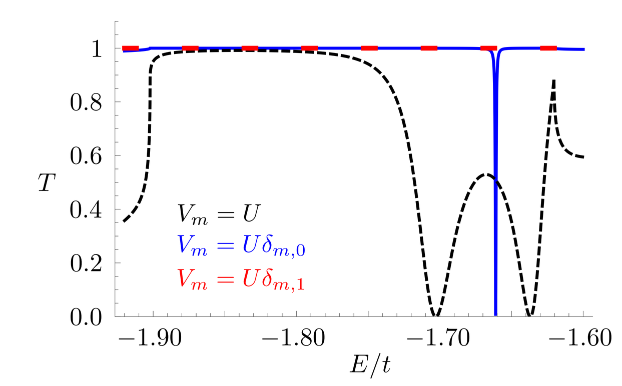
^{40}K atoms



Transmission through static impurities

Impurity localized at n=0: $V = \sum_m V_m a^\dagger_{0,m}a_{0,m}$





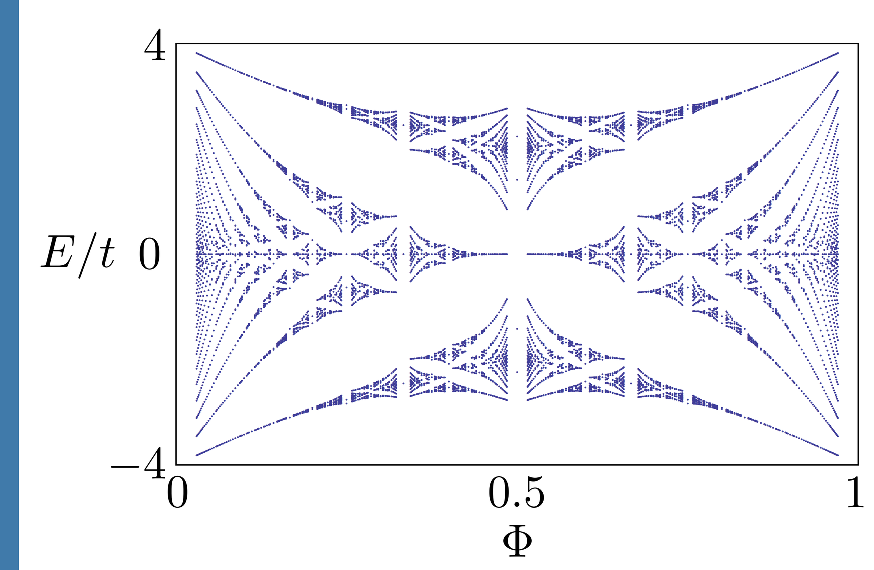
The transmission *T(E)* presents Fano resonances at energies inside the gap, due to the presence of quasi-bound states localized around the impurity. Impurities "localized on the edge", however, cause only negligible scattering.

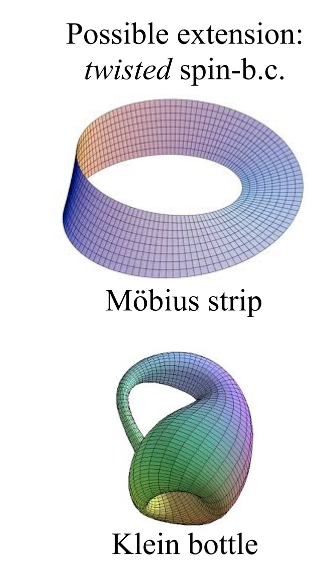
Closed spin-boundary conditions

Periodic boundary conditions along $\mathbf{e}_m$ can be induced with an extra coupling $|m=1\rangle \leftrightarrow |m=-1\rangle$ accompanied by a momentum recoil k_R along $\mathbf{e}_x$.
The system becomes periodic IFF $\gamma = 2\pi P/Q$ with *P,Q* co-prime integers.

The number of loops in the synthetic dimension required to have periodicity is *l/W* where *l=LCM(W,Q)* [thus *Q* or *Q/3* loops for *W*=3].

The spectrum has the fractal structure of a **Hofstadter butterfly**.





Interactions: SU(W)-invariant case

$$H_{\rm int} = \frac{\mathcal{U}}{2}\sum_n \mathcal{N}_n(\mathcal{N}_n - 1) \quad\text{with}\quad \mathcal{N}_n \equiv \sum_m a^\dagger_{n,m}a_{n,m}$$

Interactions are local along $\mathbf{e}_m$, but infinite in range along $\mathbf{e}_m$!

For SU(W)-invariant interactions, there exists a basis $\{c_{n,j}\}$ in which the Hamiltonian is diagonal in the spin direction. Denoting its eigenvalues by $\{\epsilon_{n,j}\}$ we can minimize the energy for fixed $\langle H_{\rm int}\rangle$ by populating only the states with lowest ϵ_{n,j_n}, as this minimizes the kinetic term $\langle H\rangle$.

Ground state of the interacting system:
- if the local (spin) ground state is not degenerate, the global ground state can be mapped to the one of a 1D uniform Bose-Hubbard chain
- if $\epsilon_{n,j}$ is minimal for two of the *W* values of *j*, the ground state of the 1D chain will possess a primitive cell containing *Q* consecutive lattice points

(degeneracy is possible only for closed spin-b.c. and rational Φ. For open b.c., the eigenvalues are independent of γ and of *n*, and never degenerate)

References

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Acknowledgments

We acknowledge funding from FRS-FNRS, ERC AdG QUAGATUA, EU IP SIQS, TOQATA, ESF POLATOM network, European Social Fund, ARO, DARPA's OLE, Atomtronics-MURI, NSF-PFC@JQI.