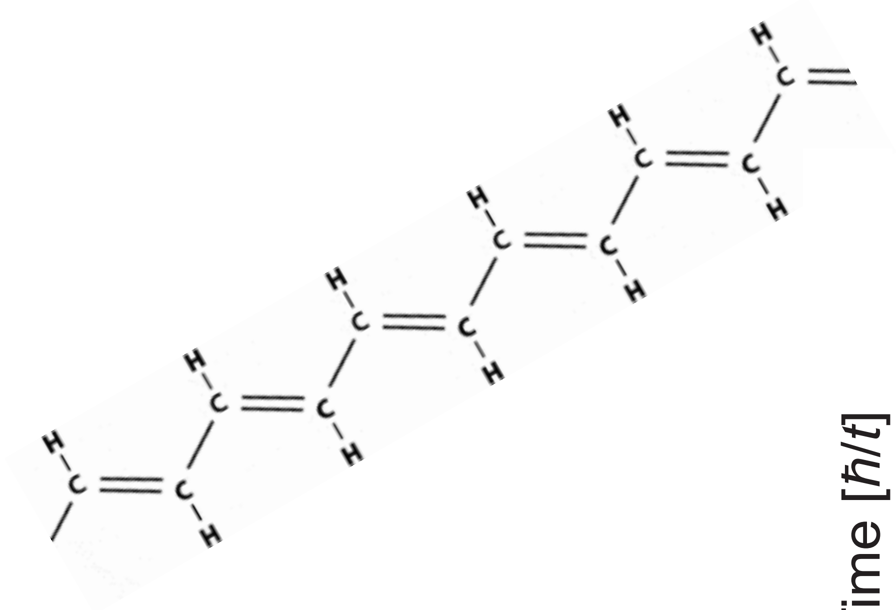
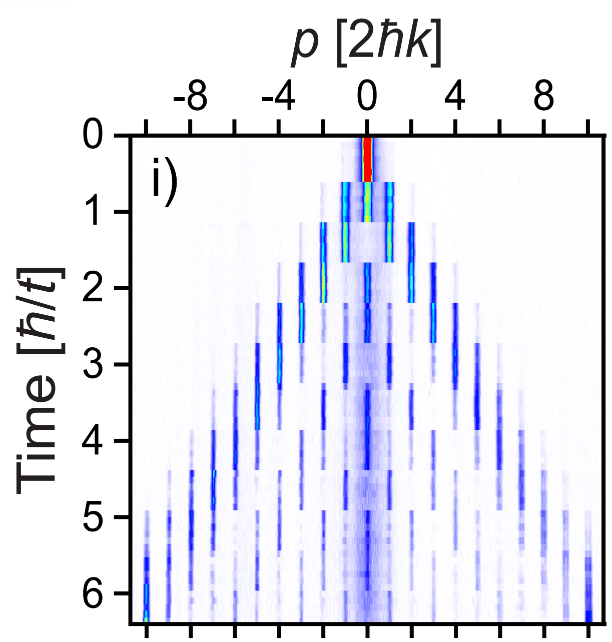
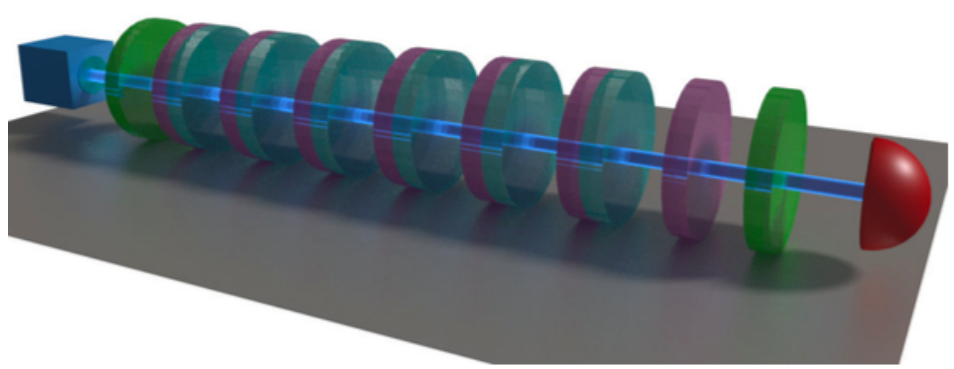
Efficient detection of topological features

Pietro Massignan







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Experiments

atomic wires


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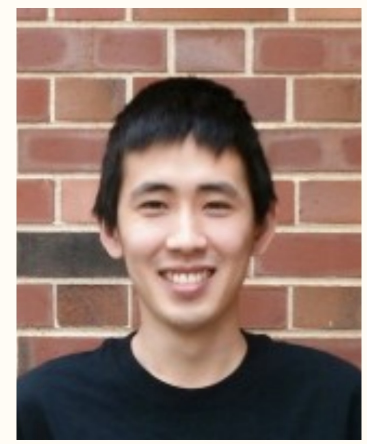
Fangzhao An

PHYSICS ILLINOIS


Bryce Gadway


Hughes Taylor

twisted photons


Filippo Cardano


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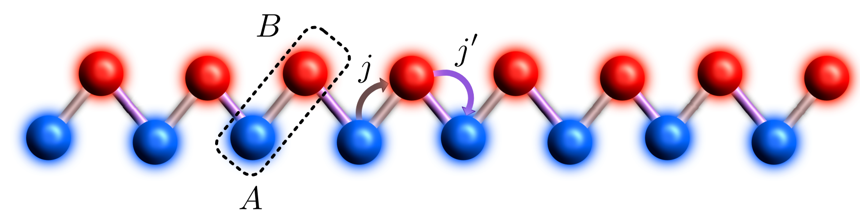
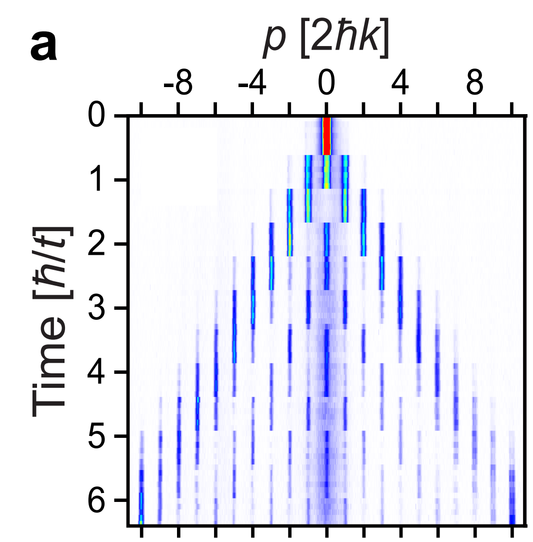
SLAM group

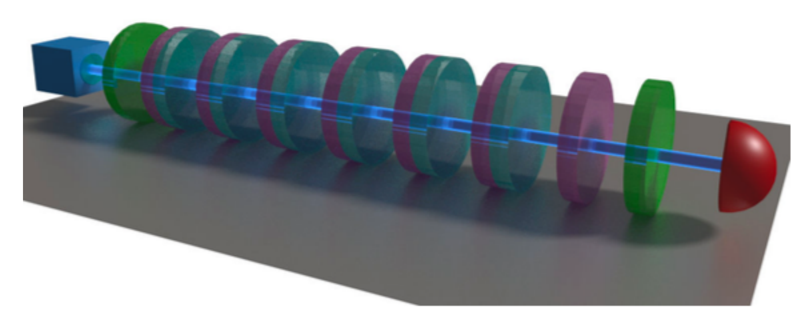

Lorenzo Marrucci

Outline

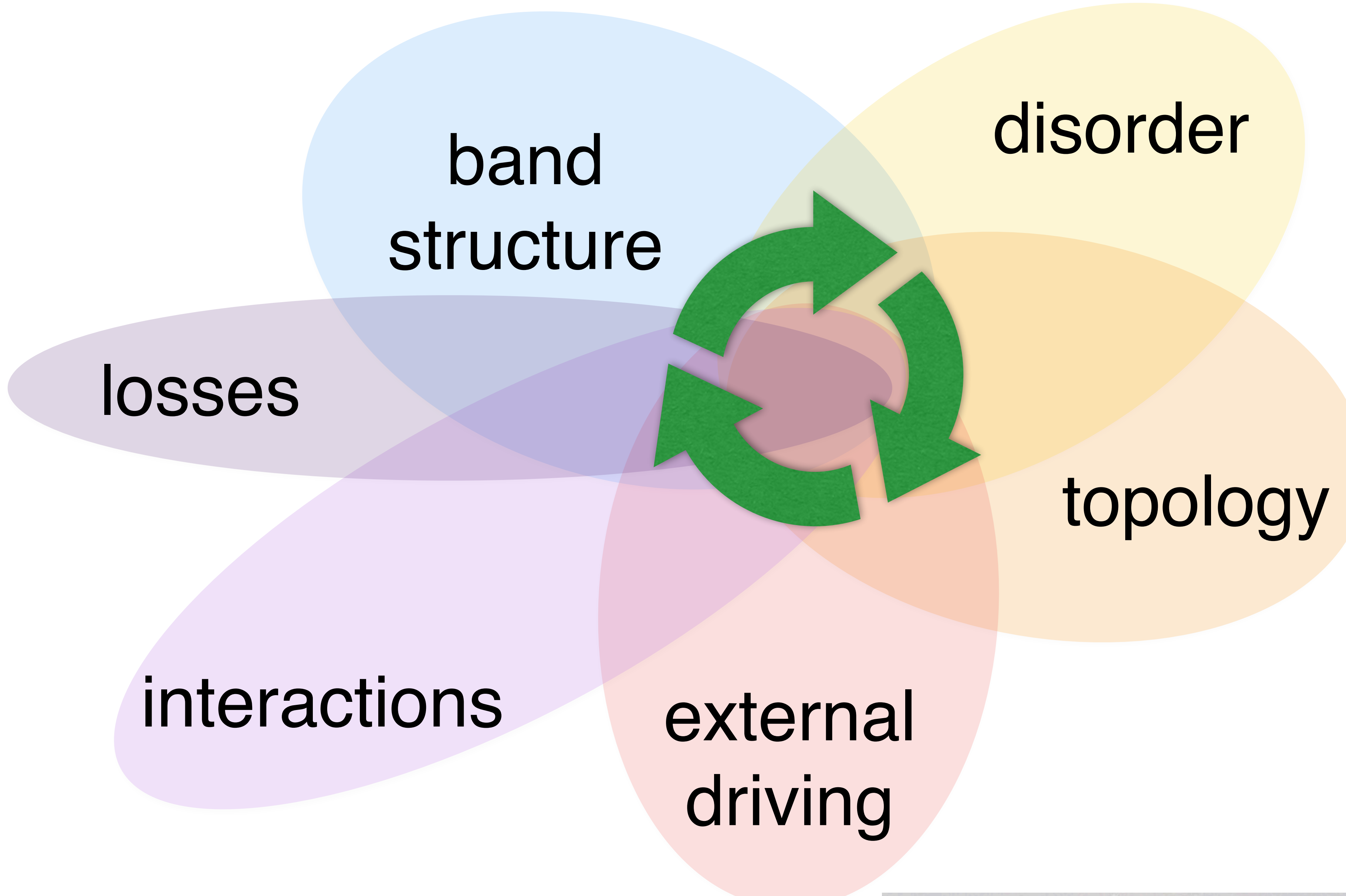
- Introduction
- One-dimensional chiral models
  - static (SSH)
  - topological Anderson transition
    (disordered atomic wires)
  - periodically-driven
    (photonic quantum walk)
- Mean Chiral Displacement (MCD)







Condensed matter



Plenty of emergent phenomena!

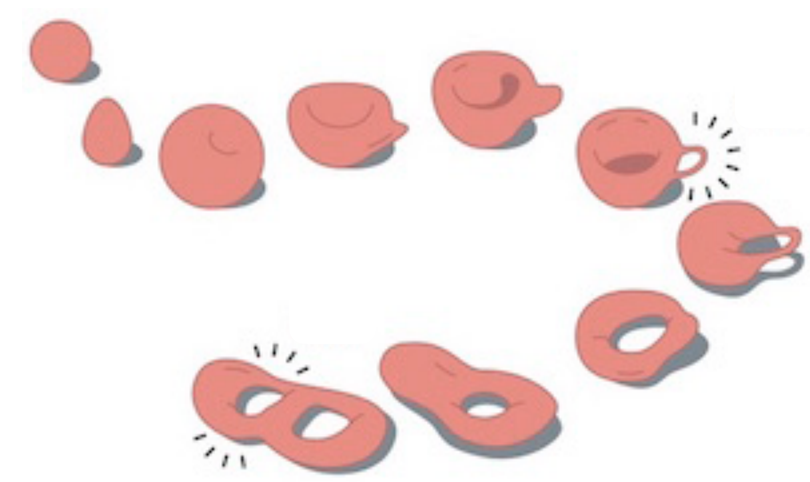
But we need to *observe* these.

E.g., how to “detect topology”?

Topology

- Geometry: classification of objects under continuous deformations

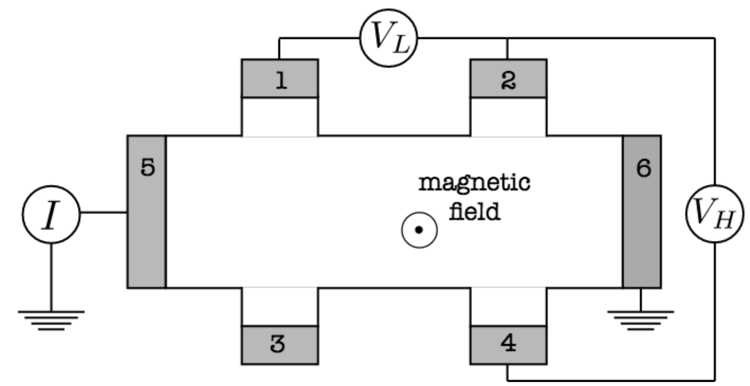
  ✓ stretch and bend
  ✘ but don't cut, puncture, or glue



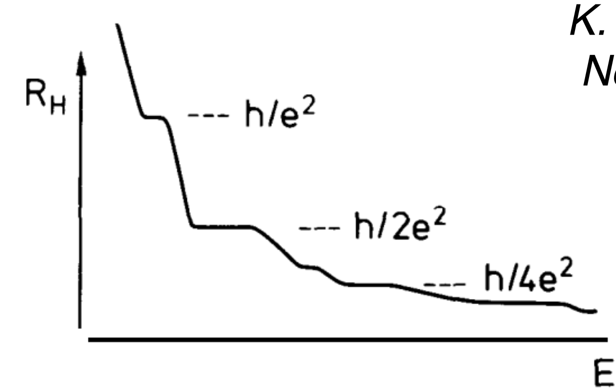
- Genus: # of holes

- Winding of a closed path:
  # of times it encircles a given point, line, …

Hall effect



- Classical Hall effect (1879):
  when current flows in a 2D material,
  in presence of an out-of-plane B field,
  there appears a transverse (Hall) current



K. Von Klitzing
Nobel lecture

- Quantum Hall effect (1980):
  at low temperatures and high-B,
  the Hall current is quantized!

- Laughlin (1982): robustness due to **topology**

- TKNN (1982): Kubo formula links conductivity to *Chern numbers*
  (topological invariants defined on the occupied bands).

Thouless, Kohmoto, Nightingale & den Nijs
Phys. Rev. Lett. (1982)

Topological insulators

- Insulators in the bulk, presenting robust current-carrying edge states
- Protected by the topology of bulk bands against local perturbations, like *disorder* and *defects*
- Enormous progresses in the last 10 years (QSH, 3D TIs., 4D QH, …)
- Characterization of non-interacting TIs in terms of discrete symmetries
  T: time-reversal
  C: charge-conjugation
  S: chiral

IQHE, Hofstadter, Chern insulators → A

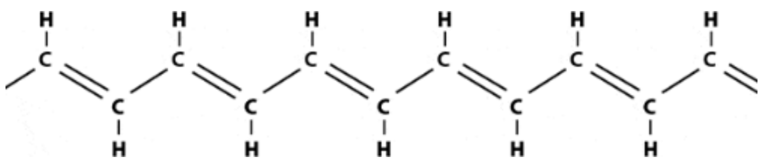
chiral → AIII, BDI

Class	T	C	S	# of dimensions 0	1	2	3	4	5	6	7
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	Chern number			$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z}$	Winding		0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Chiu, Teo, Schnyder & Ryu, Rev. Mod. Phys. (2016)

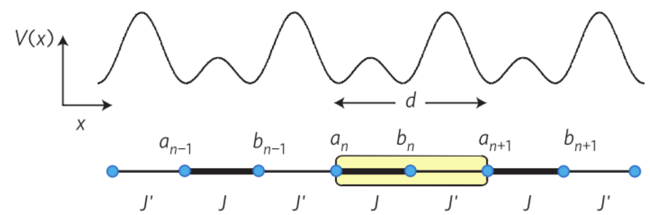
- Beyond the periodic table:
  Mott / crystalline / Anderson / Floquet TIs, …

1D chiral systems



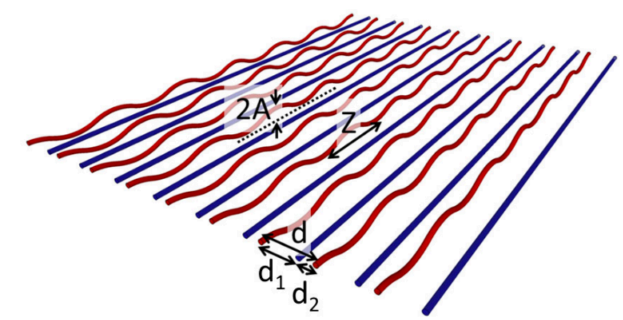
polyacetilene

[Nobel prize in Chemistry 2000]



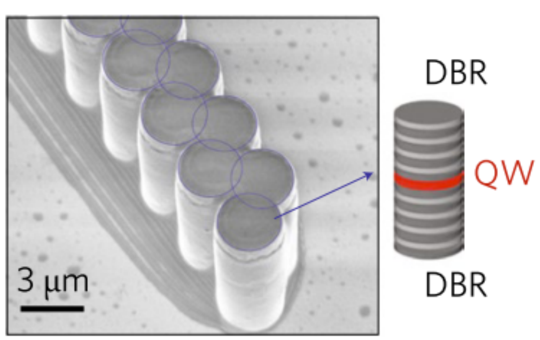
ultracold atoms
in superlattices

[M. Atala *et al.*, Nature Phys. 2013]



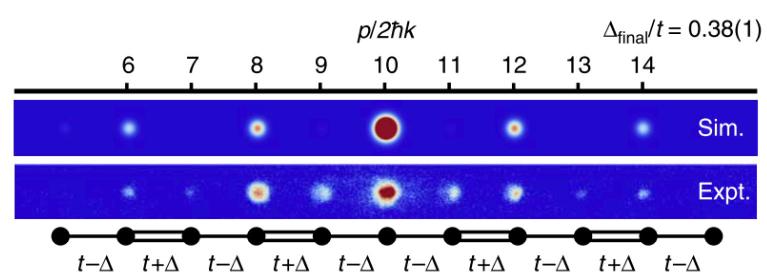
optical waveguides

[Zeuner *et al.*, PRL 2015]



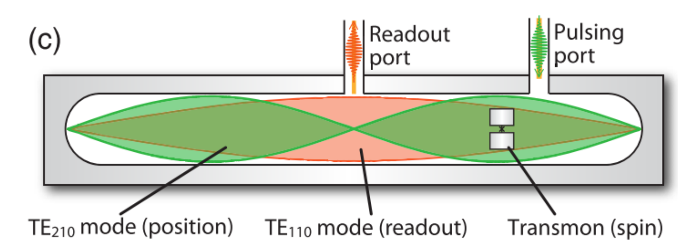
cavity polaritons

[St. Jean *et al.*, Nature Phot. 2017]



ultracold atoms
in k-space lattices

[Meier *et al.*, Nature Comm. 2016]

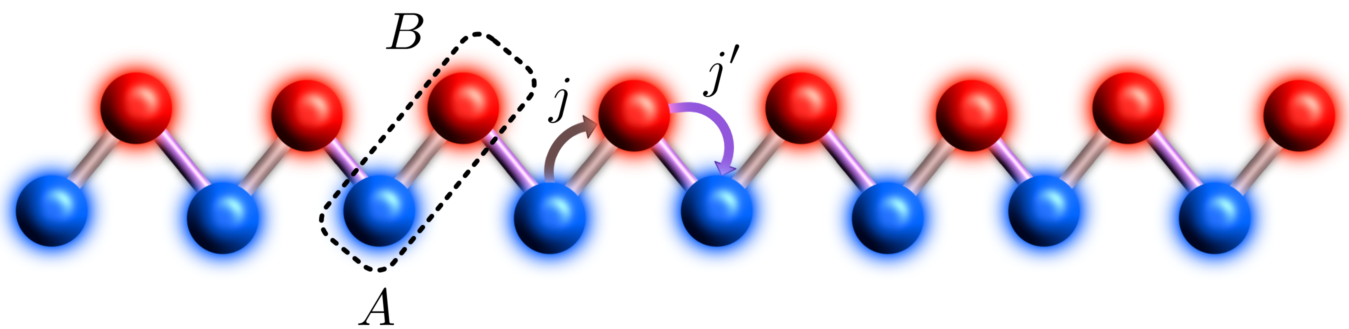


SC qubits
in mw-cavities

[Flurin *et al.*, PRX 2017]

SSH model

- Spinless fermions with staggered tunnelings:



Su, Schrieffer & Heeger
Phys. Rev. Lett. (1979)

Asbóth, Oroszlány, & Pályi
Lecture Notes in Physics (2016)

- ∃ two sublattices

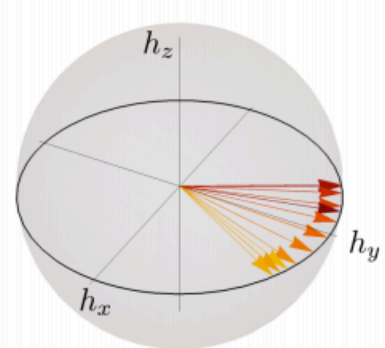
  ∃ a "canonical basis" where H is purely off-diag: $H = \begin{pmatrix} 0 & h^\dagger \\ h & 0 \end{pmatrix}$

- Chiral symmetry: $\Gamma H \Gamma = -H$ (Γ: unitary, Hermitian, local)

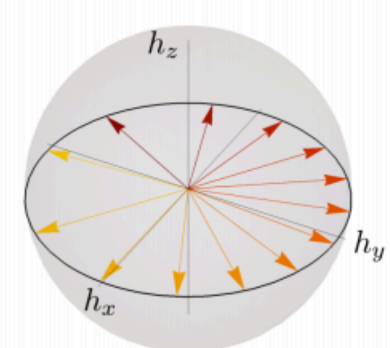
- In momentum space: $H_k = E_k \mathbf{n}_k \cdot \boldsymbol{\sigma}$

- In the canonical basis, $\mathbf{n}_k \perp \hat{\mathbf{z}}$ $\forall k$ and $\Gamma = \sigma_z$

- Winding:



$\mathcal{W} = 0$



$\mathcal{W} = 1$

The winding *W*

- $\mathcal{W}$ may be calculated:

$$H_k = E_k \mathbf{n}_k \cdot \boldsymbol{\sigma}$$

  - from n : $\mathcal{W} = \oint \frac{\mathrm{d}k}{2\pi} \left(\mathbf{n} \times \partial_k \mathbf{n}\right)_z$
  - from the *eigenstates* : $\mathcal{W} = \oint \frac{\mathrm{d}k}{\pi} \mathcal{S},$

$$\mathcal{S} = i \langle \psi_+ | \partial_k \psi_- \rangle$$

skew polarization

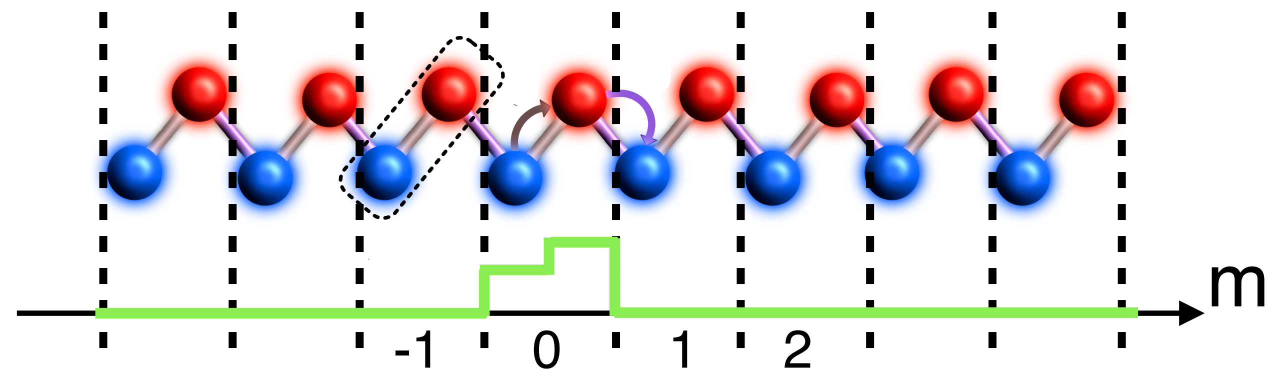
- What if the Hamiltonian is not known?
  Can one *measure* the winding?

  Yes, and it's simple!

Evolution in real time

- Initial condition

  localized on the $m=0$ cell:



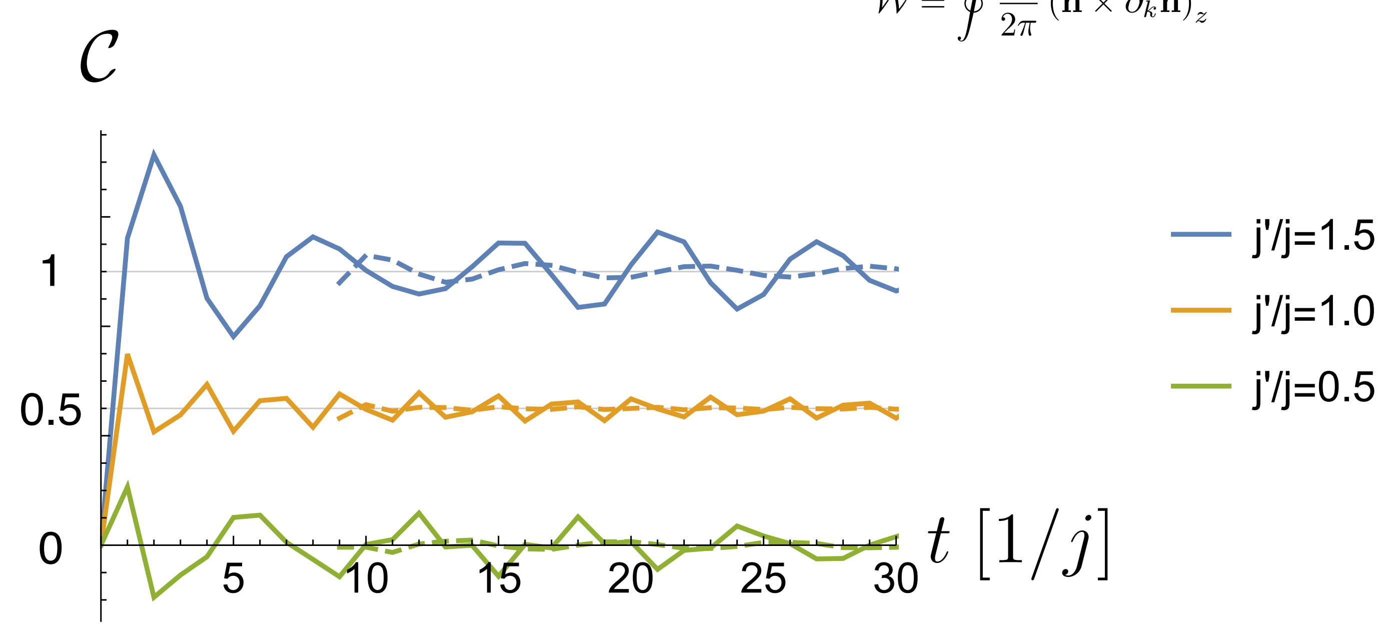
- **Mean Chiral Displacement:** $\mathcal{C}(t) \equiv 2\langle\widehat{\Gamma m}(t)\rangle = 2\Big[\langle m_A(t)\rangle - \langle m_B(t)\rangle\Big]$

$$\mathcal{C}(t) = 2\int_{-\pi}^{\pi}\frac{\mathrm{d}k}{2\pi}\,\langle U^{-t}\sigma_z(i\partial_k)U^t\rangle_{\psi_0} = 2\int_{-\pi}^{\pi}\frac{\mathrm{d}k}{2\pi}\sin^2(Et)\,|\mathbf{n}\times\partial_k\mathbf{n}| \quad \xrightarrow{t\to\infty} \quad \mathcal{W}$$

$$\mathcal{W} = \oint\frac{\mathrm{d}k}{2\pi}(\mathbf{n}\times\partial_k\mathbf{n})_z$$

- *Bulk* measurement

- Fast convergence



Cardano, D'Errico, Dauphin, Maffei, … Marrucci, Lewenstein & PM
Nature Comm. (2017)

Resistance to disorder

SSH model in the topological phase $\quad j' = 2j \quad \rightarrow \quad \begin{cases} \mathcal{W} = 1 \\ \Delta_{\text{gap}} = 2j \end{cases}$

+

independent disorder of amplitude Δ on ***all*** tunnelings

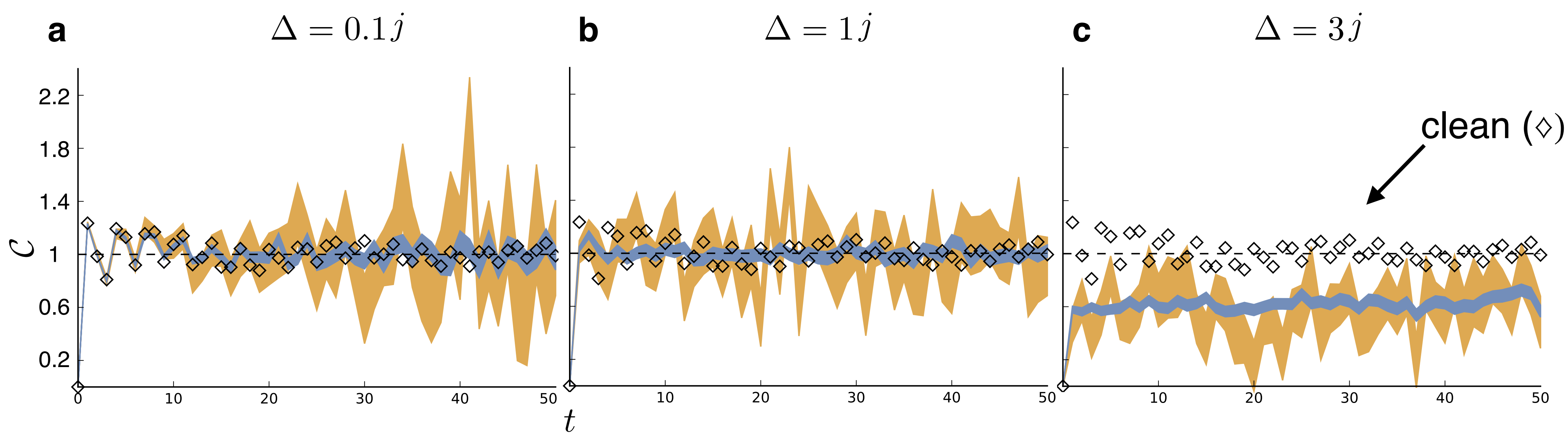
+

localized initial condition (randomly-polarized)

+

average over 50 (1000) disorder realizations

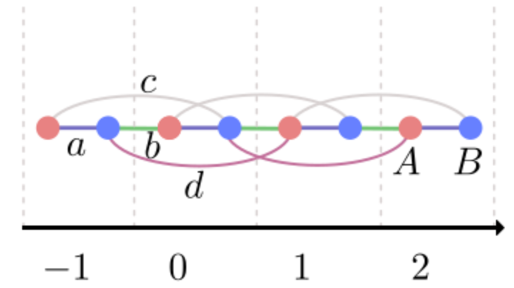
⬇

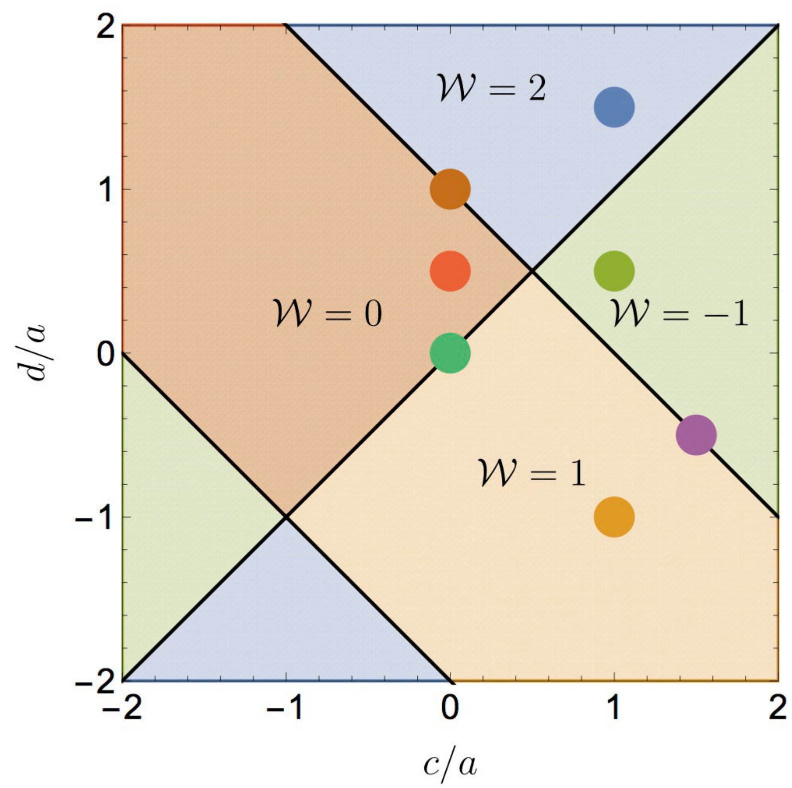


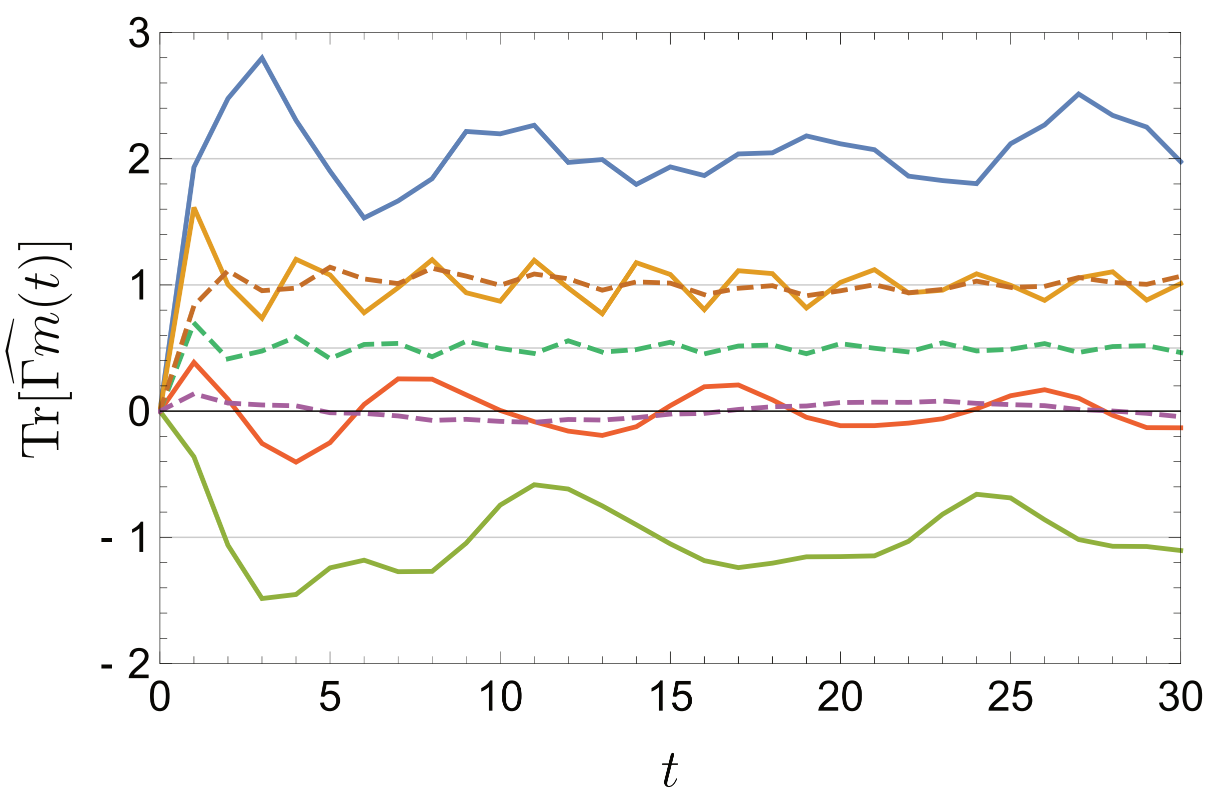
the MCD stays locked to the topological invariant as long as $\Delta < \Delta_{\text{gap}}$

Higher windings

- Extension to long-ranged models:



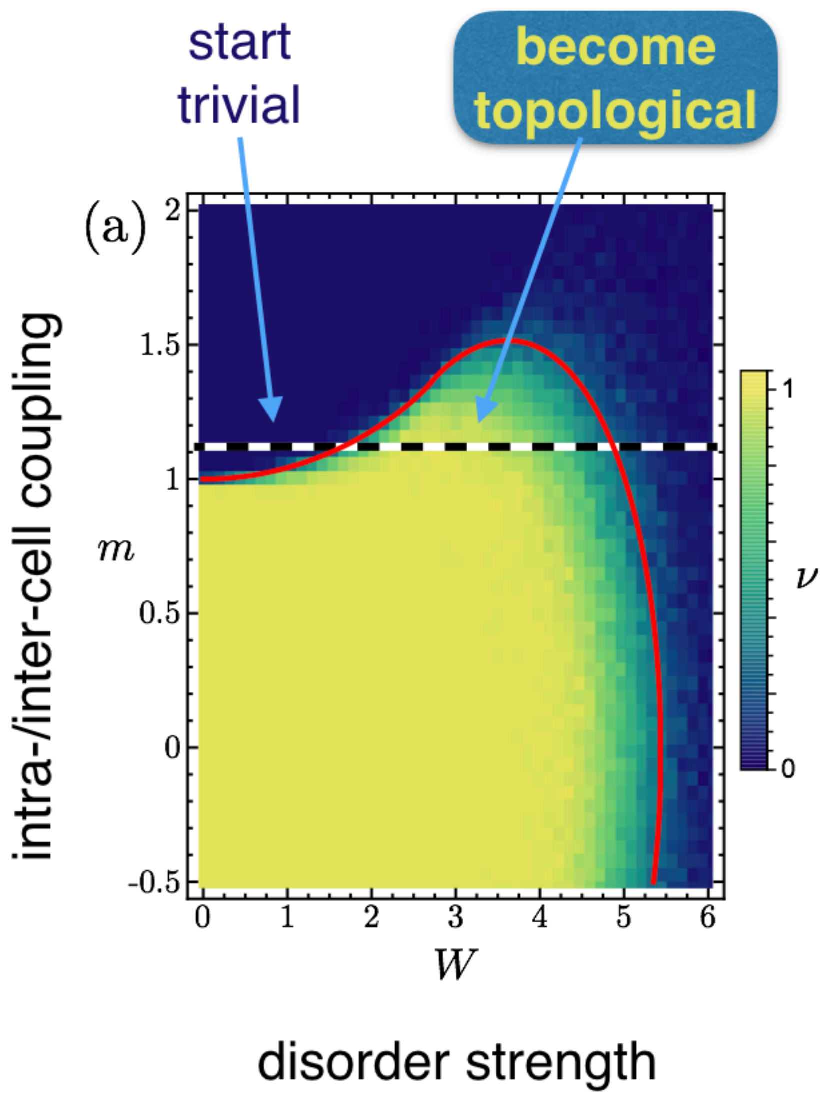




- At critical boundaries: MCD converges to the mean of the winding in the neighboring phases

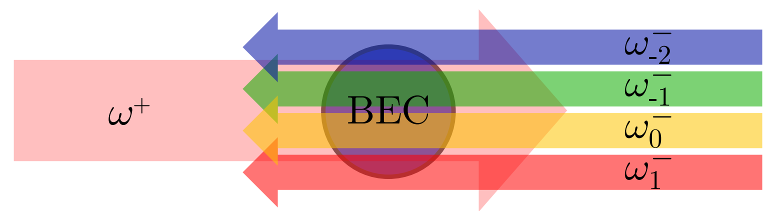
Maffei, Dauphin, Cardano, Lewenstein & PM
New J. Phys. 2018

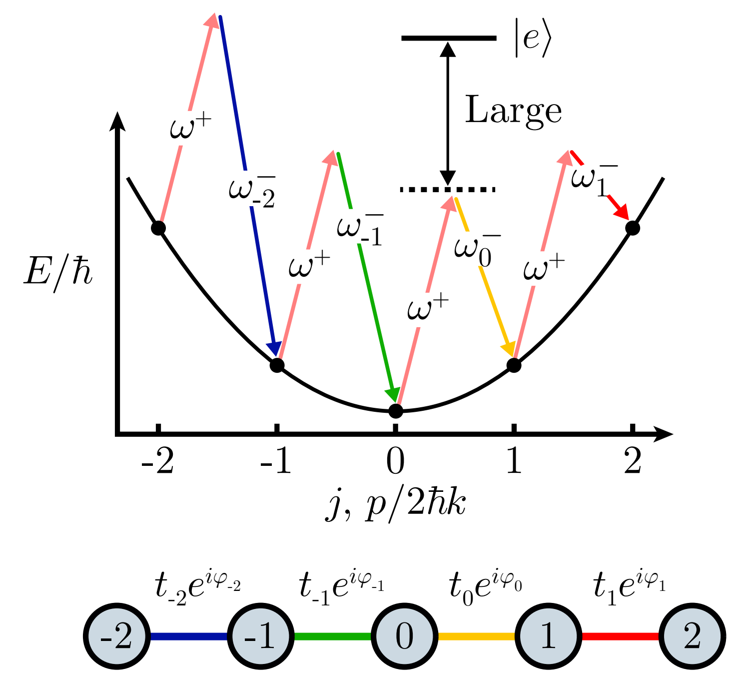
Topological Anderson insulator



Meier, An, Dauphin, Maffei, PM, Taylor and Gadway, arXiv:1802.02109

Atomic wires





- Atomic, ~ideal BEC

- Laser-driven coupling of discrete-momentum states

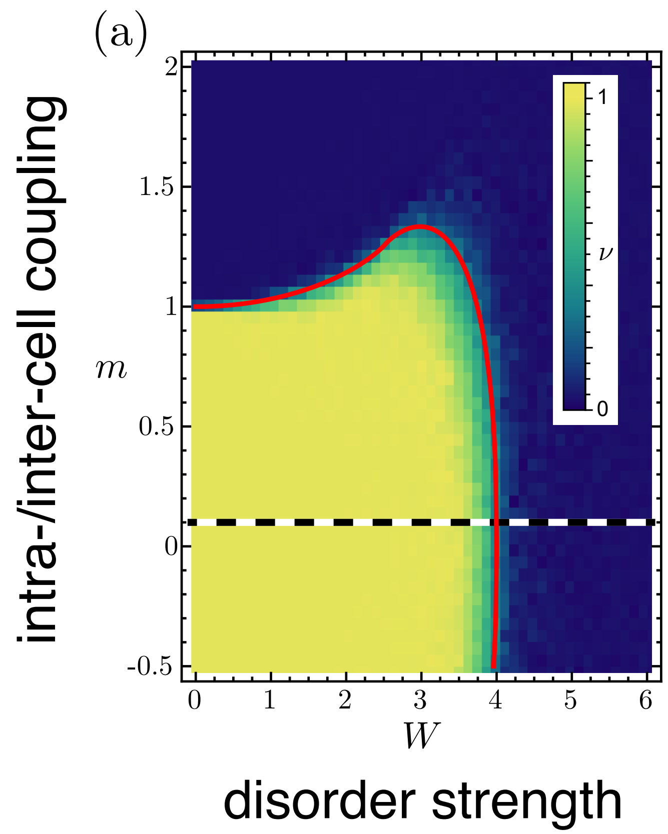
$$H_{\text{eff}} \approx \sum_j t_j (e^{i\varphi_j} |\widetilde{\psi}_{j+1}\rangle\langle\widetilde{\psi}_j| + \text{h.c.})$$

- 1D Hubbard model with full control on each tunneling strength and phase

- Built-in chiral symmetry

Detecting topology

- A topological wire becomes trivial by adding disorder

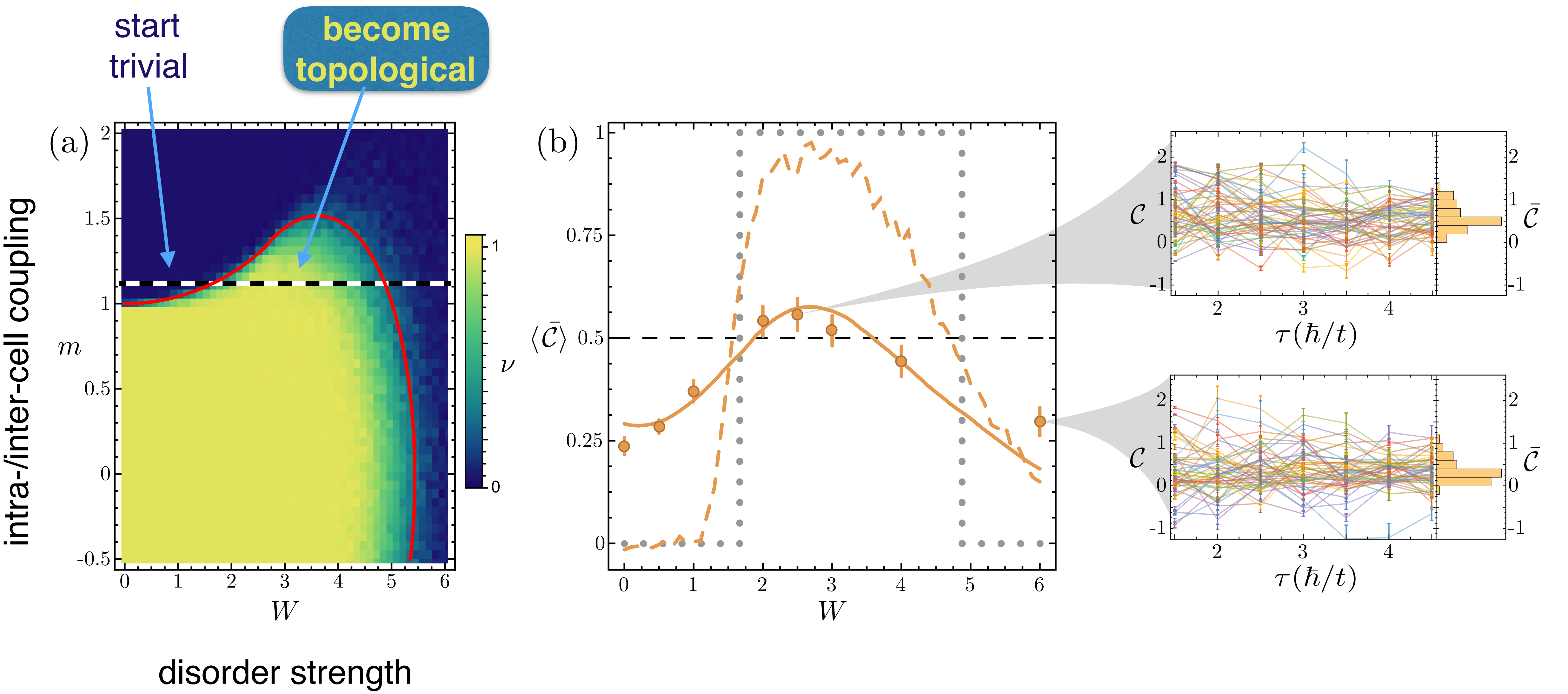


color map: real-space computation of the winding

red line: critical boundary (diverging localization length)

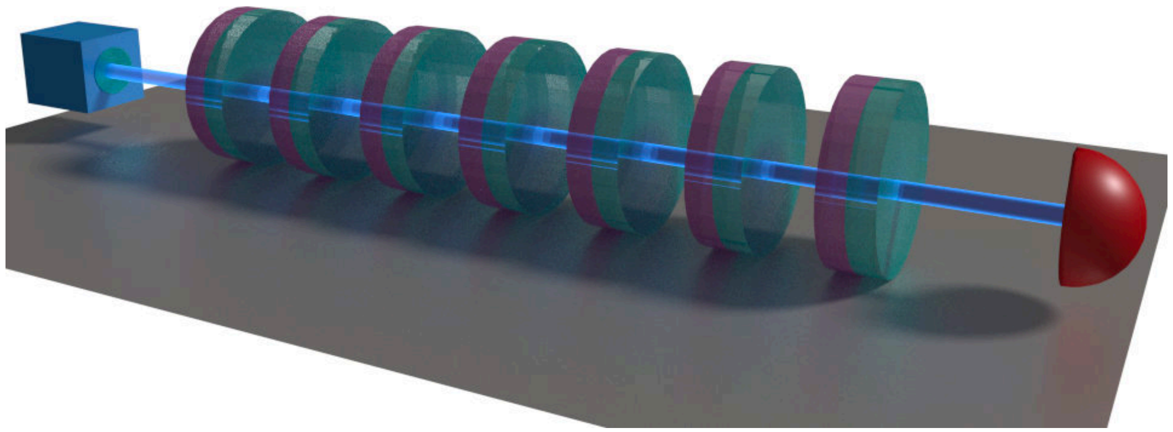
Topological Anderson transition

A trivial wire is driven into the topological phase by adding disorder



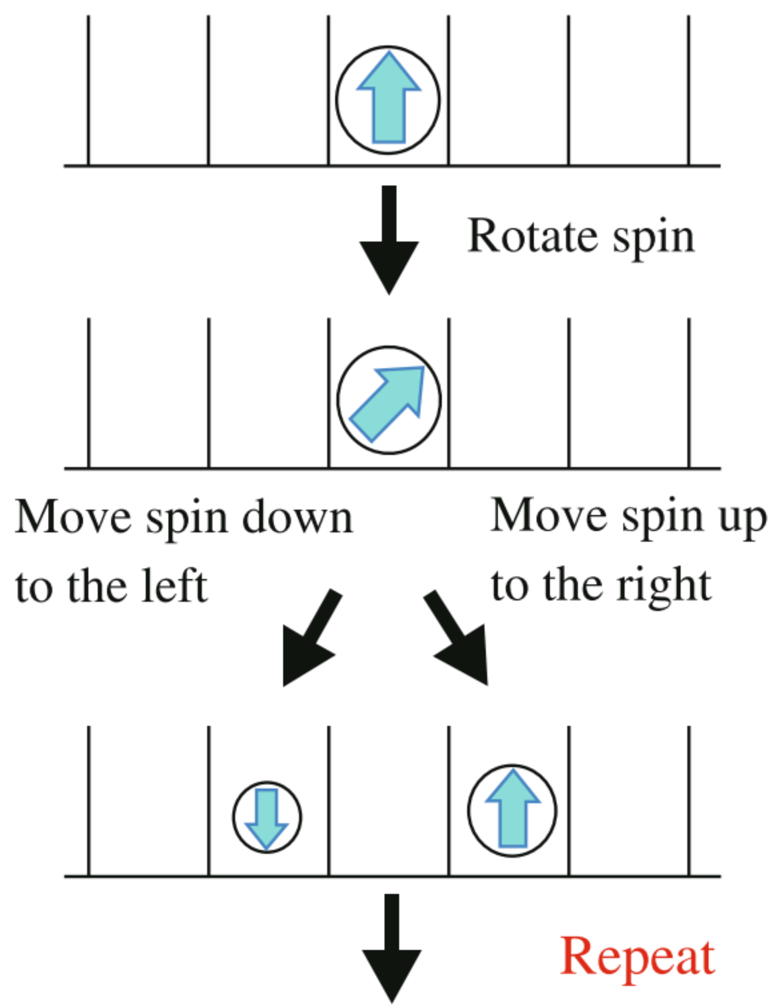
Meier, An, Dauphin, Maffei, PM, Taylor and Gadway,
arXiv:1802.02109

Floquet 1D chiral models



photonic quantum walk of *twisted* photons

Discrete-Time Quantum Walk

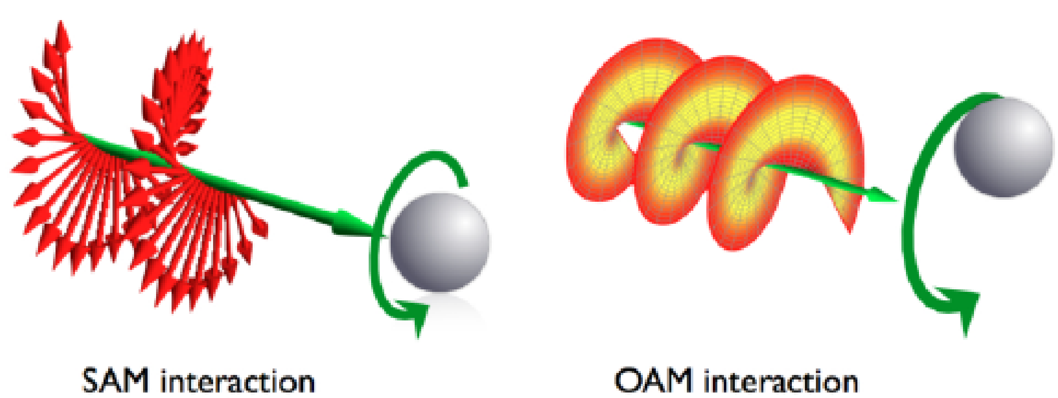


[Kitagawa, QIP (2012)]

Twisted photons

25[th] anniversary: Allen et al., PRA (1992)

- Collimated monochromatic beam propagating along $\hat{\mathbf{z}}$
- Light has linear momentum $\mathbf{p} \propto \mathbf{E}^* \times \mathbf{B}$ ("push")
- But it can also carry also *angular momentum*
- In the "paraxial approximation", $\hat{J}_z = \hat{S}_z + \hat{L}_z$
- "Spin" AM: $\hat{S}_z = \hbar \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
- Orbital AM: $\hat{L}_z = -i\hbar(\mathbf{r} \times \boldsymbol{\nabla})_z$

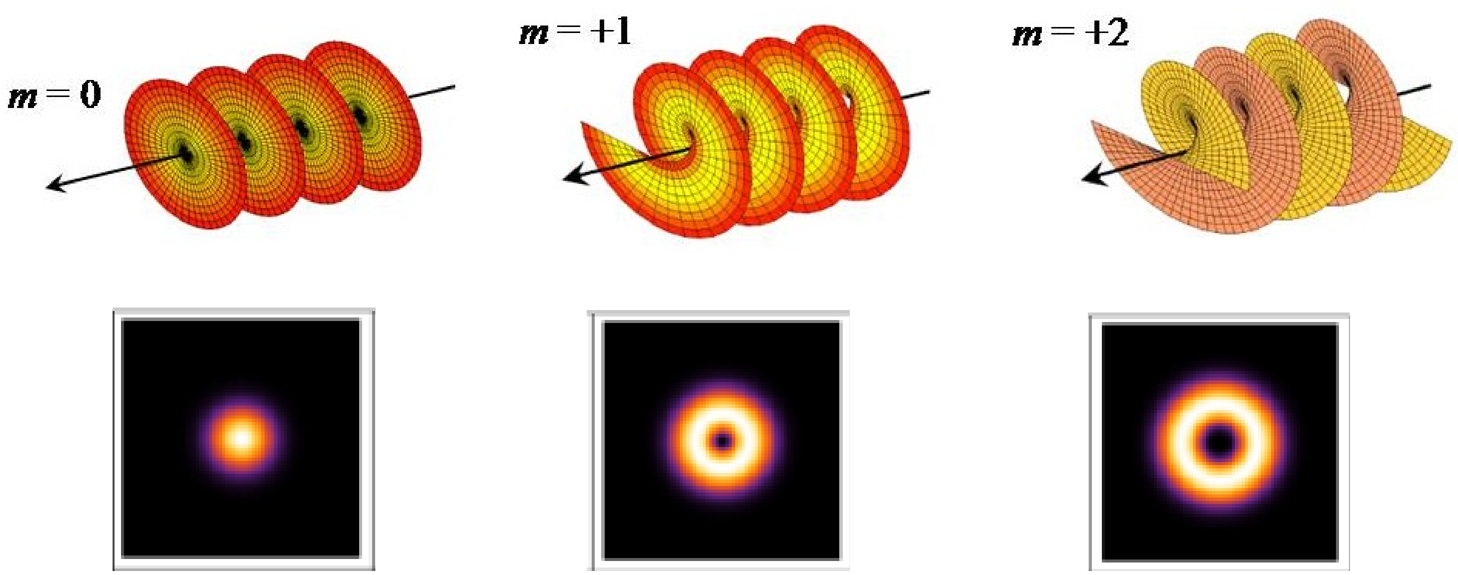


circularly polarized light interacts with the particle's spin

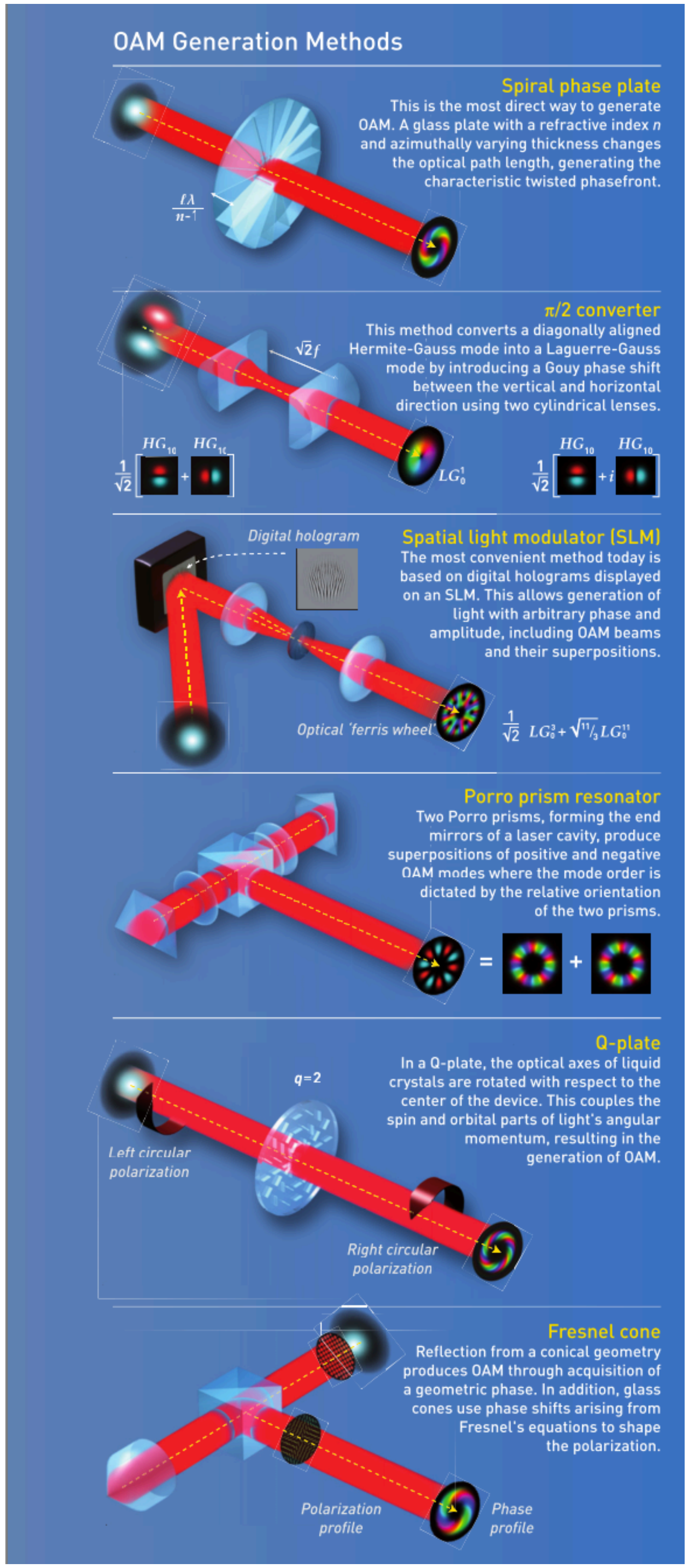
light with OAM rotates a particle around the beam axis

Twisting light

- Helical modes have a phase pattern $e^{im\phi}$
- Their OAM is quantized: $\hbar m$



Franke-Allen & Radwell
Optics&Photonics News (2017)

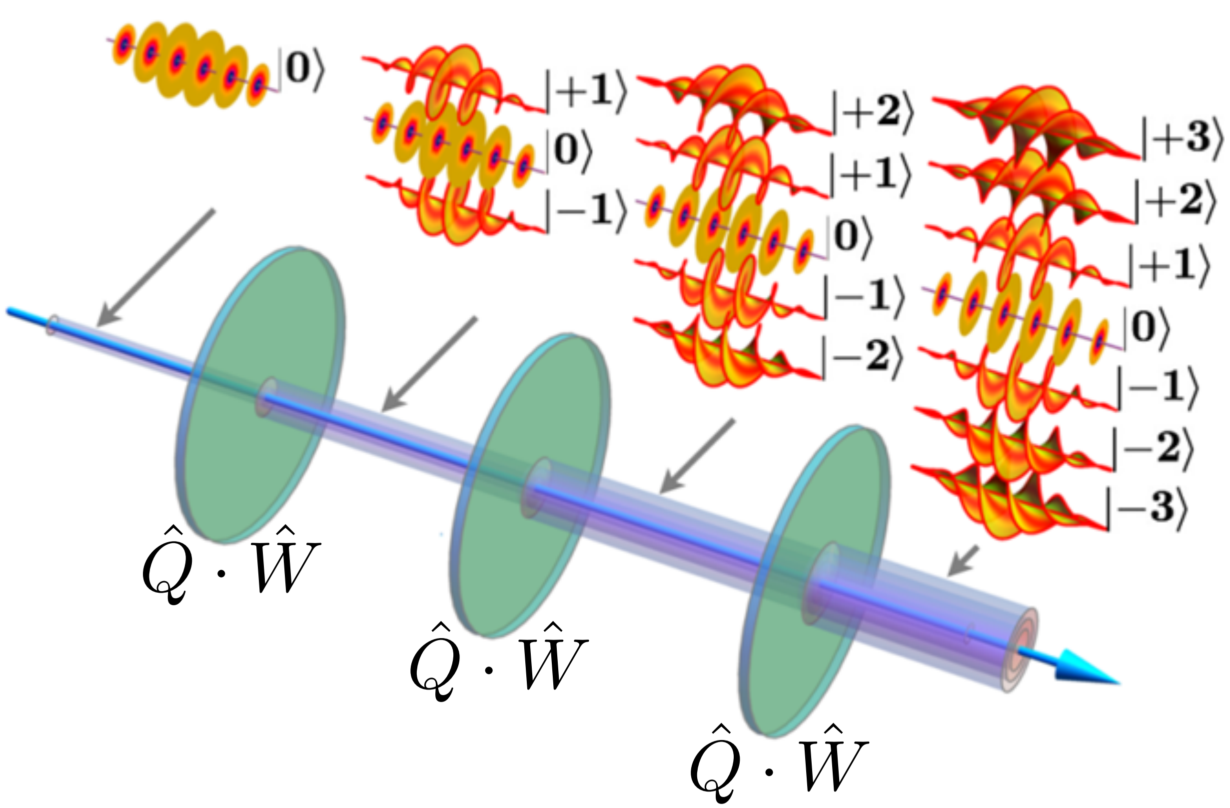


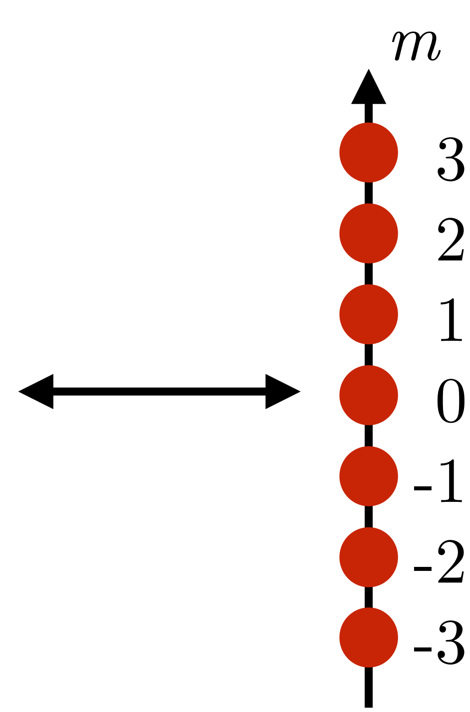
Discrete-Time Quantum Walk with twisted photons

- Cascade of Q-plates and quarter-wave plates W

$$\hat{W} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

discrete-time QW	Twisted photons
walker's position	OAM (m)
coin state (↑/↓)	polarization (↻/↺)
spin rotation	W-plate
conditional displacement	Q-plate
time	$\hat{\mathbf{z}}$

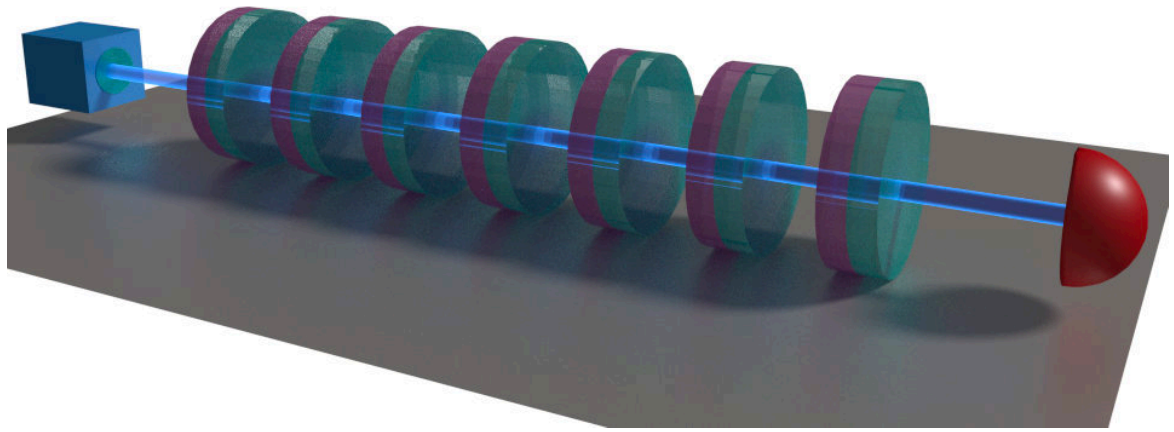


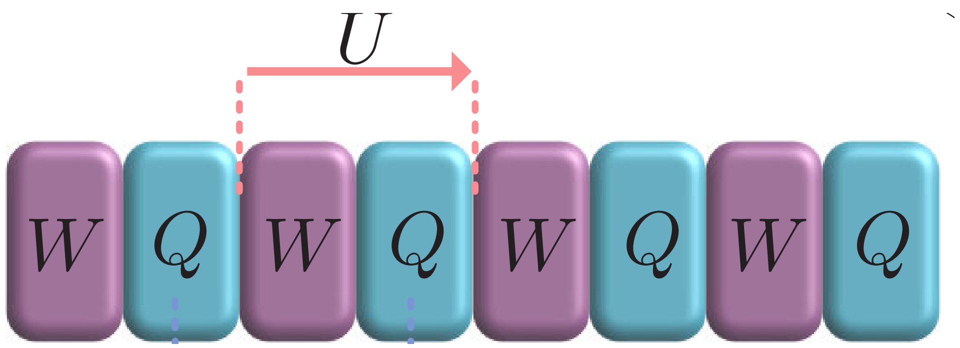


a synthetic dimension!

[Cardano et al., Science Advances (2015)]

Discrete-Time Quantum Walk



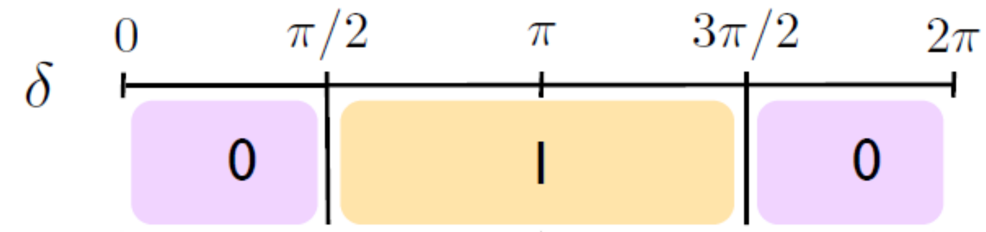


- Periodic evolution: may be treated via Floquet theory
- One-step evolution operator U ➞ $H_{\rm eff} \equiv \frac{i}{T} \log U$
- In momentum space: $H_{\rm eff}(k) = E_k \hat{\mathbf{n}}_k \cdot \boldsymbol{\sigma}$
- The spectrum of $H_{\rm eff}$ is 2π-periodic (quasi-energies E_k)
- T+C+S symmetries: BDI class —> same invariant as the static SSH model

Cardano, D'Errico, Dauphin, Maffei, … Marrucci, Lewenstein & PM
Nature Comm. (2017)

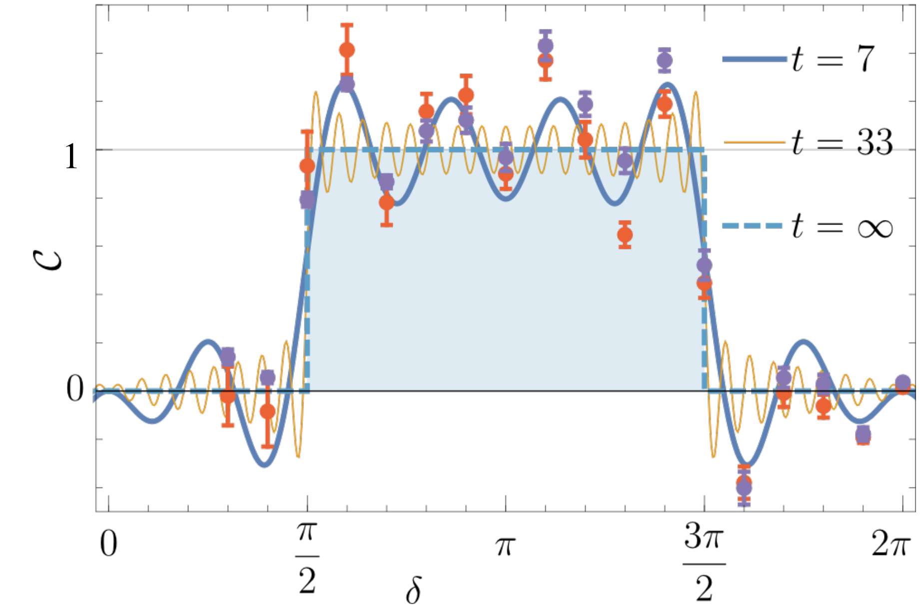
Detecting the invariant

- Winding: $\mathcal{W} = \oint \frac{\mathrm{d}k}{2\pi} \left(\mathbf{n} \times \partial_k \mathbf{n}\right)_z$



- Experimental measurement of the MCD after 7 timesteps of the DTQW with twisted photons:

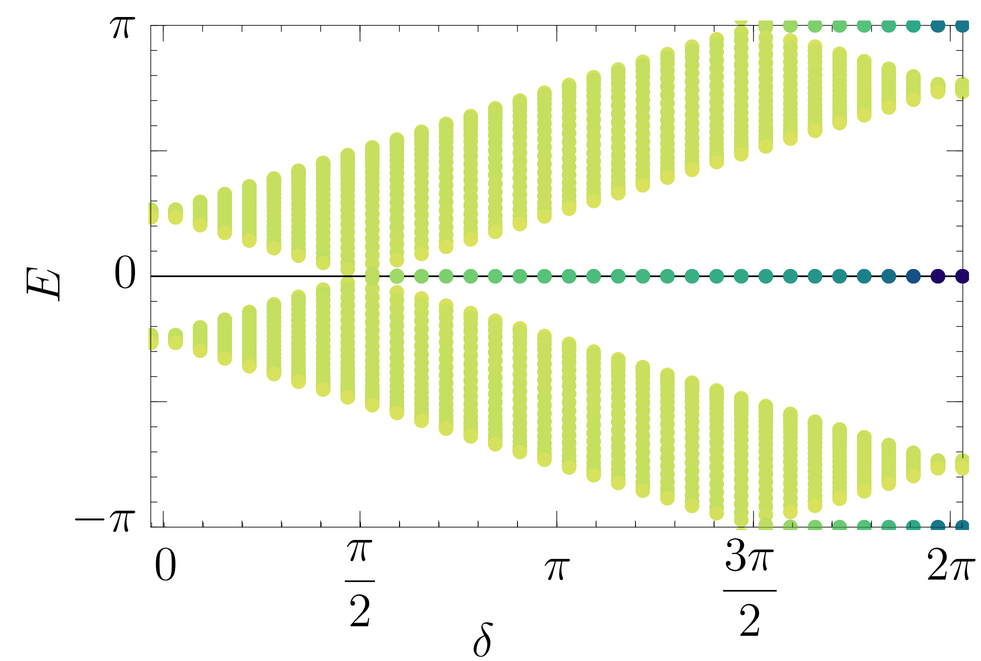
(●/●): different initial polarizations



- Check bulk-boundary correspondence

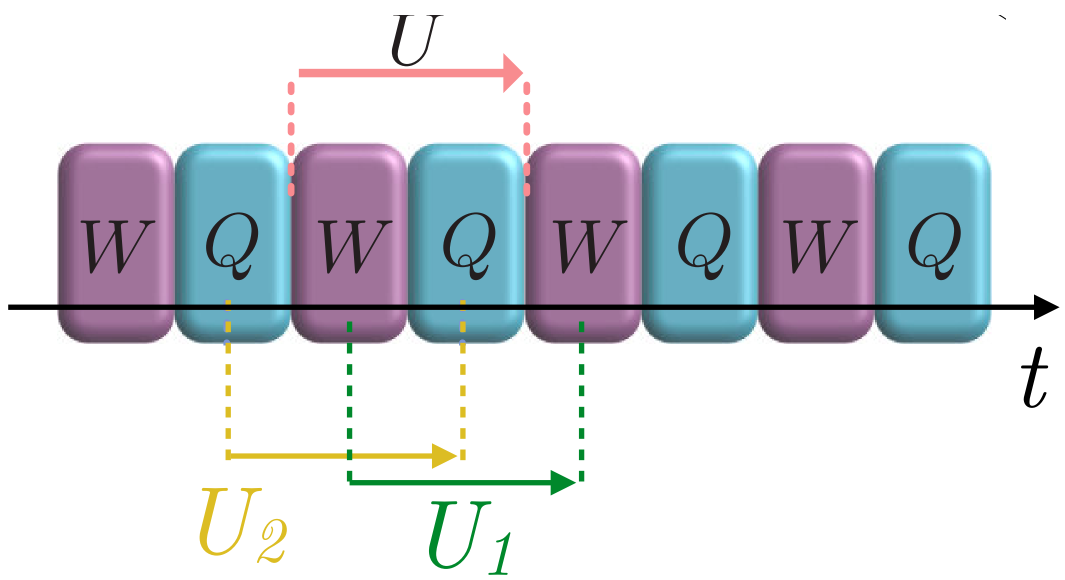
- Spectrum with edges:

darker colors: “edgier” states



- Bulk-boundary correspondence violated?

Timeframes

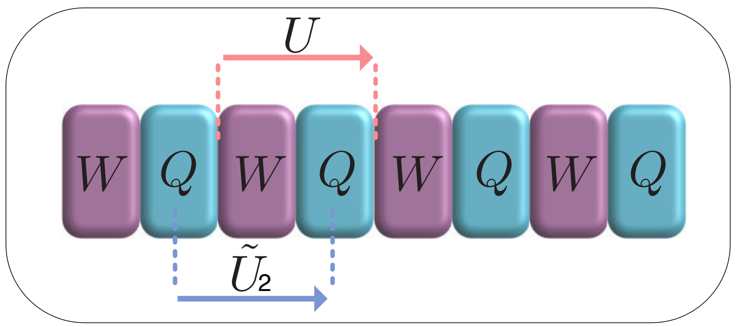


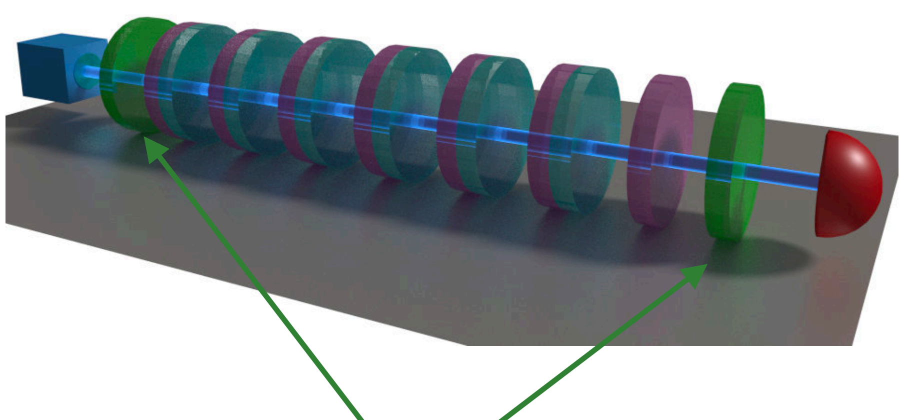
- Different initial t_0 lead to different U
- Eigenvalues of H_{eff} don't depend on t_0
- Eigenstates instead do! And so does the winding: $\mathcal{W} = \mathcal{W}_1 \neq \mathcal{W}_2$
- Timeframes invariant under time-reflection (U_1 and U_2) are special
- # of 0-energy edge states: $C_0 = (\mathcal{W}_1 + \mathcal{W}_2)/2$
- # of π-energy edge states: $C_\pi = (\mathcal{W}_1 - \mathcal{W}_2)/2$

[Asboth and Obuse, PRB (2013)]

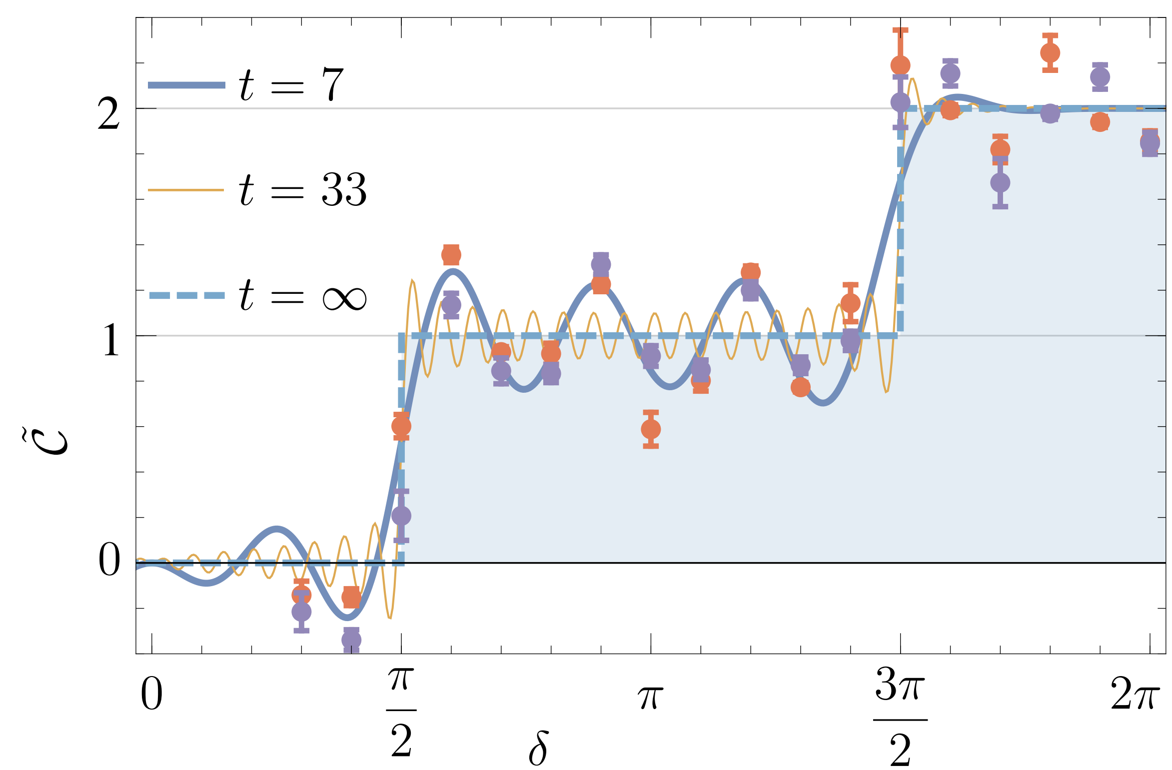
Winding in an alternative timeframe

Measurement of the MCD with protocol U_2:





$$\sqrt{Q_\delta} = Q_{\delta/2}$$

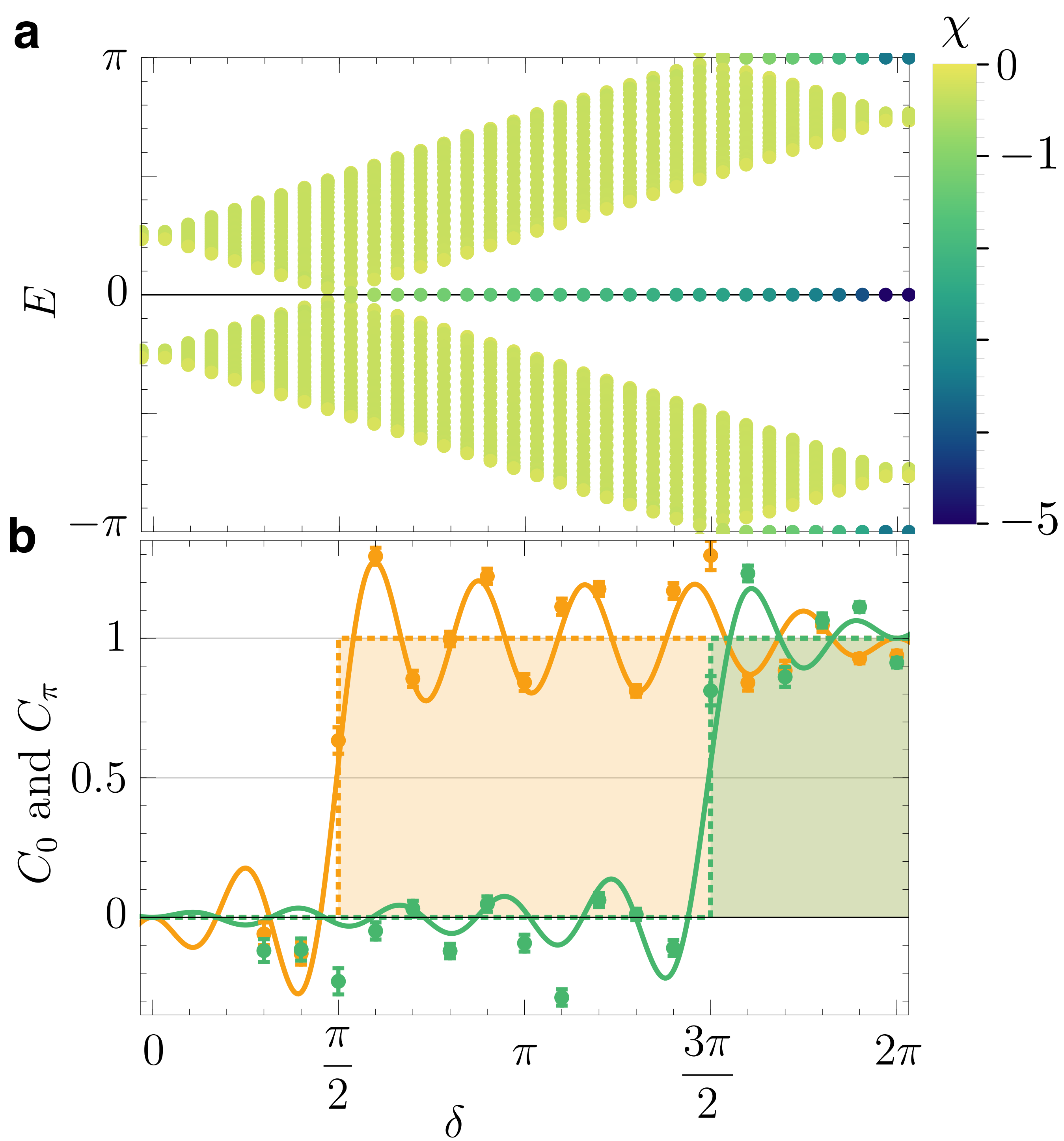


(●/●): different initial polarizations

Bulk-boundary correspondence

Theory:

Measurements:



Conclusions

- The *mean chiral displacement* captures the winding of 1D chiral systems (static, periodically driven, and disordered)
- Detection of MCD is *simple, rapid, and robust*
- Experimental observation of a **topological Anderson transition**
- Complete **topological characterization of Floquet systems** by studying *different timeframes*

- Dynamical observables for *other topological classes*?
- Extension to interacting systems?

Cardano *et al.*, Nature Comm. 2017
Maffei *et al.*, New J. Phys. 2018
Meier *et al.*, arXiv 2018

Thank you!