The fate of an impurity in a Fermi gas: polaron or molecule?

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Many-body systems

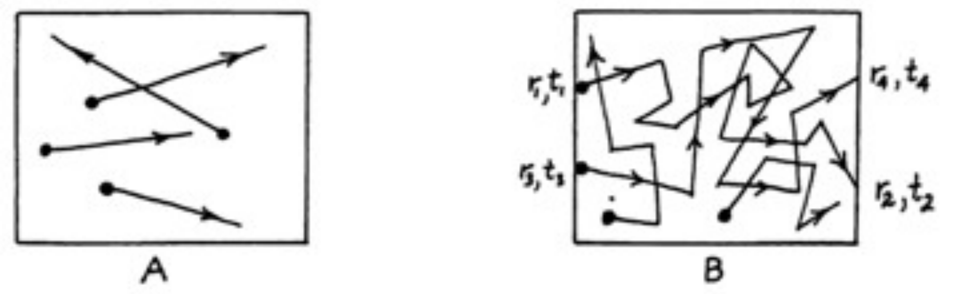


Fig. 0.2 A. *Non-interacting Particles*
B. *Interacting Particles*

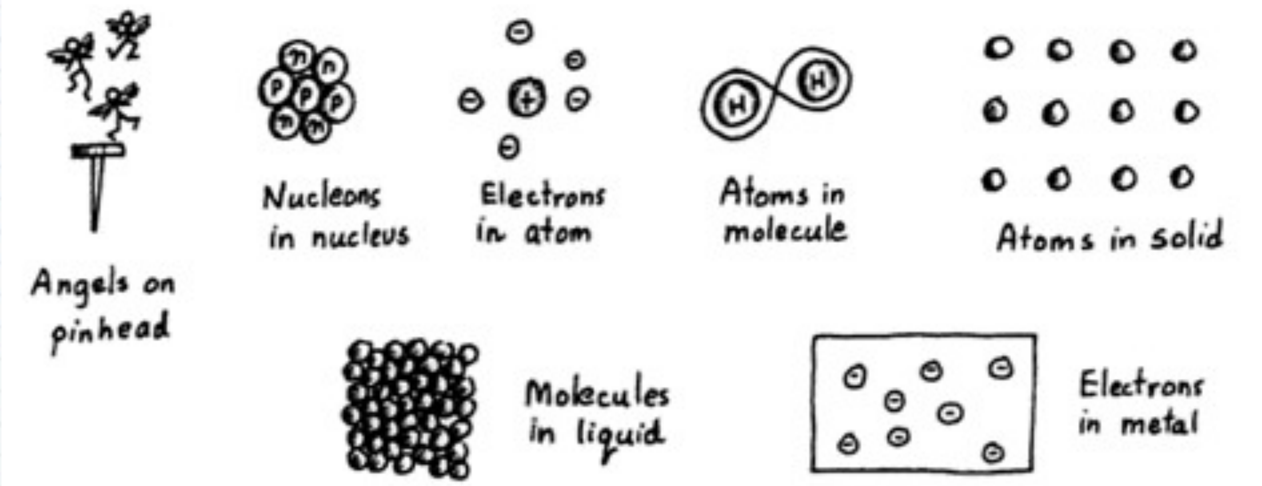
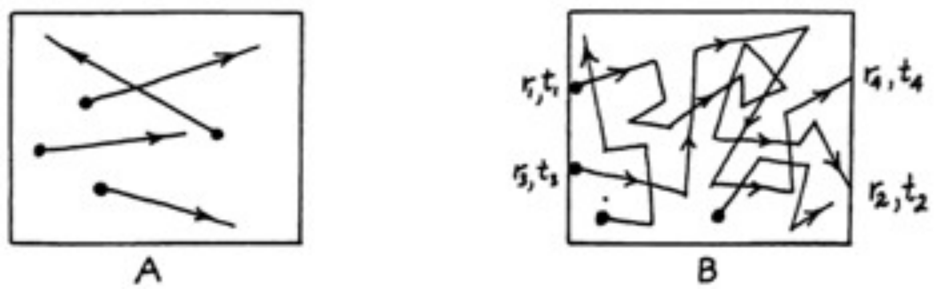


Fig. 0.1 *Some Many-body Systems*

(from Richard Mattuck's book)

Many-body systems



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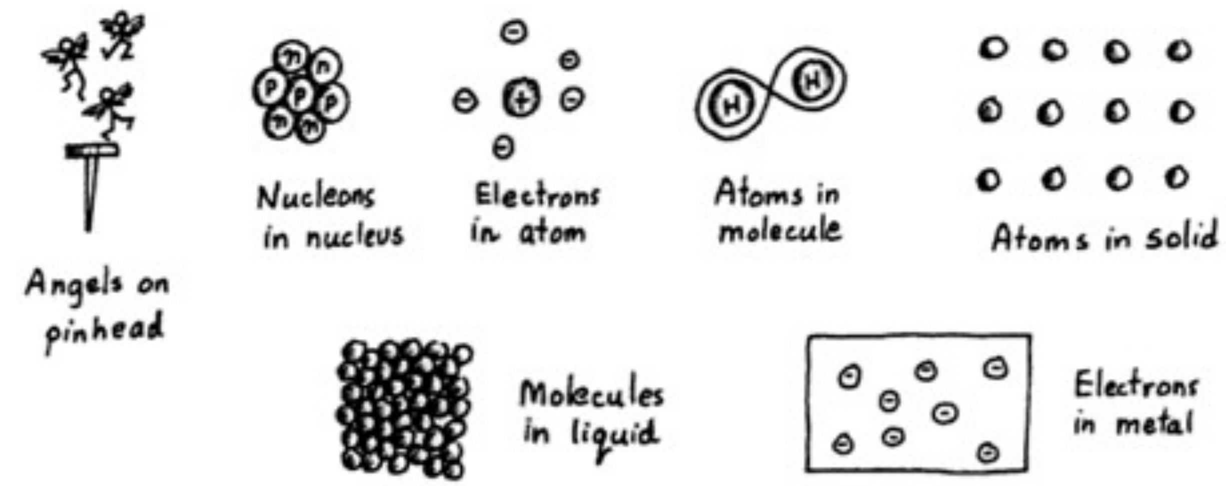
2 A GUIDE TO FEYNMAN DIAGRAMS [0.0



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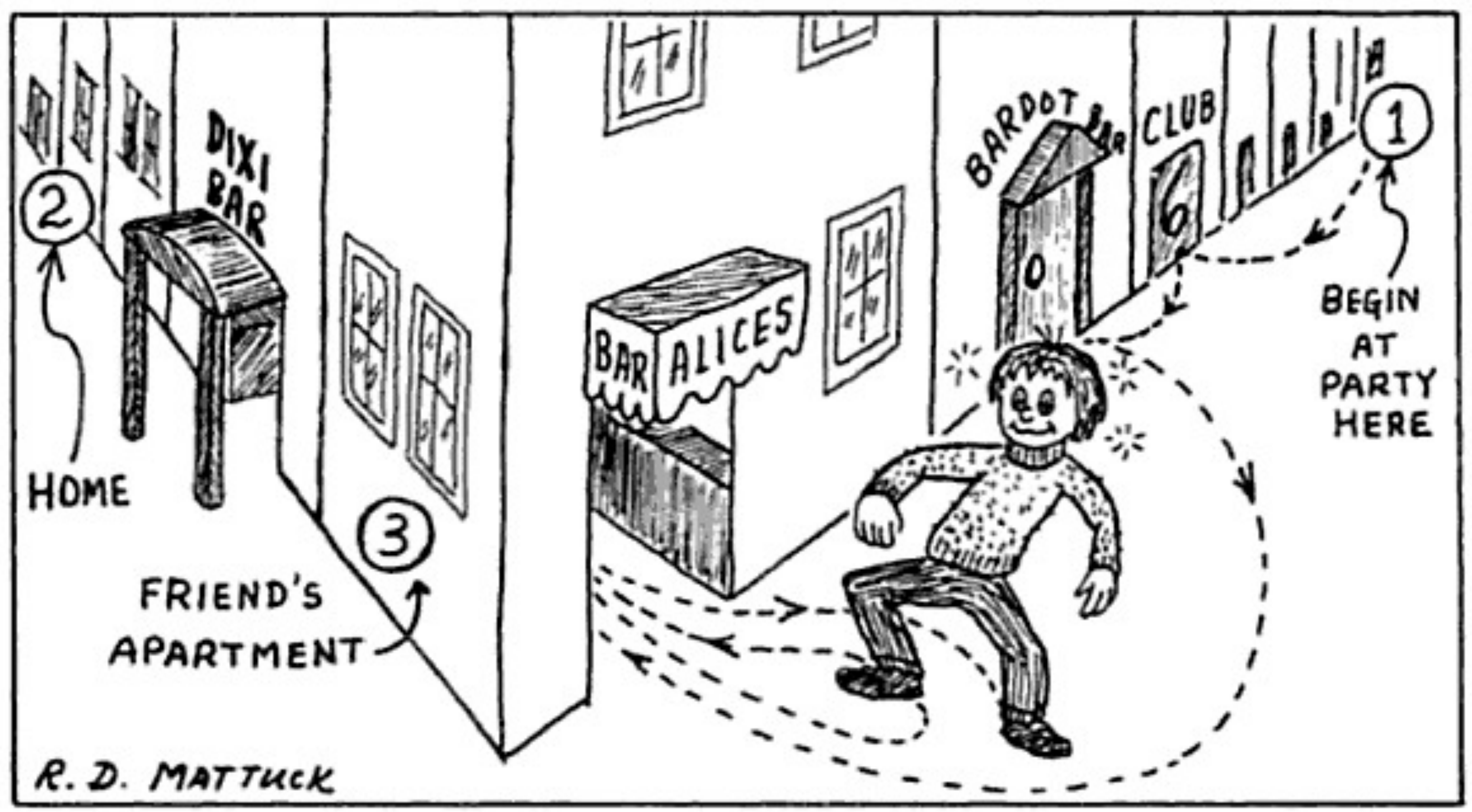


Fig. 1.1 *Propagation of Drunken Man*

Quasi-Particles

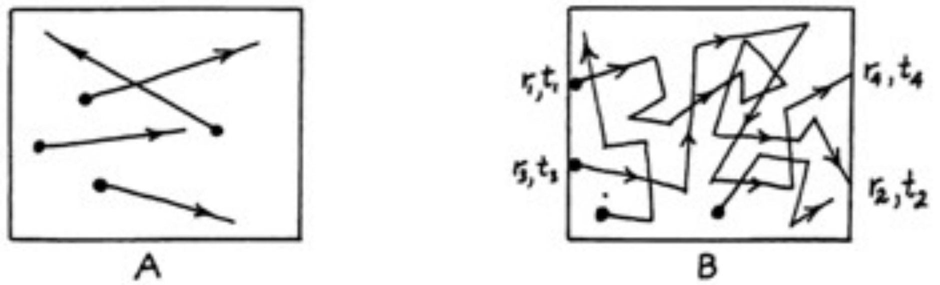


Fig. 0.2 A. *Non-interacting Particles*
B. *Interacting Particles*

Landau's idea:
who cares about real particles?

Excitations behave
as quasi-particles!

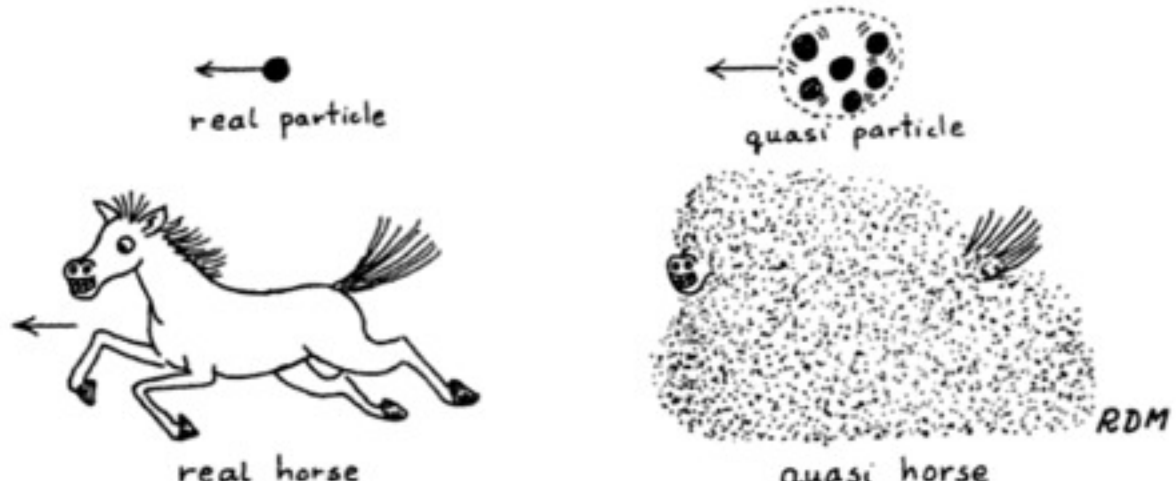


Fig. 0.4 *Quasi Particle Concept*

a QP is a "free particle" with:

@ renormalized mass
@ self-energy
@ shielded interactions
@ lifetime

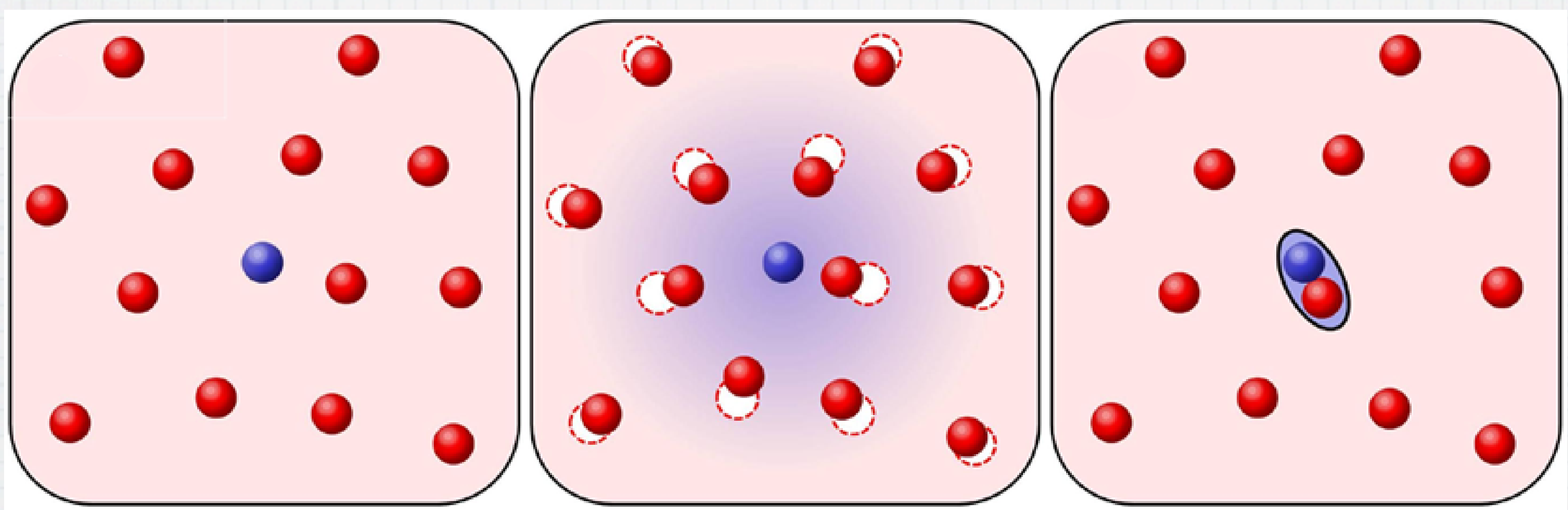
The impurity problem

- non-interacting ↑ Fermi sea (N>>1)
- a single ↓ impurity

Attraction strength

BCS ⟶ BEC

$(k_F a)^{-1} < 0$



$(k_F a)^{-1} > 0$

free particle | QP (polaron) | molecule

Polaron Ansatz

(F. Chevy, PRA 2006)

the ↓ impurity

$$|\psi_{\mathbf{p}}\rangle = \phi_0 |\mathbf{p}\rangle_\downarrow |0\rangle_\uparrow + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{qk}} |\mathbf{p}+\mathbf{q}-\mathbf{k}\rangle_\downarrow \, c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{q}\uparrow} |0\rangle_\uparrow$$

non-interacting ↑ Fermi sea

Particle-Hole dressing

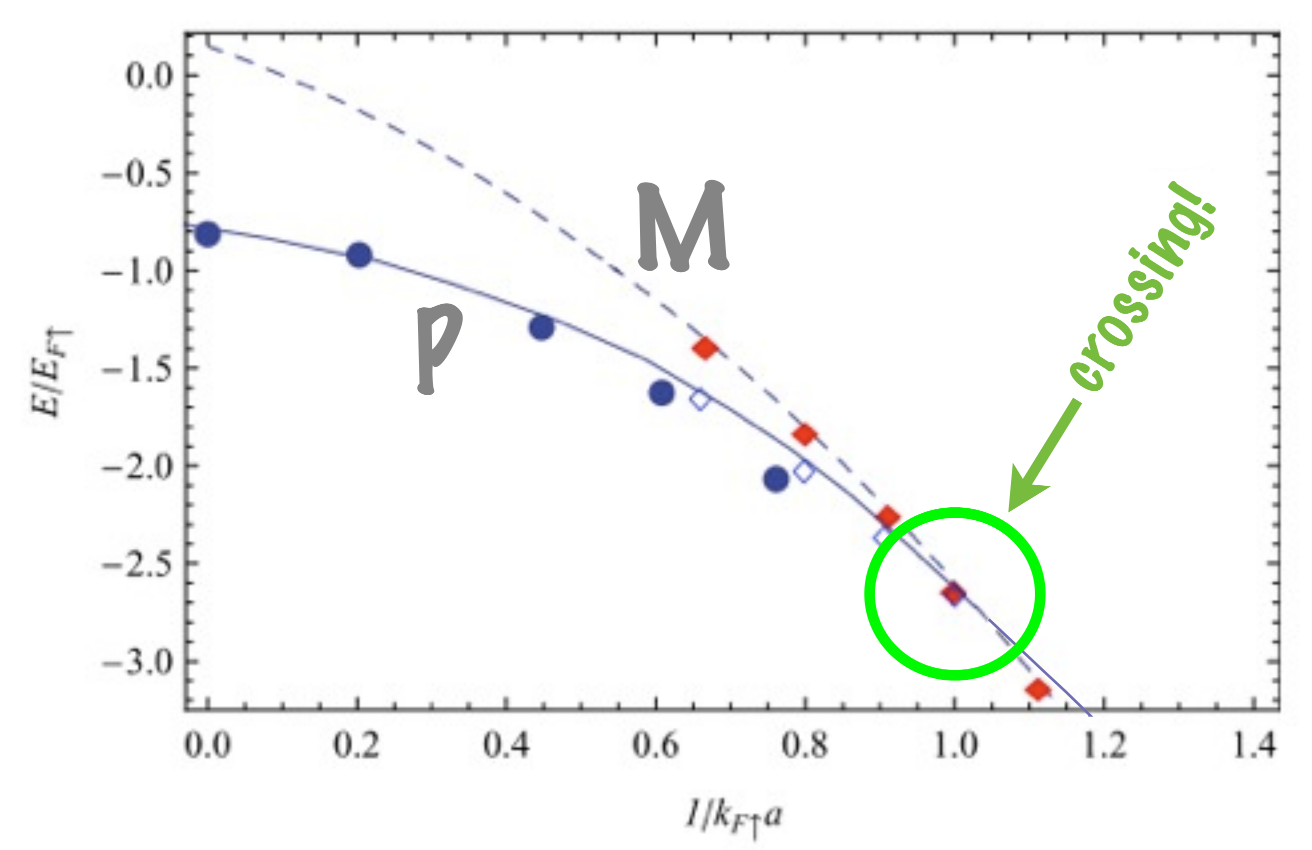
This Ansatz gives a very good agreement with MC results for the energy and m*, even at unitarity.

The variational treatment has a diagrammatic equivalent.
It corresponds to the forward scattering, or ladder, approximation.

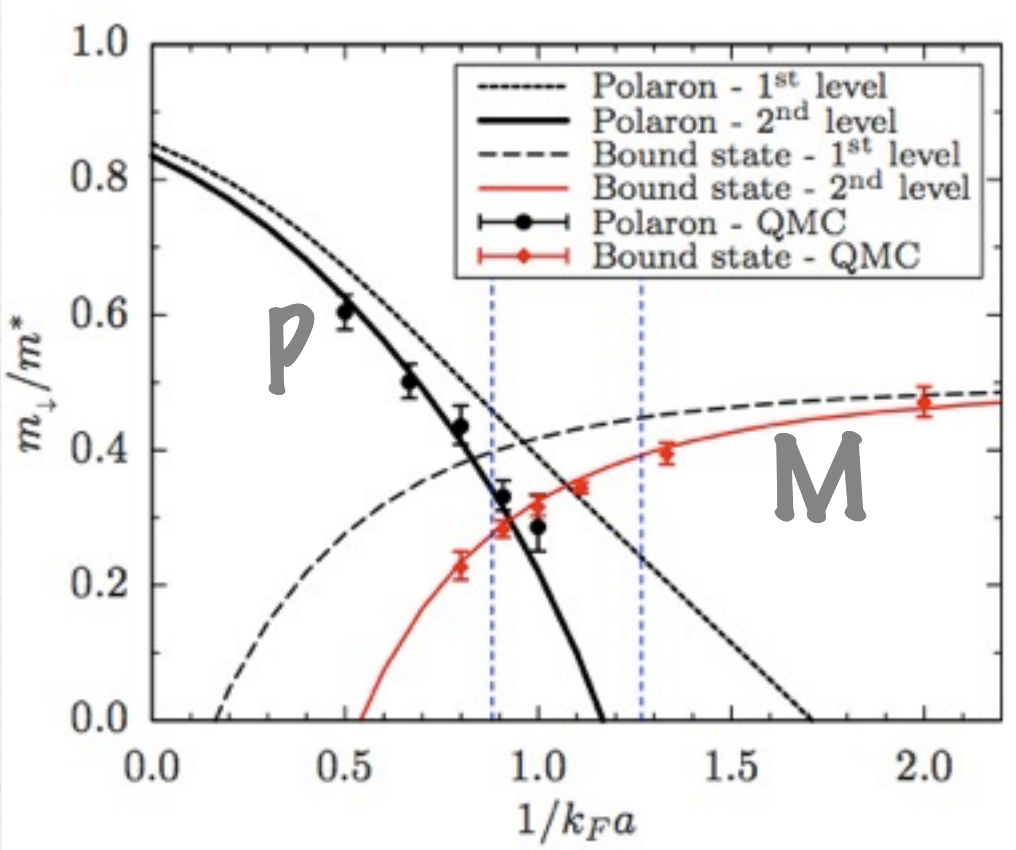
(Combescot et al., PRL 2007)

QP parameters

Self-energy Σ



Effective mass m*



Theory (QMC, variational, diagrammatic): Prokof'ev&Svistunov, Chevy, Recati, Lobo, Stringari, Combescot, Leyronas, Massignan&Bruun, Zwerger, Punk, Stoof, Mora,...

Experiments: MIT, ENS

P-P Interactions: Mora&Chevy, PRL 2010; Zhenhua, Zöllner&Pethick, arXiv:1006.4723

P&M lifetimes: Bruun&Massignan, PRL 2010

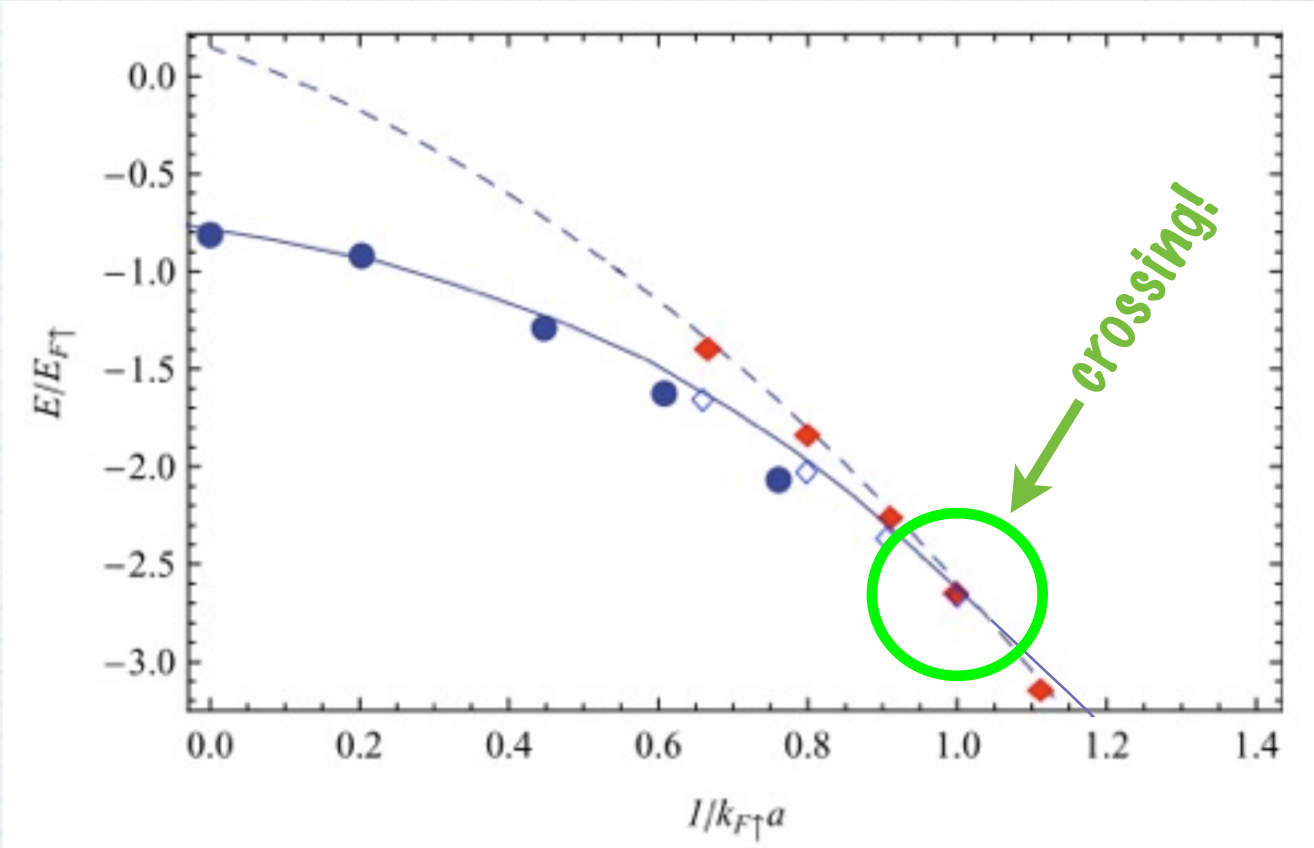
Bottom line: long life to quasi-particles!

$$\Gamma_P \sim Z_M(k_F a)\,(m^*_M)^{3/2}\,(\Delta\omega)^{9/2}$$

$$\Gamma_M \sim Z_P(k_F a)\,(m^*_P)^{3/2}\,(|\Delta\omega|)^{9/2}$$

$$\Delta\omega = \omega_P - \omega_M$$

Long lifetimes
~ 10-100ms



Bruun&Massignan, PRL 2010

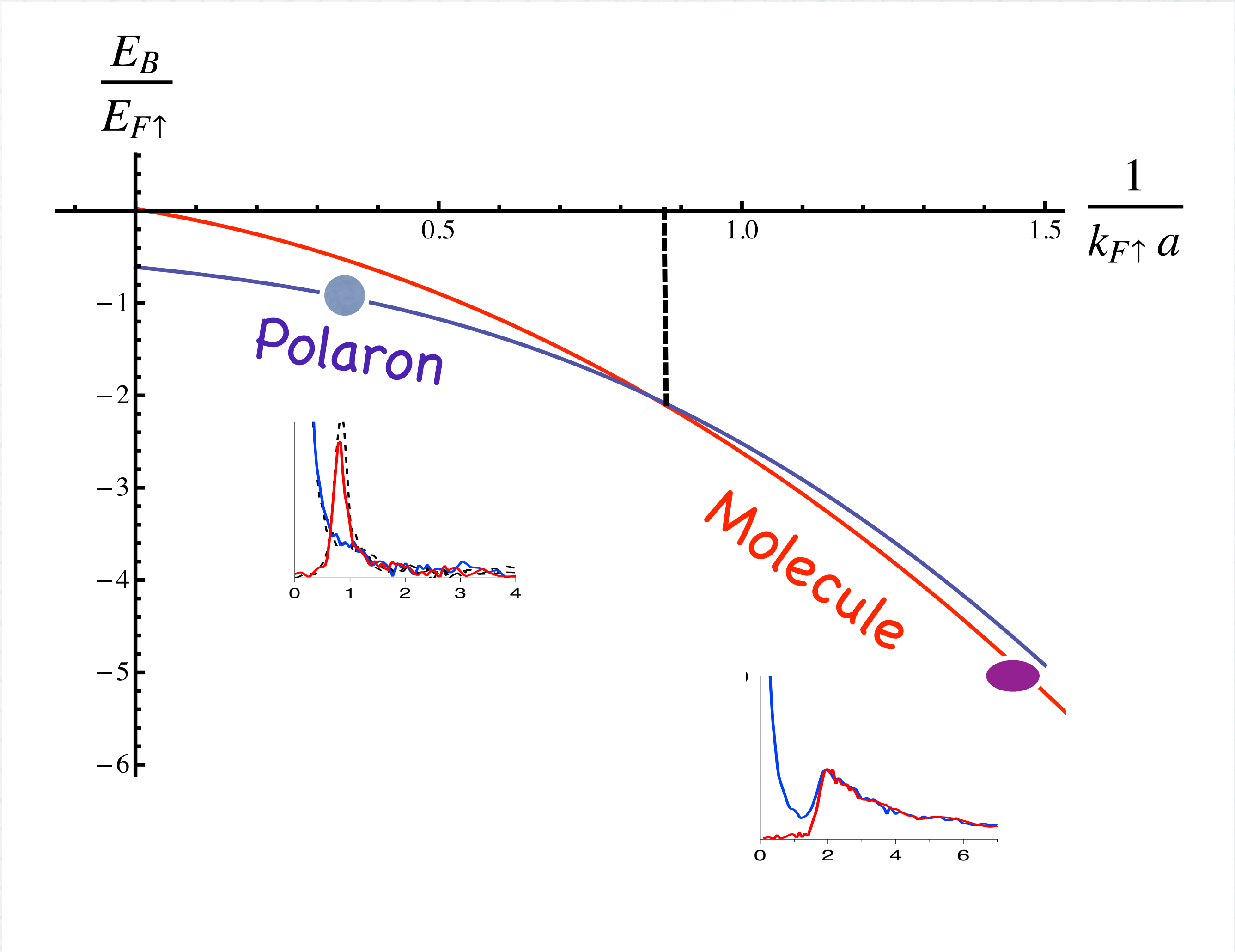
Experimental observation

Methods:

- RF spectra
- Collective modes to measure m* vs. time

Issues:

- No decay to deeply bound molecular states
- Phase separation?
  - stabilized by finite T
  - work with $m_\downarrow \neq m_\uparrow$
  - use bosonic impurities



Pol→Mol decay

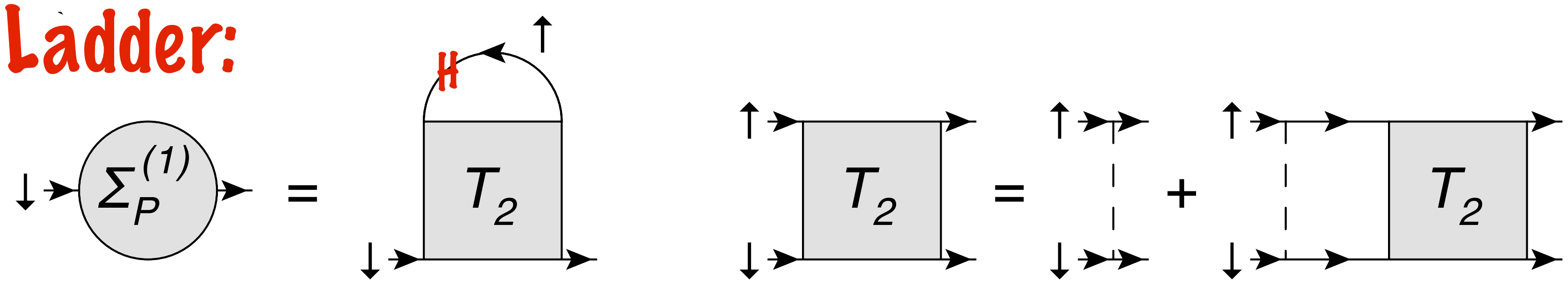
$$\Delta\omega = \omega_P - \omega_M > 0$$

Polaron: $G_\downarrow(\mathbf{p}, z)^{-1} = G^0_\downarrow(\mathbf{p}, z)^{-1} - \Sigma_P(\mathbf{p}, z)$

Decay rate: $\Gamma_P = -\mathrm{Im}\Sigma_P(p = 0, \omega_P)$

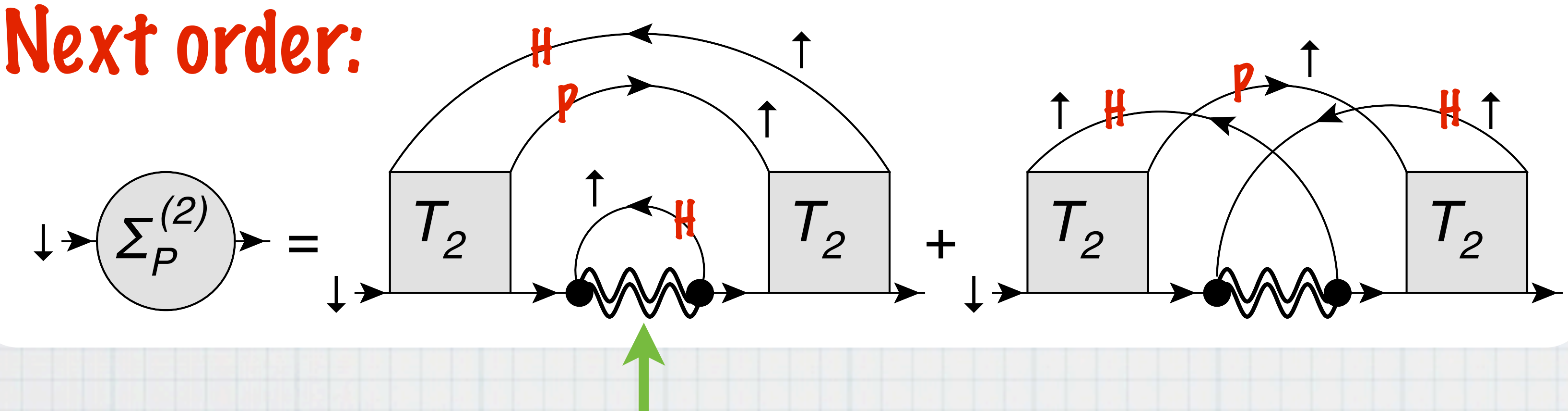
Hole expansion: $\Sigma_P(\mathbf{p}, z) = \Sigma^{(1)}_P(\mathbf{p}, z) + \Sigma^{(2)}_P(\mathbf{p}, z) + \ldots$

Ladder:



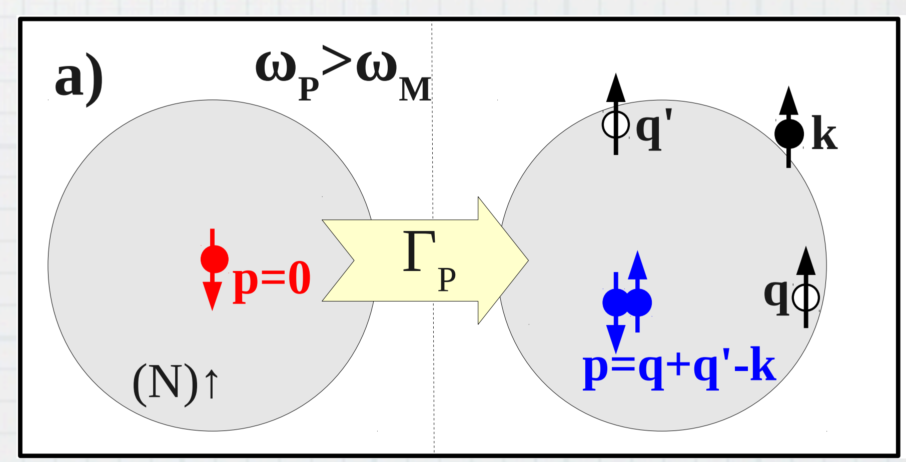
no damping
in the ladder approx.

Next order:



dressed molecule

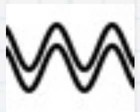
3-body process



molecule w.f. in vacuum:

$$\phi_q = \frac{\sqrt{8\pi a^3}}{1+q^2a^2} \quad \text{or} \quad \phi_r \propto \frac{e^{-r/a}}{r}$$

dressed molecule:



$$D(\mathbf{p},\omega) \simeq \frac{Z_M}{\omega - \omega_M - p^2/2m_M^*}.$$

atom-molecule coupling:

$$\bullet = \frac{1}{g(\mathbf{p},z)} = \int \frac{d^3q}{(2\pi)^3}\phi_q \frac{1-f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z-\xi_{\mathbf{p}-\mathbf{q}\uparrow}-\xi_{q\downarrow}} \sim -\sqrt{\frac{m_r^2 a}{2\pi}}$$

(Bruun&Pethick, PRL 2004)

$$\Gamma_P = \frac{g^2 Z_M}{2}\int \frac{d^3k\ d^3q\ d^3q'}{(2\pi)^9}\left[F(\mathbf{q},\mathbf{k},\omega_P) - F(\mathbf{q}',\mathbf{k},\omega_P)\right]^2 \delta\left(\Delta\omega + \xi_{q\uparrow} + \xi_{q'\uparrow} - \xi_{k\uparrow} - \frac{(\mathbf{q}+\mathbf{q}'-\mathbf{k})^2}{2m_M^*}\right)$$

$$q, q' < k_F \ , \ k > k_F$$

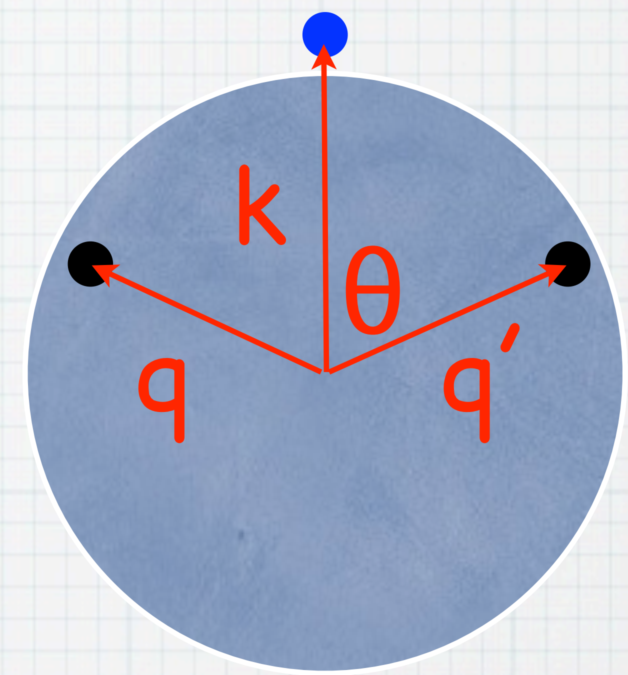
$$F(\mathbf{q},\mathbf{k},\omega) = T_2(\mathbf{q},\omega+\xi_{q\uparrow})G^0_\downarrow(\mathbf{q}-\mathbf{k},\omega+\xi_{q\uparrow}-\xi_{k\uparrow})$$

$\Delta\omega \ll \epsilon_F$

$q \simeq k \simeq k' \simeq k_F$

In the neighborhood of the P-M crossing,

$$\int \frac{d^3k\ d^3q\ d^3q'}{(2\pi)^9}\delta(\ldots) \sim (m_M^*)^{3/2}(\Delta\omega)^{7/2}$$



The P+H+H form an equilateral triangle, since $q + q' - k \sim 0$

At the crossing, Fermi antisymmetry yields a vanishing of the matrix element!

$$F(\mathbf{q}, \mathbf{k}, \omega_P) - F(\mathbf{q}', \mathbf{k}, \omega_P)$$

the angular dependence of F is only on θ

Expand matrix element around the equilateral shape to get an extra factor of Δω:

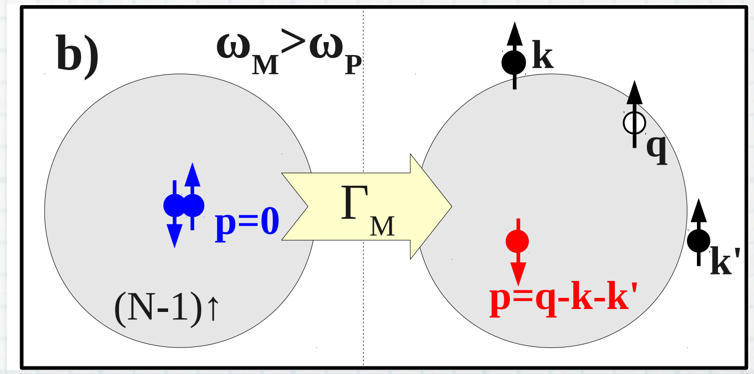
$$\Gamma_P \sim Z_M(k_F a)\,(m_M^*)^{3/2}\,(\Delta\omega)^{9/2}$$

1st order transition between the P&M states (no coupling at the crossing)

Mol→Pol decay

$$\Delta\omega = \omega_P - \omega_M < 0$$

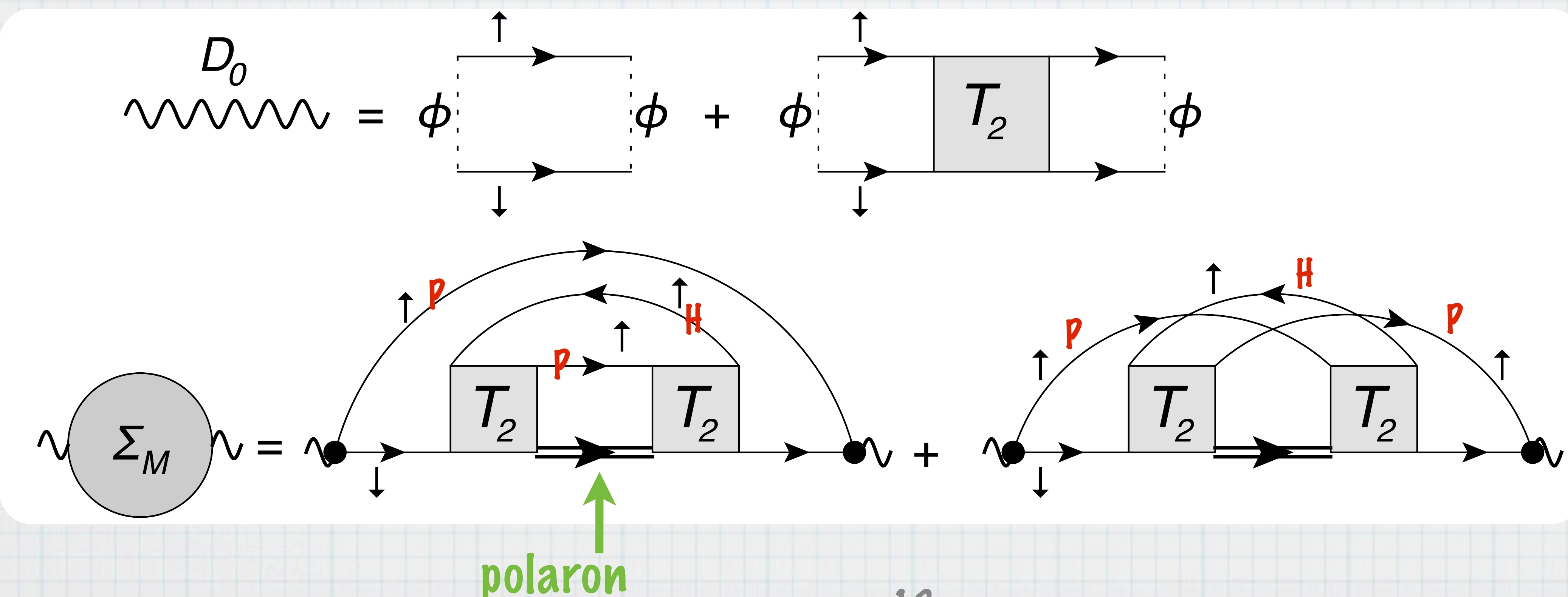
Molecule: $D(\mathbf{p},z)^{-1} = D_0(\mathbf{p},z)^{-1} - \Sigma_M(\mathbf{p},z)$



Decay rate: $\Gamma_M = -\mathrm{Im}\Sigma_M(p=0,\omega_M)$

Vacuum: $$D_0(\mathbf{p},z) = \int d^3\check{q}\phi_q^2 \frac{1-f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z-\xi_{\mathbf{p}-\mathbf{q}\uparrow}-\xi_{q\downarrow}} + \frac{T_2(\mathbf{p},z)}{g(\mathbf{p},z)^2}$$

3-body process



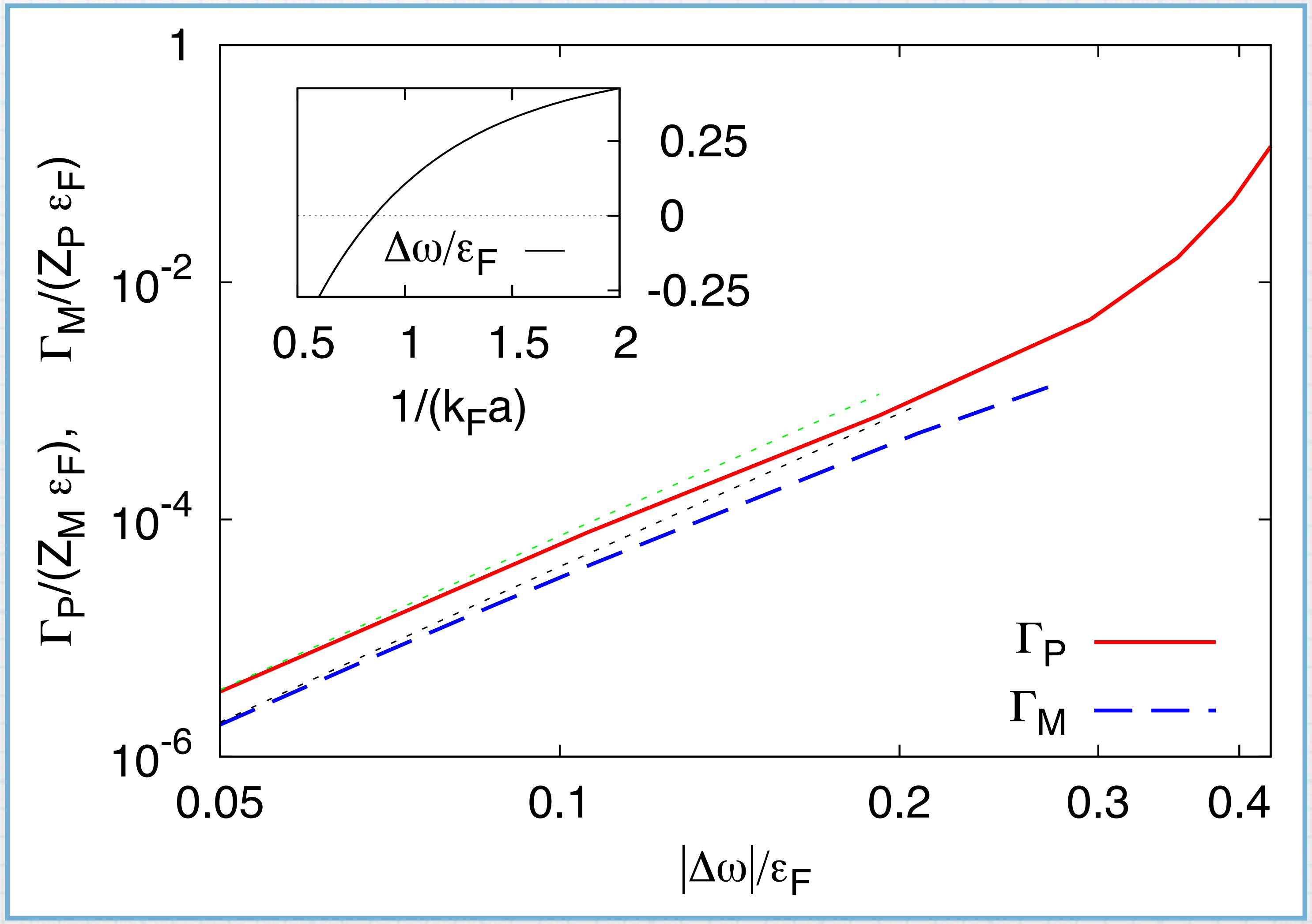
$$\Gamma_M = \frac{g^2 Z_P}{2} \int \frac{d^3k\ d^3k'\ d^3q}{(2\pi)^9} \left[C(\mathbf{q},\mathbf{k},\omega_M) - C(\mathbf{q},\mathbf{k}',\omega_M)\right]^2 \delta\left(|\Delta\omega| + \xi_{q\uparrow} - \xi_{k\uparrow} - \xi_{k'\uparrow} - \frac{(\mathbf{q}-\mathbf{k}-\mathbf{k}')^2}{2m_P^*}\right)$$

In the neighborhood of the M-P crossing, $\Gamma_M \sim Z_P(k_F a)\,(m_P^*)^{3/2}\,(|\Delta\omega|)^{9/2}$

For both decay processes,
very **long lifetimes** are ensured by:

- limited phase-space
- Fermi antisymmetry

much longer than usual Fermi liquids



In the numerics:

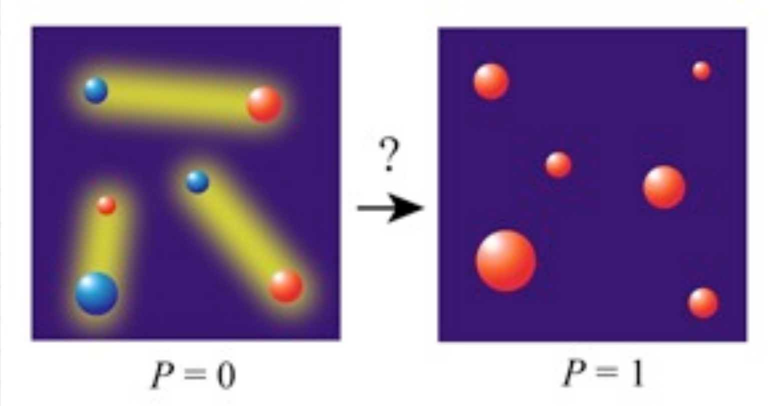
$$\omega_M = -\frac{\hbar^2}{2m_r a^2} - \epsilon_F + g_3 n_\uparrow$$

$$a_3 = 1.18a$$

$$T_2(\mathbf{p},\omega) = \frac{2\pi a/m_r}{1 - \sqrt{2m_r a^2\left(\frac{p^2}{2m_M} - \omega - \epsilon_F + g_3 n_\uparrow\right)}}$$

Conclusions

- The impurity problem contains
  a sharp Polaron-Molecule transition
- At small momenta, the process coupling molecules and polarons requires at least 3-bodies
- The P-M decay is strongly suppressed due to a combination of small final density of states and Fermi statistics
- Expected lifetimes ~ 10-100ms



G. M. Bruun and P. Massignan, Phys. Rev. Lett. 105, 020403 (2010).

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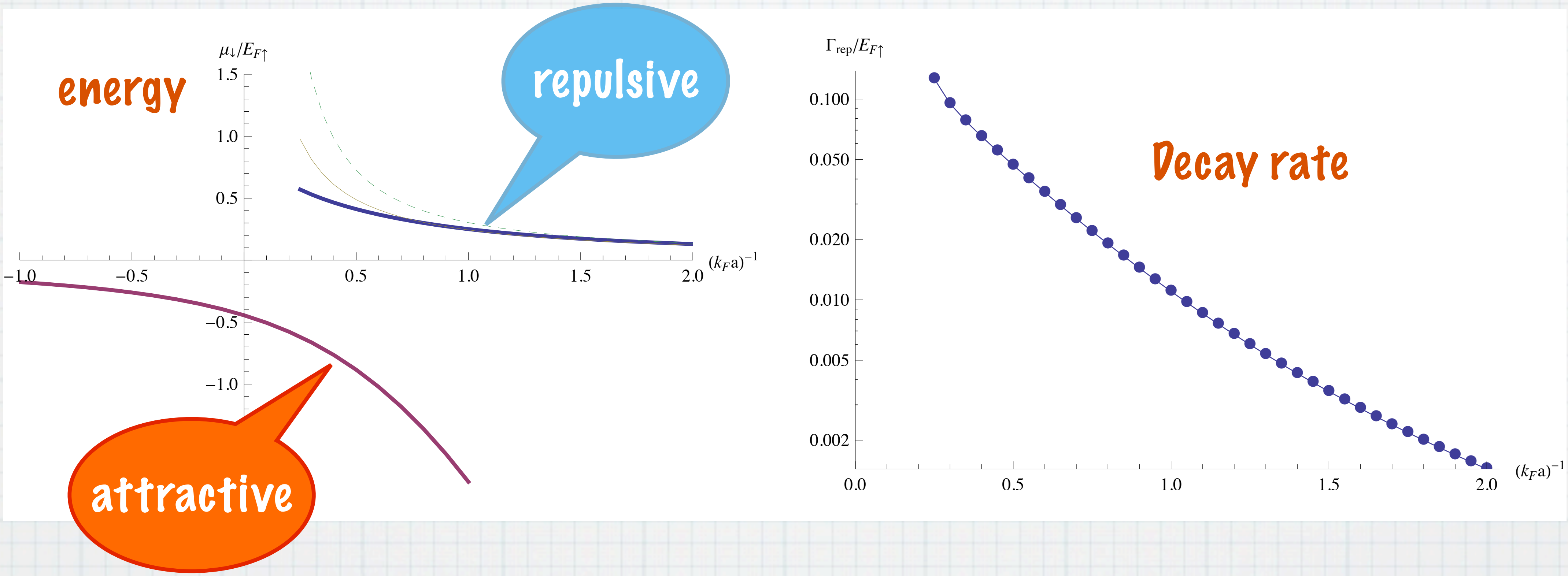
Vacuum:

$$D_0(\mathbf{p},z) = \int d^3\check{q}\phi_q^2 \frac{1-f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z-\xi_{\mathbf{p}-\mathbf{q}\uparrow}-\xi_{q\downarrow}} + \frac{T_2(\mathbf{p},z)}{g(\mathbf{p},z)^2}$$

Repulsive polaron



A ^{40}K impurity in a Fermi sea of ^{6}Li

Impurity spectral function

$A = -\mathrm{Im}[G_{\uparrow}(k=0, \omega)]$