Polarons and molecules close to narrow Feshbach resonances

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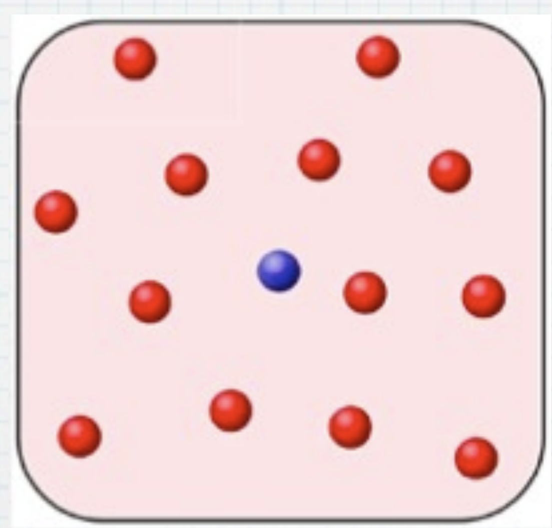
$N \gg 1$



normal Fermi liquid

ICFO
Institut de Ciències Fotòniques

European Science Foundation
SETTING SCIENCE AGENDAS FOR EUROPE
FerMix - EuroQuam

in collaboration with Georg Bruun and the FeLiKx team (Innsbruck)

Quasi-Particles

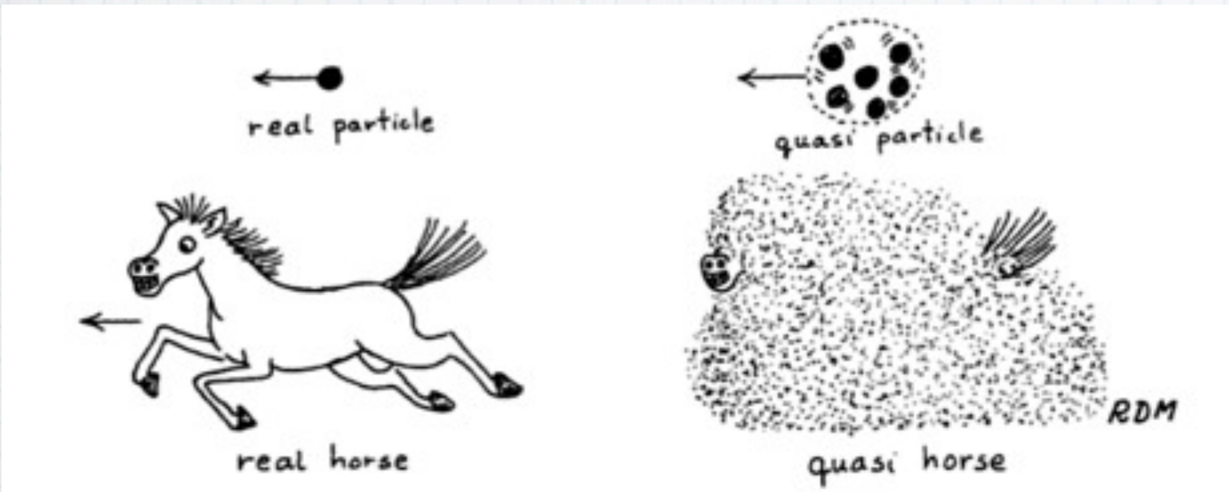
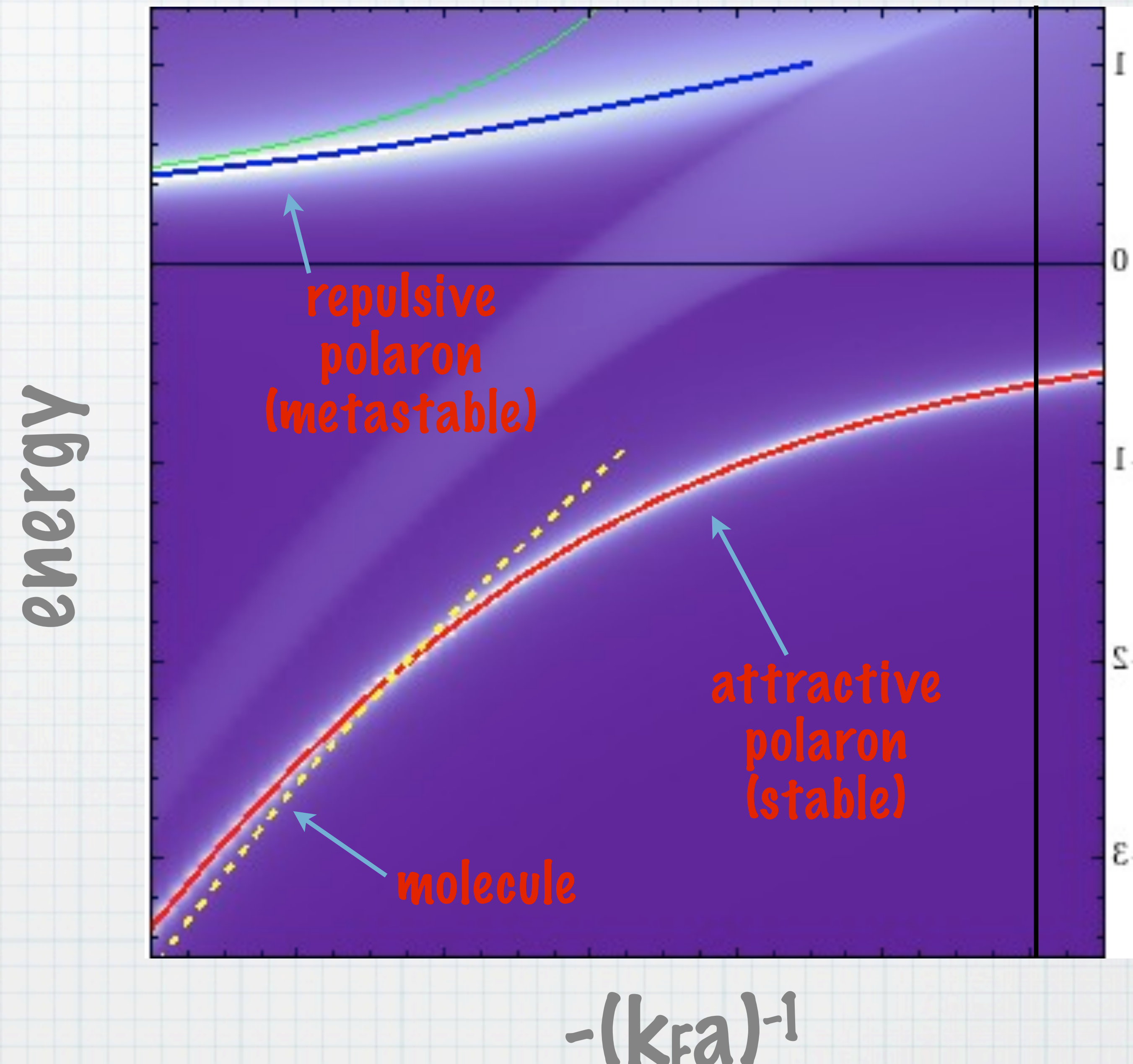


Fig. 0.4 *Quasi Particle Concept*

(Richard Mattuck)

QP properties:

@ renormalized mass
@ chemical potential
@ shielded interactions
@ q. numbers (charge, spin, ...)
@ lifetime



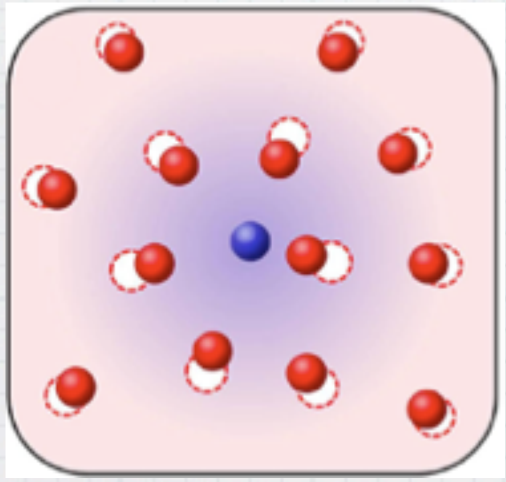
Polaron: variational Ansatz

the $\downarrow$ impurity

$$|\psi_{\mathbf{p}}\rangle = \phi_0 c^\dagger_{\mathbf{p}\downarrow}|0\rangle_\uparrow + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{qk}}\, c^\dagger_{\mathbf{p}+\mathbf{q}-\mathbf{k}_\downarrow} \boxed{c^\dagger_{\mathbf{k}_\uparrow}\, c_{\mathbf{q}_\uparrow}\, |0\rangle_\uparrow}$$

non-interacting $\uparrow$ Fermi sea

Particle-Hole dressing



minimize $\langle\psi|\hat{H} - E|\psi\rangle$

Dressed Molecules

$$|\Phi_{\mathbf{p}=0}\rangle = \left(\beta_0^{(0)} b_0^\dagger + \sum_k \beta_{\mathbf{k}}^{(1)} d_{-\mathbf{k}}^\dagger u_{\mathbf{k}}^\dagger + \sum_{\mathbf{k},\mathbf{q}} \beta_{\mathbf{k},\mathbf{q}}^{(2)} b_{\mathbf{q}-\mathbf{k}}^\dagger u_{\mathbf{k}}^\dagger u_{\mathbf{q}} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \beta_{\mathbf{k},\mathbf{k}',\mathbf{q}}^{(3)} d_{\mathbf{q}-\mathbf{k}-\mathbf{k}'}^\dagger u_{\mathbf{k}'}^\dagger u_{\mathbf{k}}^\dagger u_{\mathbf{q}} \right) |FS_{N-1}\rangle.$$

$$H = \sum_{\mathbf{p}} \left[\xi_{\mathbf{p},\uparrow} u_{\mathbf{p}}^\dagger u_{\mathbf{p}} + \xi_{\mathbf{p},\downarrow} d_{\mathbf{p}}^\dagger d_{\mathbf{p}} + (\xi_{\mathbf{p},M} + \nu_0)\, b_{\mathbf{p}}^\dagger b_{\mathbf{p}} \right] + \frac{g_0}{V} \sum_{\mathbf{p},\mathbf{p}'} \left(b_{\mathbf{p}}^\dagger u_{\mathbf{p}'} d_{\mathbf{p}-\mathbf{p}'} + h.c. \right)$$

renormalization conditions: $$\frac{1}{T} = \frac{m_r}{2\pi a} + \cancel{\frac{\nu_0}{g_0^2}} = \frac{1}{V} \sum_{\mathbf{p}} \frac{1}{2\xi_{\mathbf{p}}}$$

universal case considered by:
Punk&Dumitrescu&Zwerger, Mora&Chevy, Combescot&Giraud&Leyronas (2009)

minimize $\langle \Phi | \hat{H} - E | \Phi \rangle$

Narrow Feshbach Resonances

Scattering amplitude: $f = -\left[a^{-1} + ik + R^* k^2 + \dots\right]^{-1}$

$$R^* = -\frac{r_e}{2} = \frac{\hbar^2}{2m_r a_{\mathrm{bg}} \Delta B \delta\mu}$$

Molecule energy: $E_M = -\frac{\hbar^2}{2m_r (a_*)^2}$ with $a^* = \frac{2R^*}{\sqrt{1+4R^*/a}-1}$

$a \gg R^* : \quad a^* \sim a$

$a \ll R^* : \quad a^* \sim \sqrt{aR^*}$

a FR is broad if $R^* \ll R_{VdW}$ or $k_F R^* \ll 1$

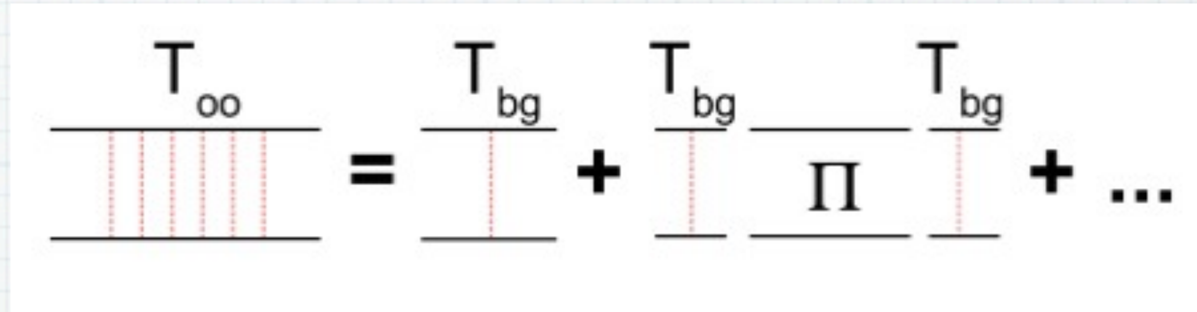
No broad heteronuclear FR found yet.

Many-body description of narrow FR

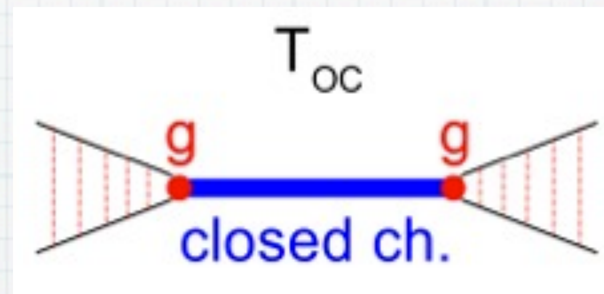
Bruun, Jackson & Kolomeitsev, PRA 2005
Massignan & Stoof, PRA 2008

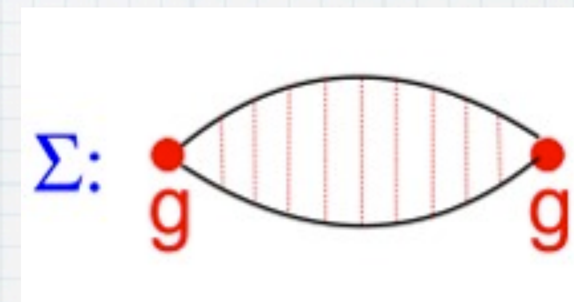
$$T = T_{\mathrm{OO}} + T_{\mathrm{OC}}$$

OO: open channel only



OC: involves coupling between open and closed channels





$$T_{OC} = \left(\frac{g}{1 - T_{bg}\Pi(E)}\right)^2 \frac{1}{E - \Delta\mu(B - B_0) - \Sigma(E)}$$

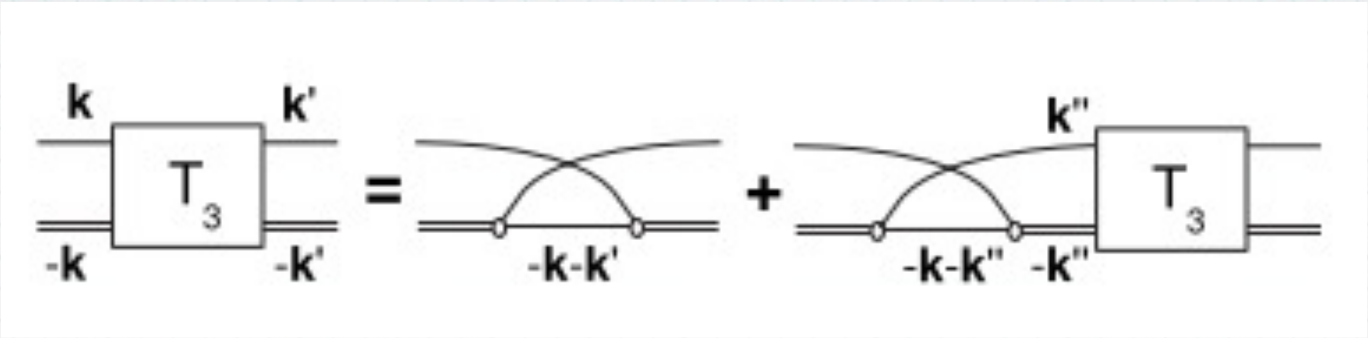
$$T = -\frac{2\pi\hbar^2}{m_r} f \quad \text{with} \quad f = -\left\{\left[a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0 - E_{\mathrm{CM}}/\delta\mu}\right)\right]^{-1} - \frac{2\pi\hbar^2}{m_r}\Pi(\mathbf{p}, E_{\mathrm{CM}})\right\}^{-1}$$

low energy expansion:

$$a^*(B) = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

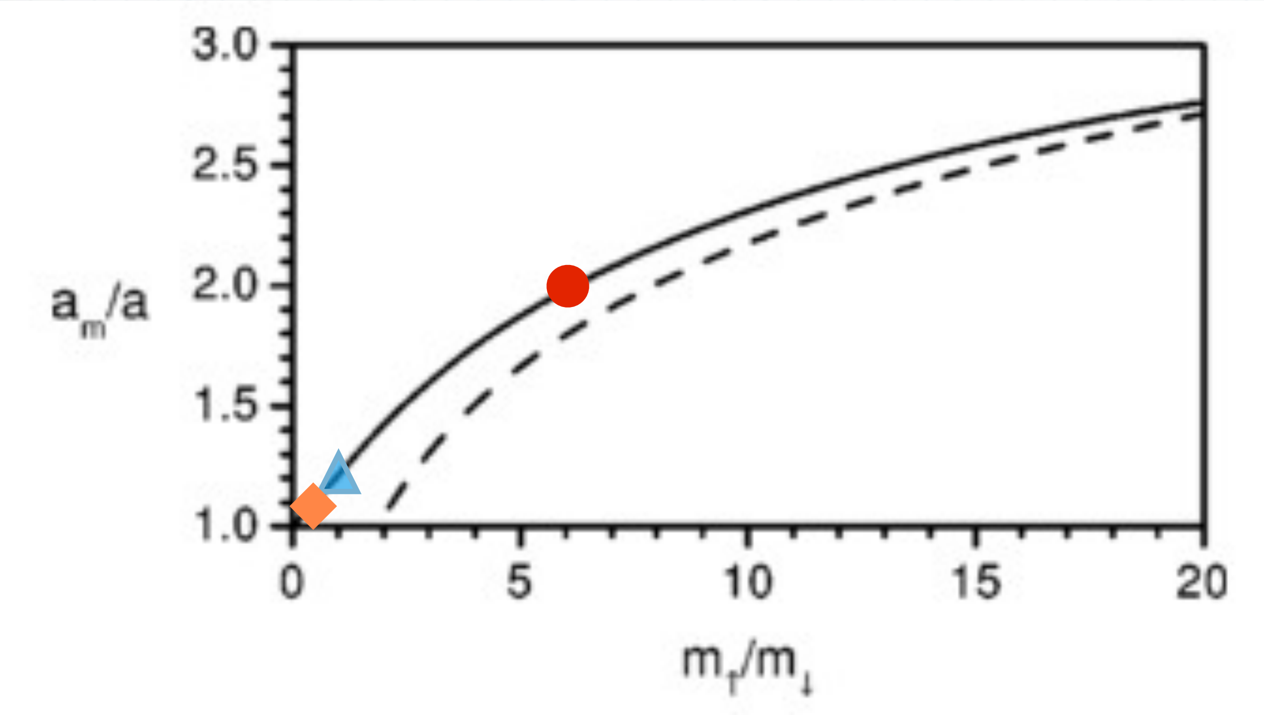
$$R^*(B) = \frac{\hbar^2 \Delta B}{2 m_r a_{\mathrm{bg}} (B - B_0 - \Delta B)^2 \delta\mu}$$

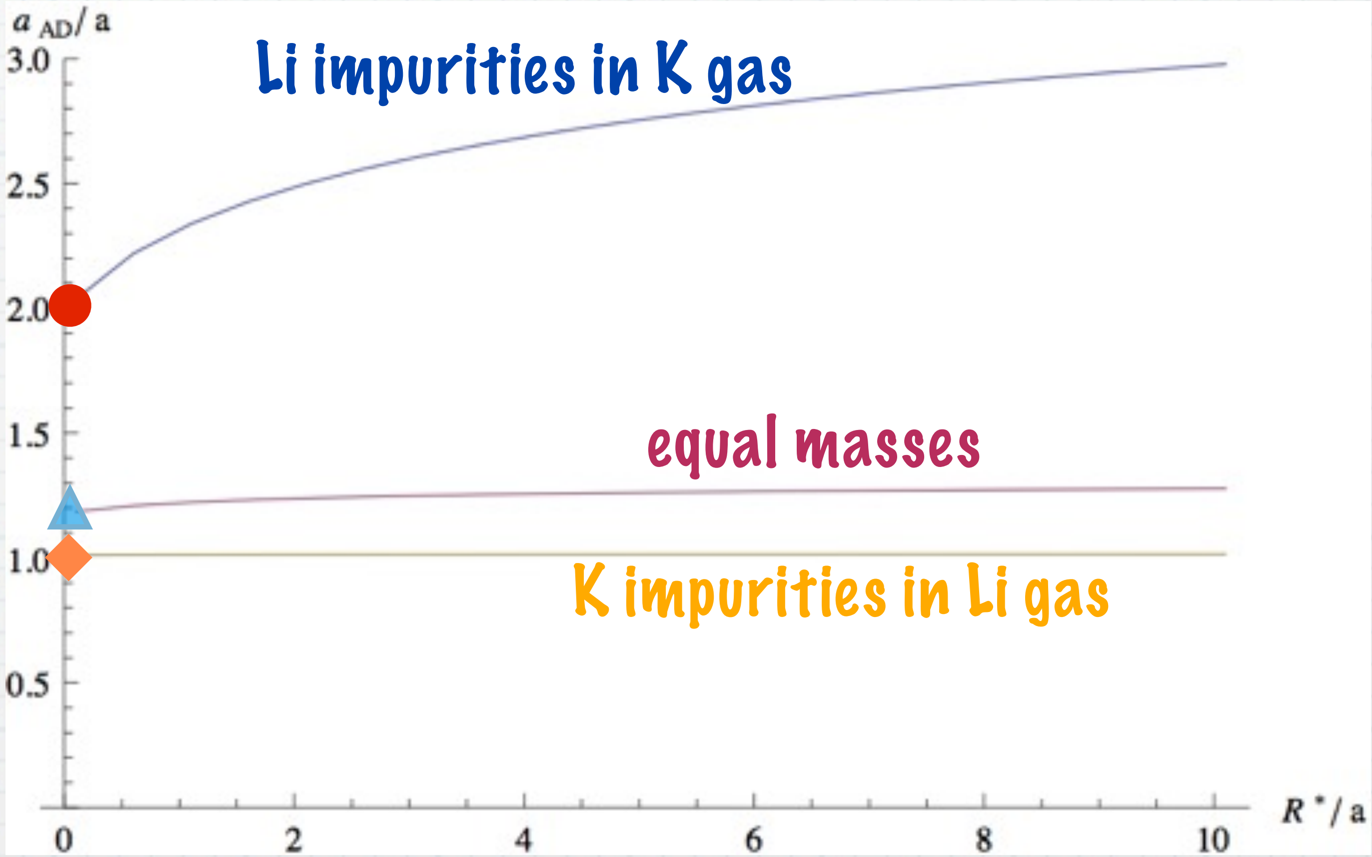
WarmUp: Atom-Dimer scattering



Broad FR

Narrow FR

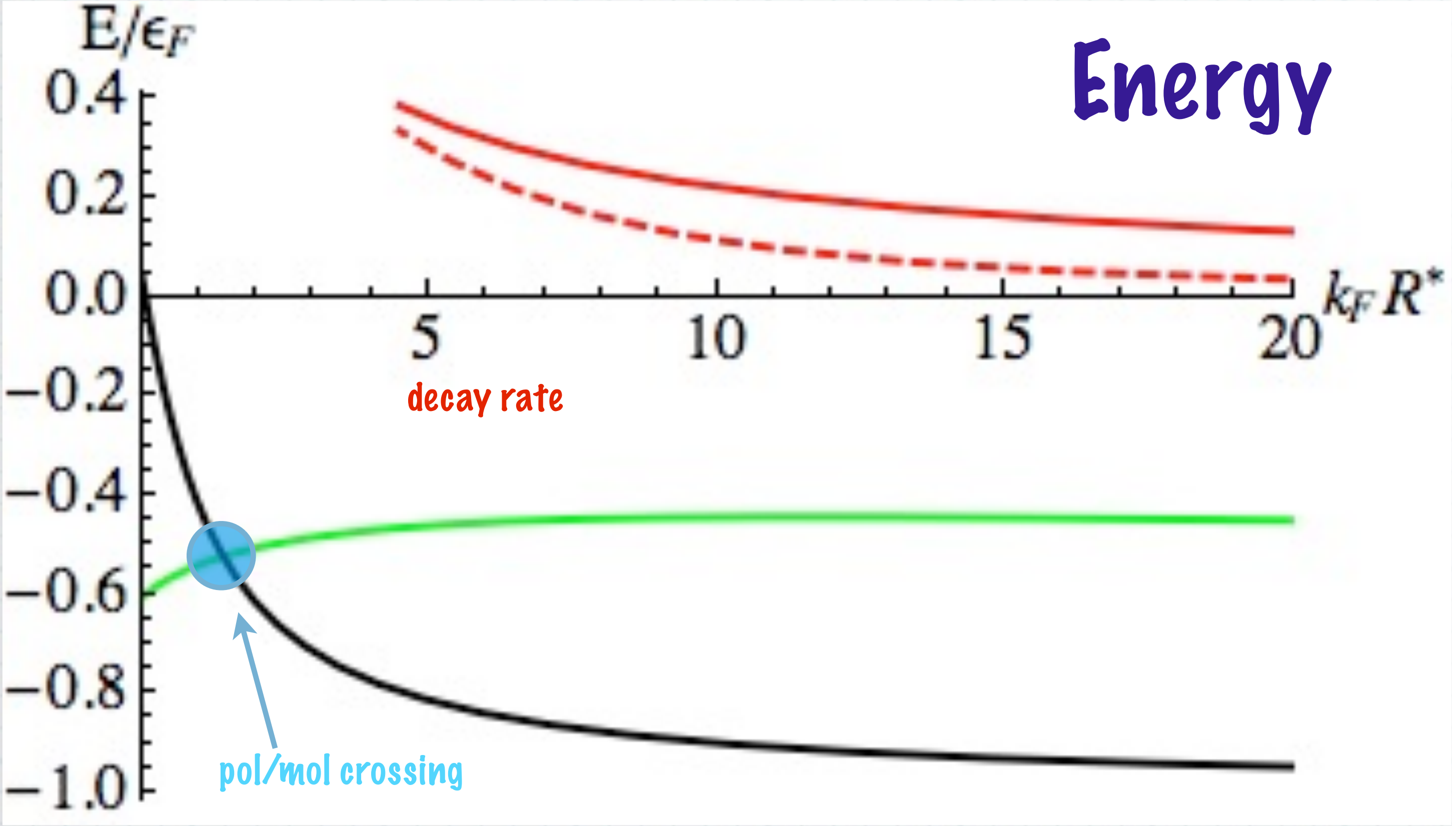


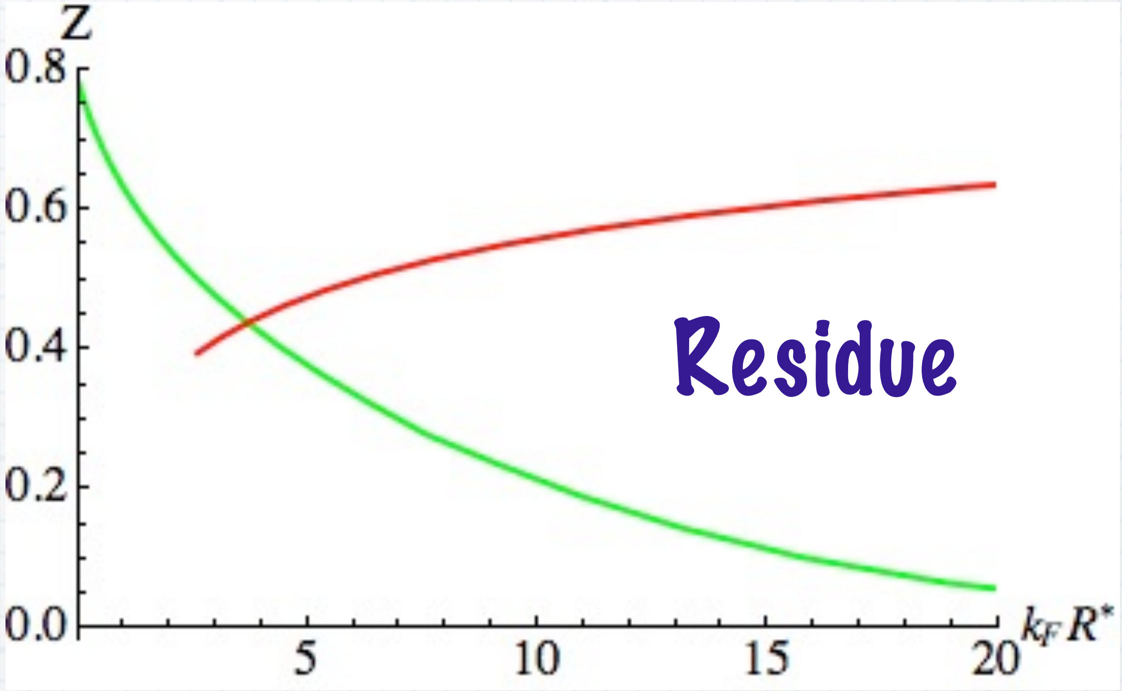


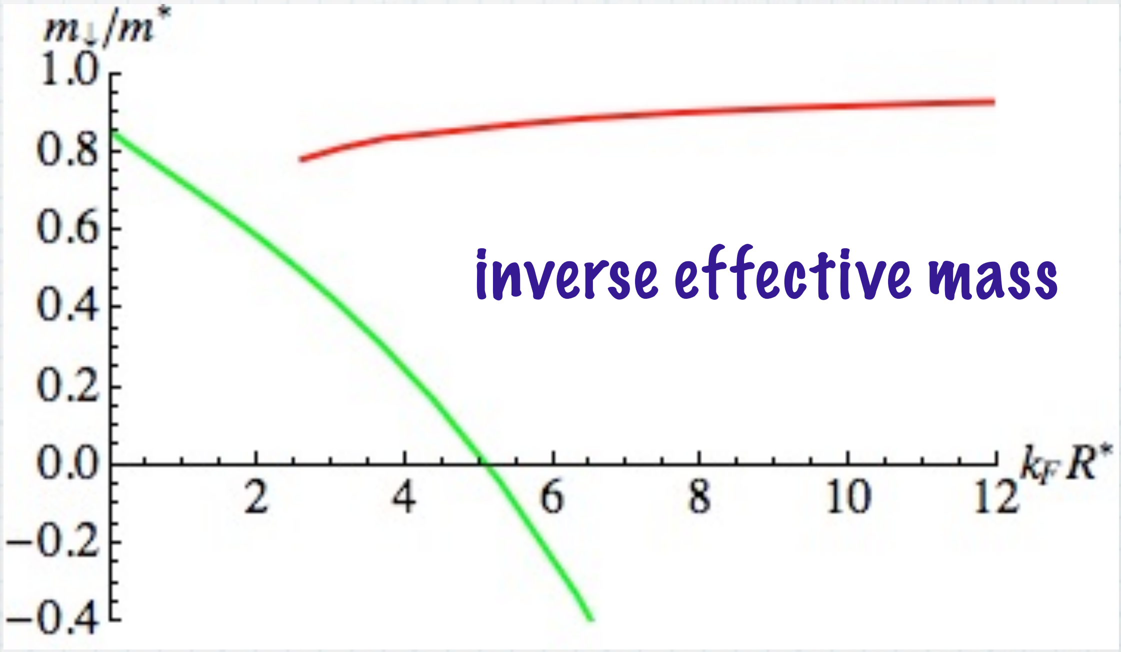
agrees with real-space calculation (Petrov, PRA 2003; Petrov&Levinsen, arXiv: 1101.5979)

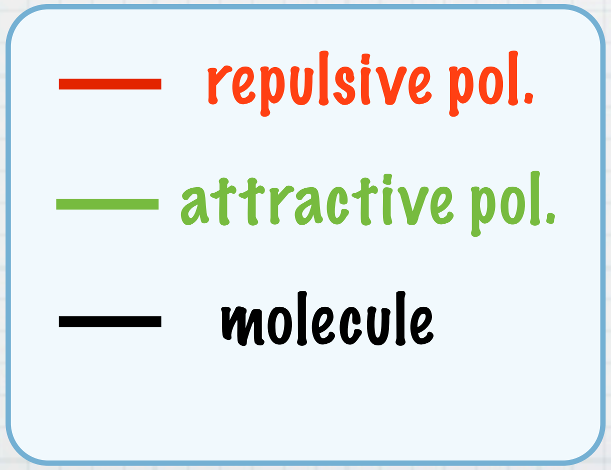
“Narrow” quasiparticles

$(k_F a)^{-1}=0$ [at resonance]
equal masses

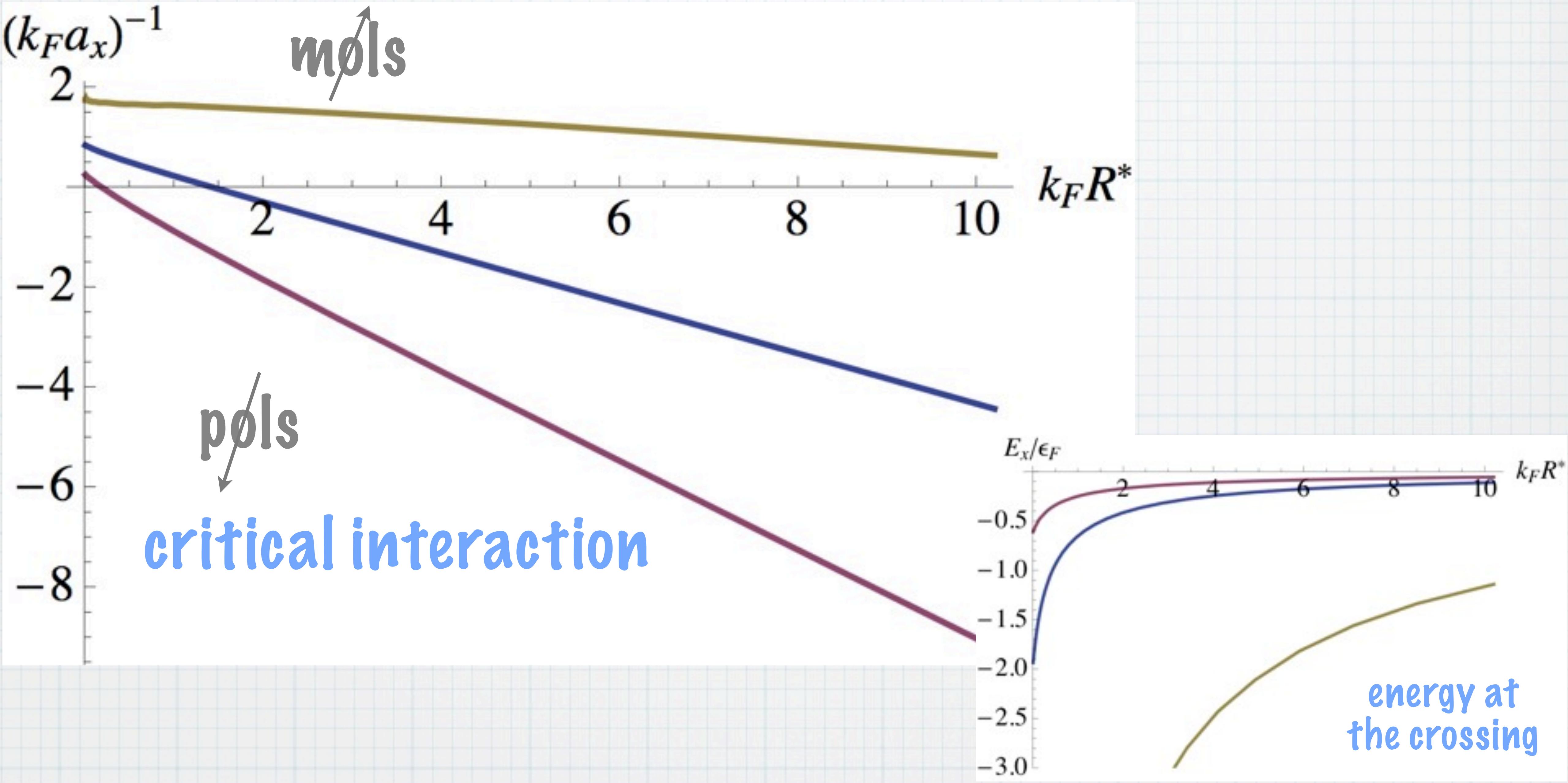








Pol/Mol crossing at a narrow FR



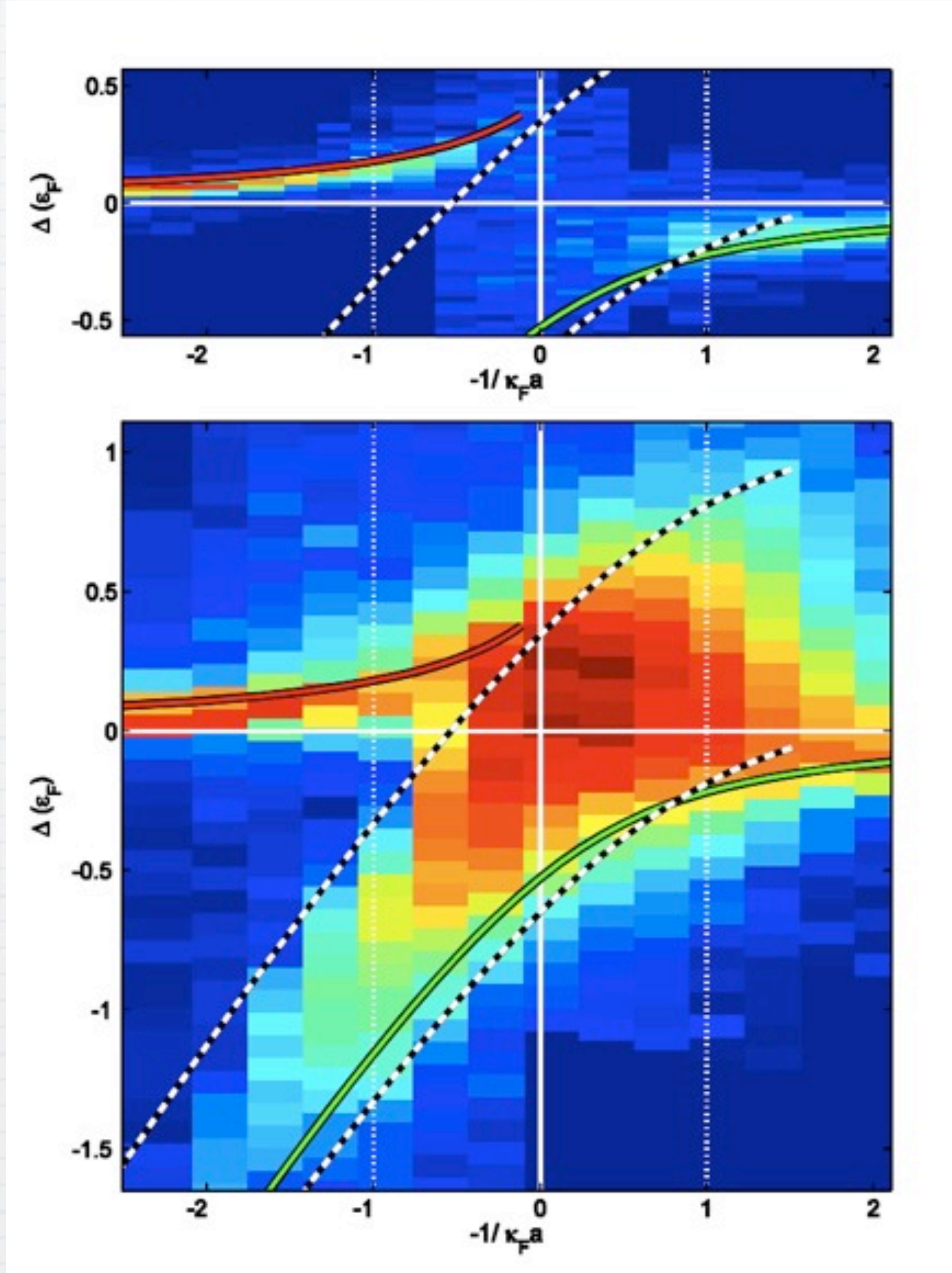
for impurity/gas mass ratios: 6/40, 1, 40/6

Comparison with experiment

low power RF:

high power RF:

high power is needed to couple to the MH continuum, due to a small FC overlap



repulsive pol.
attractive pol.
molecule+hole continuum

$k_F R^* \simeq 1$

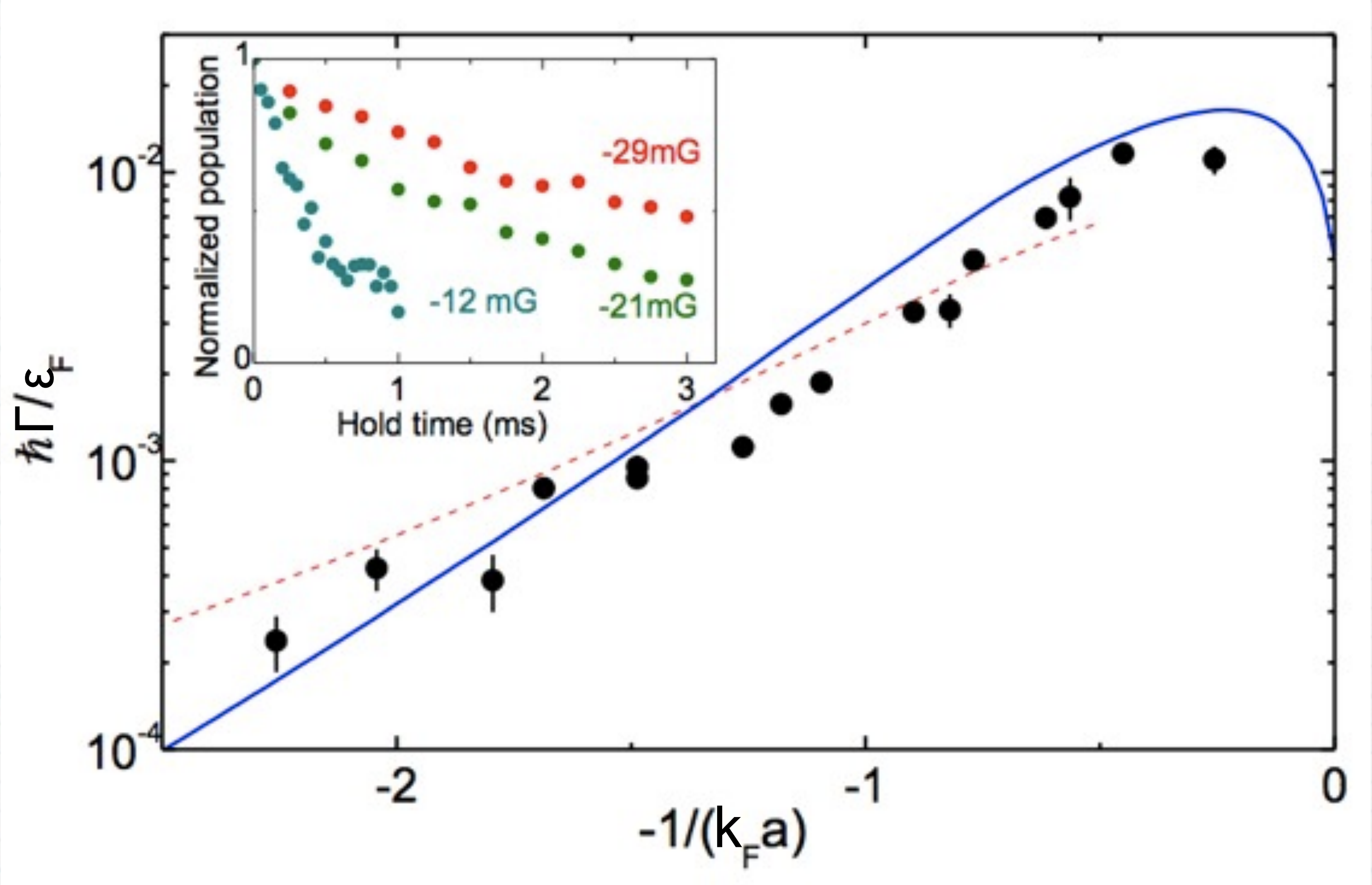
C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, PM, G. Bruun, F. Schreck & R. Grimm, in preparation.

Decay rate of repulsive polarons

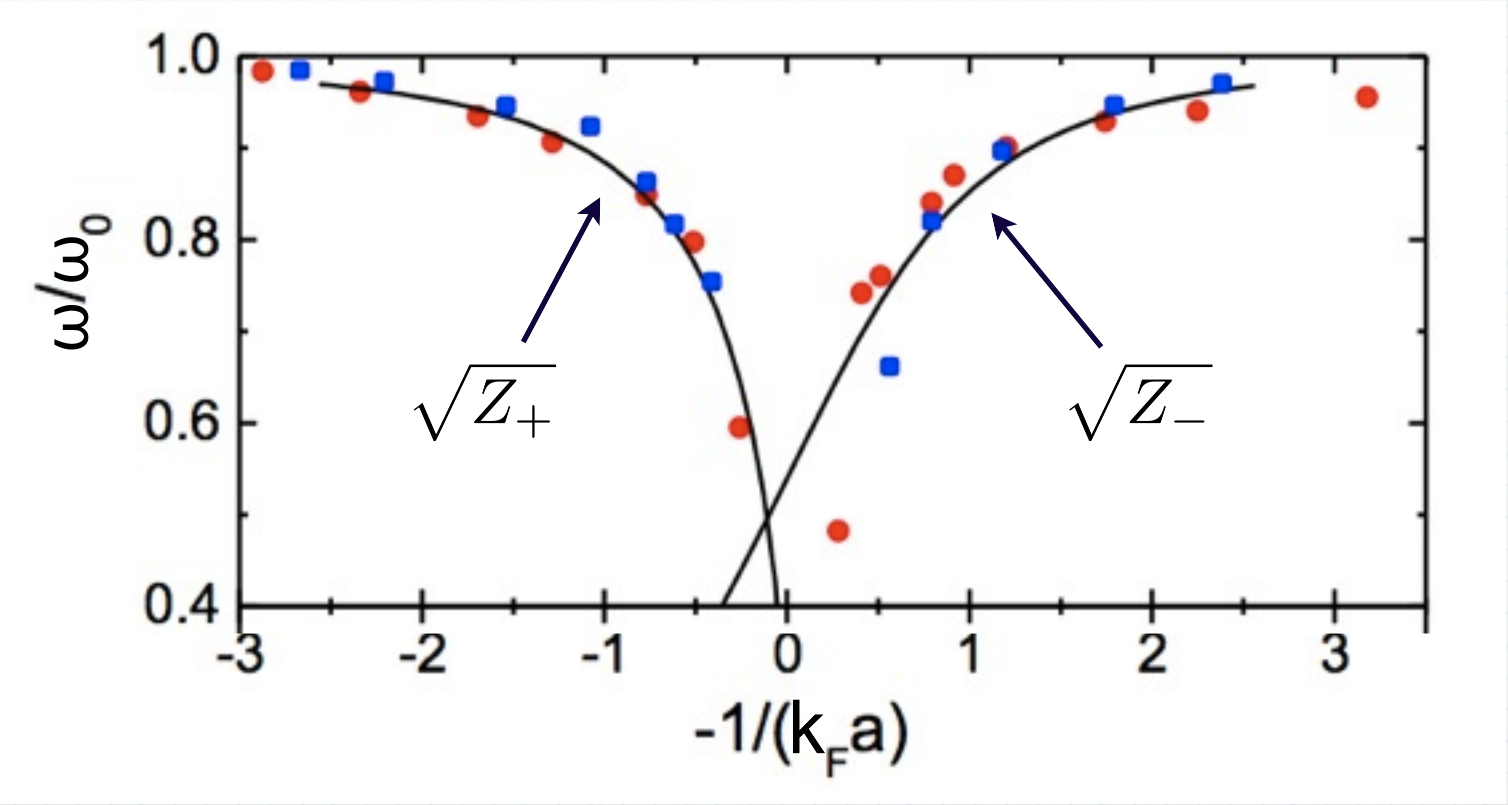
exp. data

vs.

theor. calculation for Pol→Pol and Pol→Mol processes



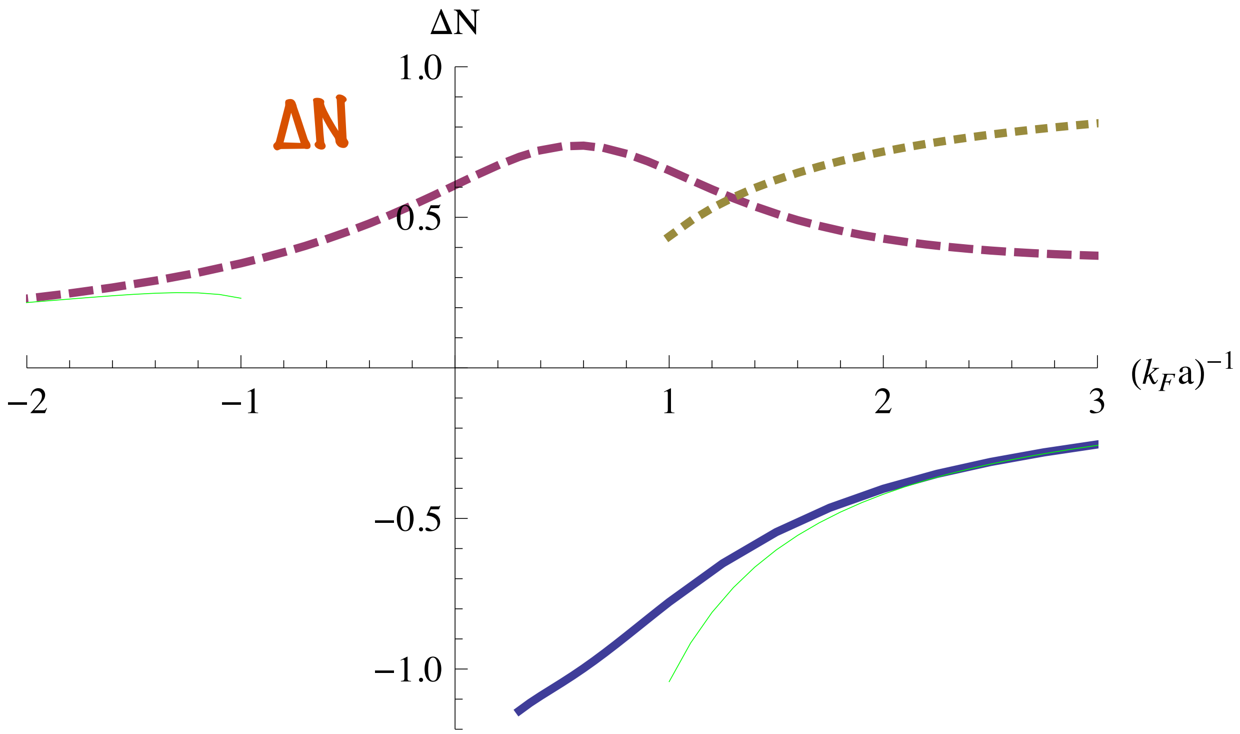
Rabi frequency as a measure of polaron residues



of particles in the dressing cloud

$$\delta\mu_\uparrow = \frac{\partial^2 \varepsilon}{\partial n_\uparrow \partial n_\downarrow} + \frac{\partial^2 \varepsilon}{(\partial n_\uparrow)^2}\Delta N = 0$$

$$\Delta N = -\left(\frac{\partial \mu_\downarrow}{\partial n_\uparrow}\right)_{n_\downarrow} \Big/ \left(\frac{\partial \mu_\uparrow}{\partial n_\uparrow}\right)_{n_\downarrow} \approx -\left(\frac{\partial \mu_\downarrow}{\partial \epsilon_F}\right)_{n_\downarrow}$$



weak coupling: $\Delta N = -\frac{2}{\pi}k_F a - \frac{4}{\pi^2}(k_F a)^2 + \ldots$

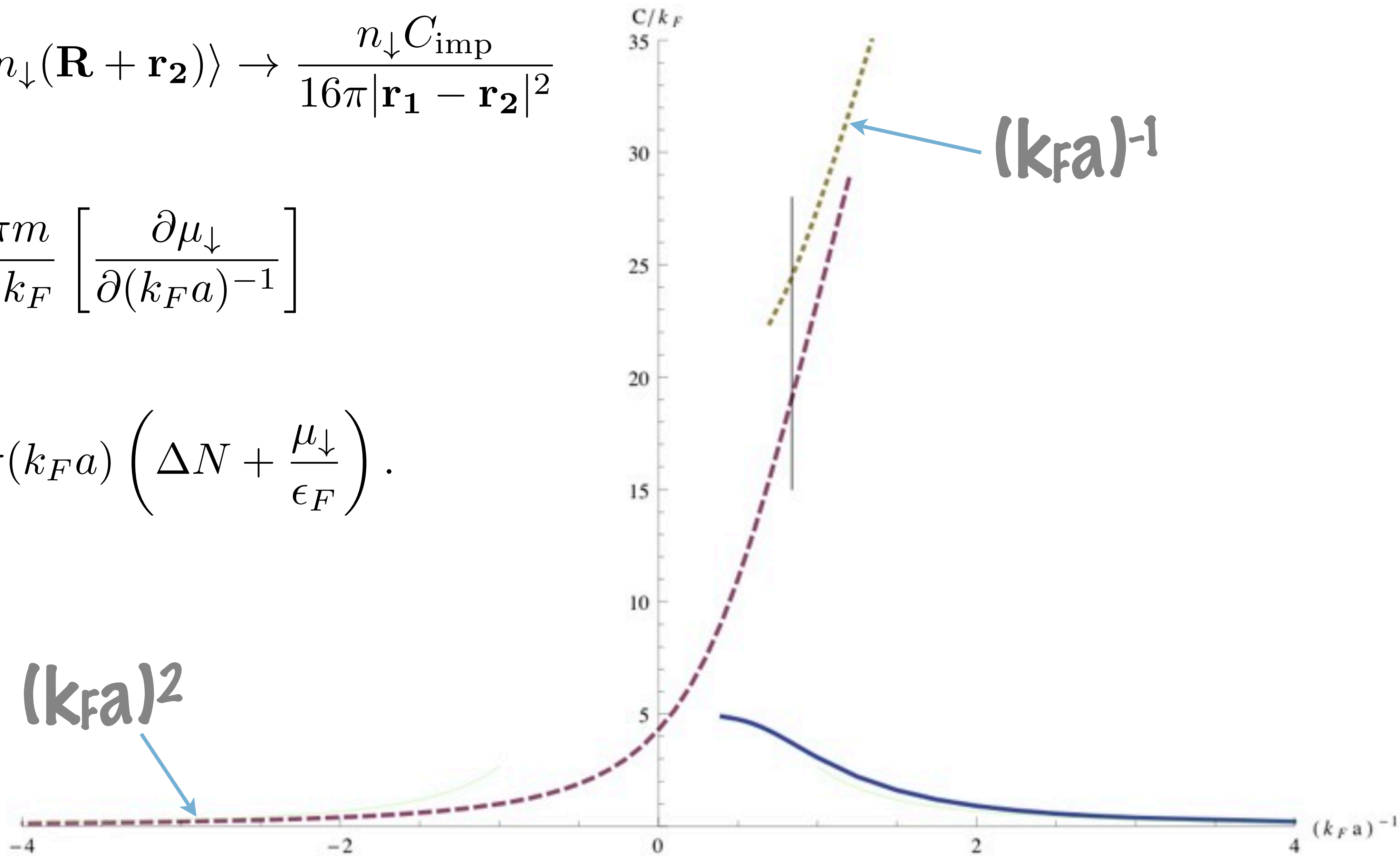
Tan's contact

C_{imp}: contact density per impurity

$$\langle n_\uparrow(\mathbf{R}+\mathbf{r_1})n_\downarrow(\mathbf{R}+\mathbf{r_2})\rangle \to \frac{n_\downarrow C_{\rm imp}}{16\pi|\mathbf{r_1}-\mathbf{r_2}|^2}$$

$$C_{\rm imp} = -\frac{4\pi m}{\hbar^2 k_F}\left[\frac{\partial \mu_\downarrow}{\partial (k_F a)^{-1}}\right]$$

$$\frac{C_{\rm imp}}{k_F} = -4\pi(k_F a)\left(\Delta N + \frac{\mu_\downarrow}{\epsilon_F}\right).$$



Decay rates

effect of mass ratio and resonance width

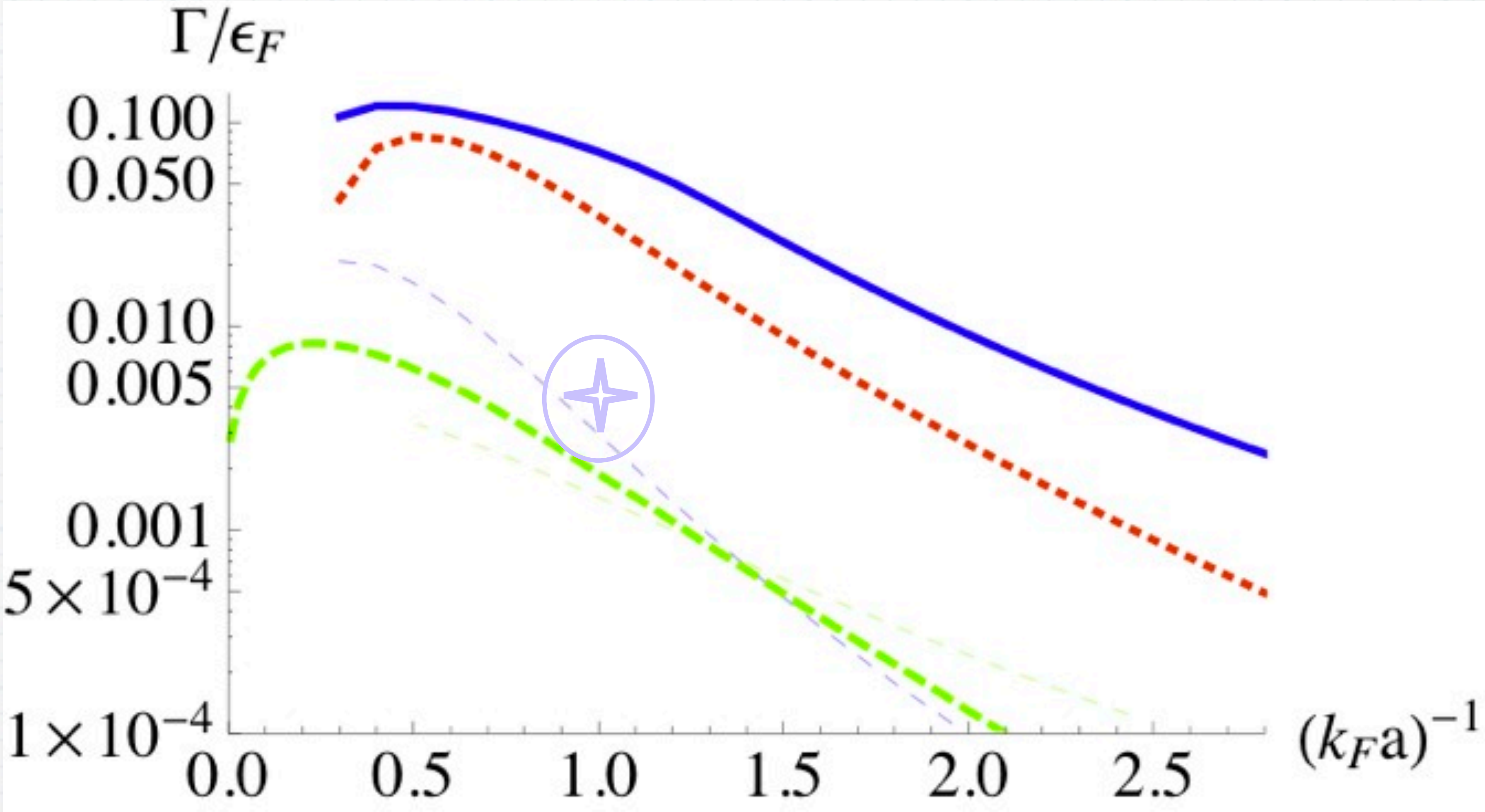
thick: Pol→Pol

thin: Pol→Mol

Universal limit $\propto (k_F a)^6$

(agrees with Petrov, PRA 2003)

Narrow limit $\propto (a)^{9/2} (R^*)^{3/2}$



MIT data point (C. Sanner et al., arXiv:1108.2017)

for impurity/gas mass ratios: 1_{univ}, $40/6_{univ}$, $40/6_{narrow}$

Conclusions

- A new strongly interacting quantum state: the repulsive polaron
  - energy, residue, decay rate, m^*, ΔN, contact
- Many-body physics at narrow FR
  - polaron/molecule crossing and quasiparticle properties vs. width of the resonance
- Excellent comparison with experimental data

I) G. Bruun and PM, PRL (2010)
II) K. Sadeghzadeh, G. Bruun, C. Lobo, PM, and A Recati, New J. of Phys. 13, 055011 (2011).
III) PM and G. Bruun, Eur. Phys. J. D (2011); in press, available as DOI: 10.1140/epjd/e2011-20084-5.
IV) C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, PM, G. Bruun, F. Schreck & R. Grimm, in preparation.
V) PM, in preparation.