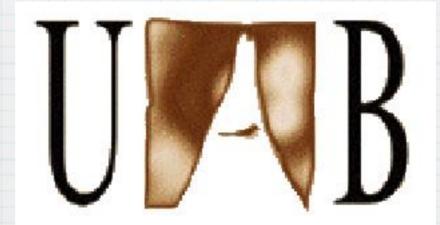
# imapurities inafermisea

Pietro Massignan (UAB&ICFO-Barcelona)

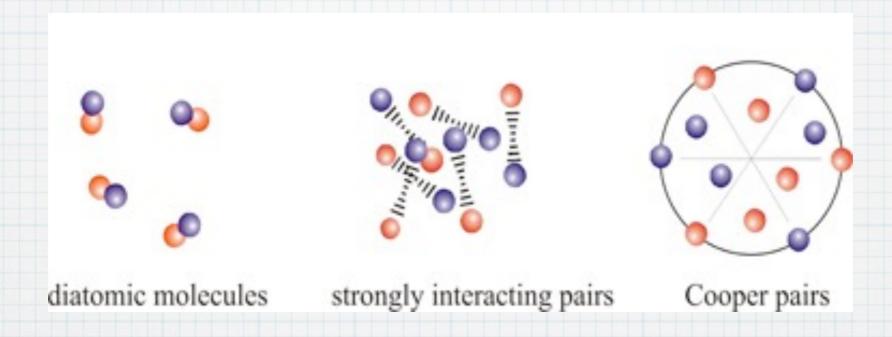




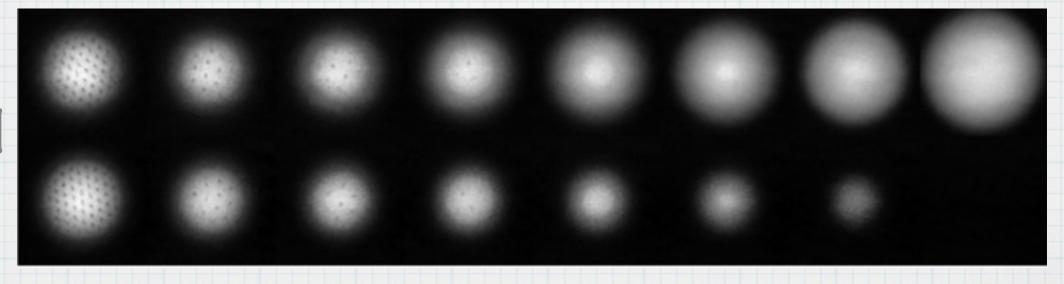


#### Fermi mixtures

BEC-BCS



SF-normal



N=N

N>>N

#### in collaboration with:



Georg Bruun (Aarhus)



Carlos Lobo (Southampton)



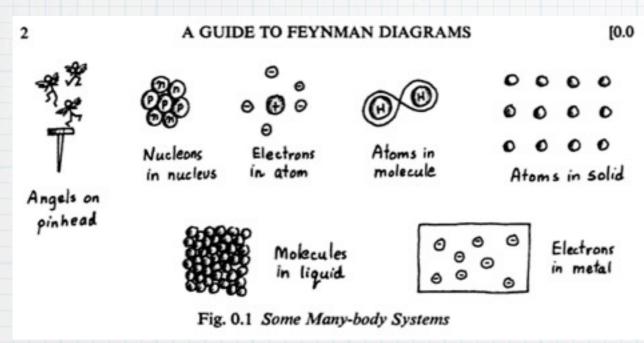
Kayvan Sadegzadeh (Cambridge)



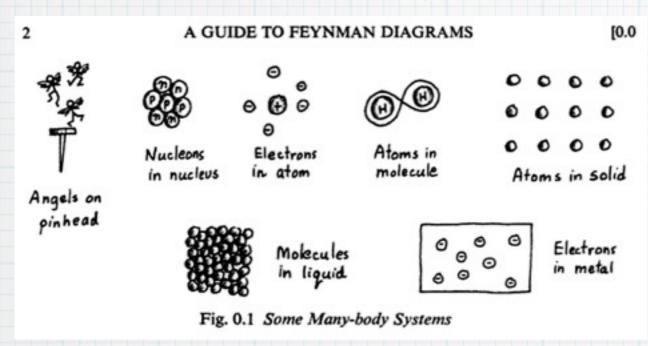
Alessio Recati (Trento)

#### Outline

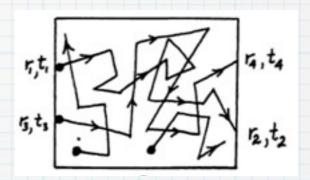
- \* Quasi-particles in many-body systems
- \* The MIT "impurity" experiment
- \* Polarons and molecules
- \* Decay rates
- \* Repulsive polaron

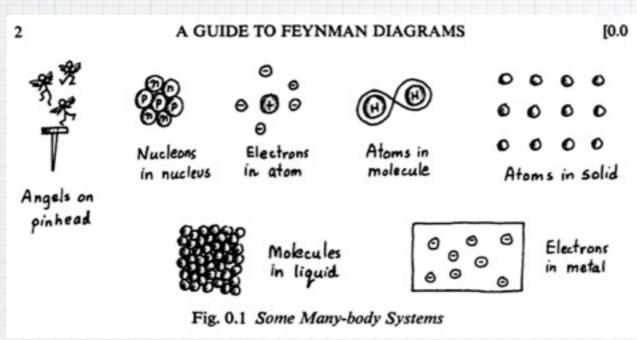


(from Richard Mattuck's book)



(from Richard Mattuck's book)

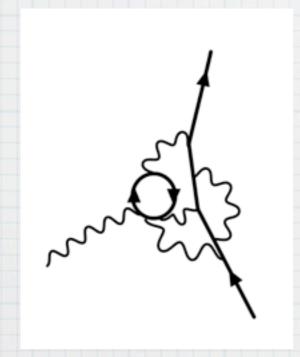


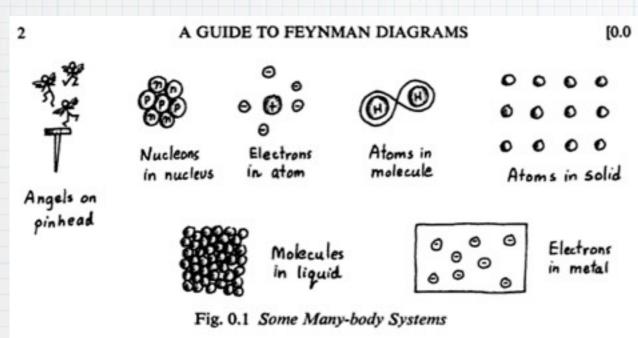


(from Richard Mattuck's book)

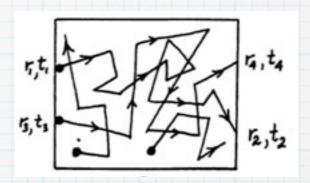


#### Feynman diagrams:

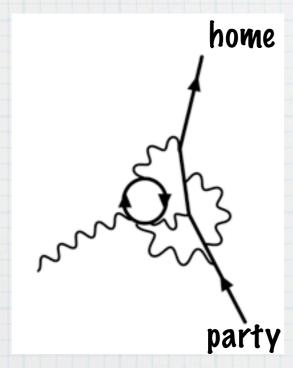




(from Richard Mattuck's book)



#### Feynman diagrams:



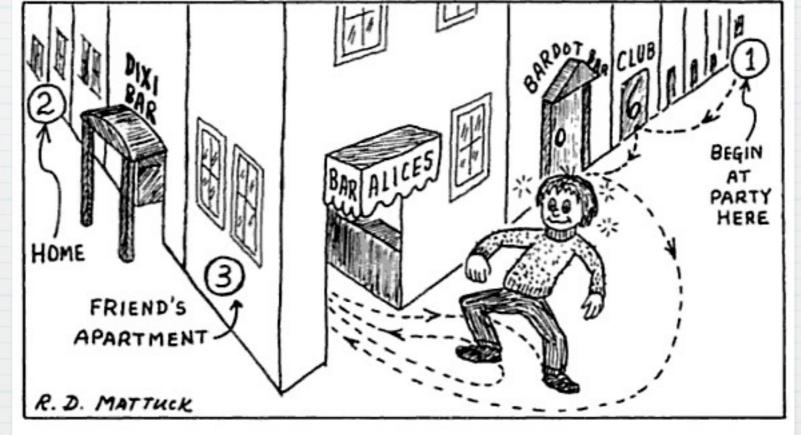
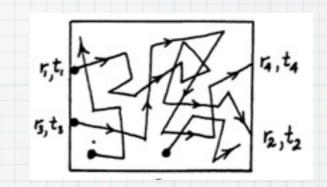
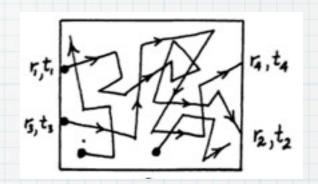


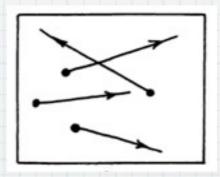
Fig. 1.1 Propagation of Drunken Man

Landau's idea: who cares about real particles?

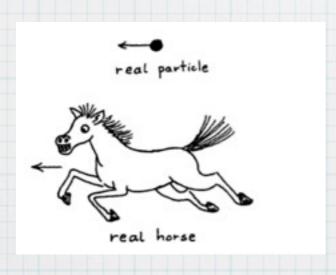


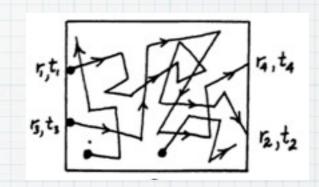
Landau's idea: who cares about real particles?

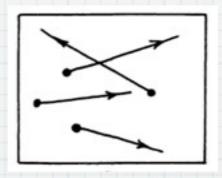




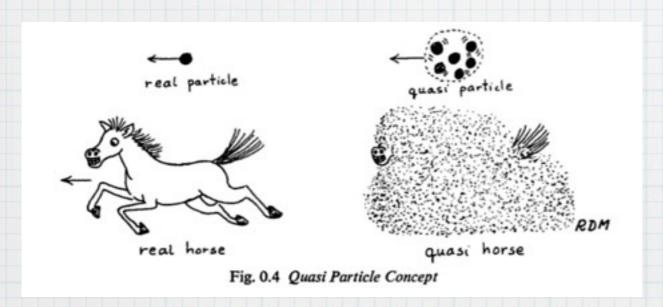
Landau's idea: who cares about real particles?

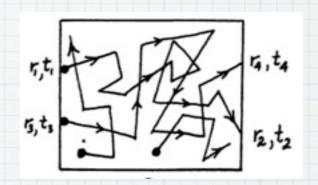


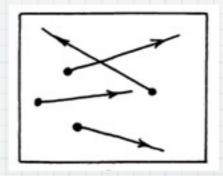




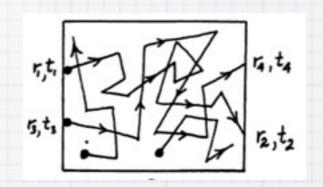
Landau's idea: who cares about real particles?

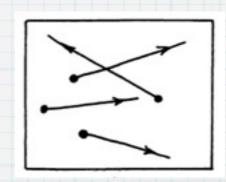


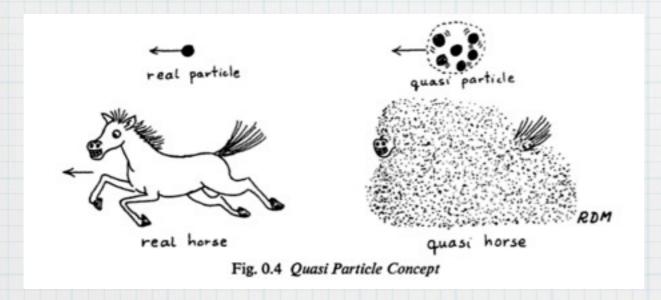




Landau's idea: who cares about real particles?







- a QP is a "free particle" with:
- @ renormalized mass
- @ chemical potential
- @ shielded interactions
- @ q. numbers (charge, spin, ...)
- @ lifetime

#### The MIT experiment

Schirotzek, Wu, Sommer & Zwierlein, PRL 2009

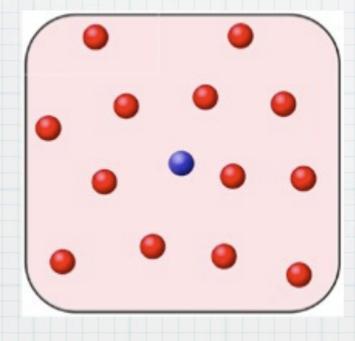
- non-interacting Fermi sea (N>>1)
- a single impurity

BCS

Attraction strength

BEC

(kfa)-1<0



free particle

(kfa)-1>0

#### The MIT experiment

Schirotzek, Wu, Sommer & Zwierlein, PRL 2009

- non-interacting Fermi sea (N>>1)
- a single impurity

BCS Attraction strength BEC

(kfa)-1<0

 $(k_{F}a)^{-1}>0$ 

free particle

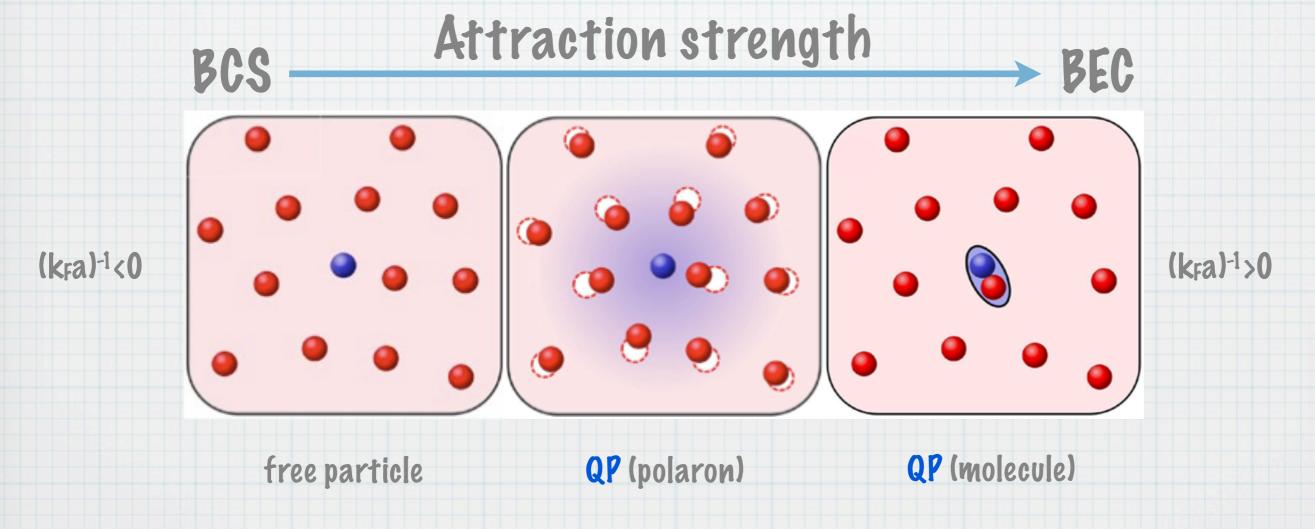
QP (polaron)

-

#### The MIT experiment

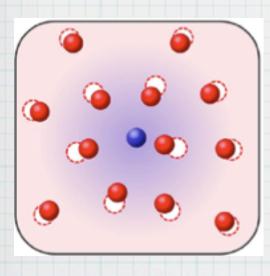
Schirotzek, Wu, Sommer & Zwierlein, PRL 2009

- non-interacting Fermi sea (N>>1)
- a single impurity



P-M transition: Prokof'ev&Svistunov, PRB 2008

(F. Chevy, PRA 2006)

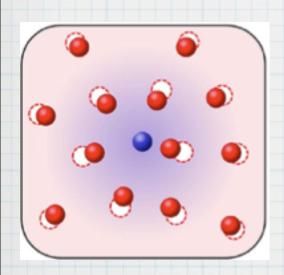


(F. Chevy, PRA 2006)

the impurity

$$|\psi_{\mathbf{p}}\rangle = \phi_0 c_{\mathbf{p}_{\downarrow}}^{\dagger} |0\rangle_{\uparrow} + \sum_{q < k_F}^{k > k_F} \phi_{\mathbf{q}\mathbf{k}} \ c_{\mathbf{p}+\mathbf{q}-\mathbf{k}_{\downarrow}}^{\dagger} \left(c_{\mathbf{k}_{\uparrow}}^{\dagger} c_{\mathbf{q}_{\uparrow}} |0\rangle_{\uparrow}\right)$$

non-interacting Fermi sea



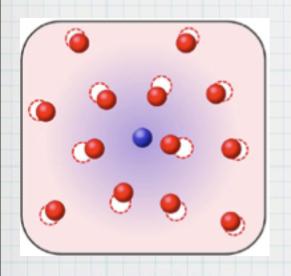
Particle-Hole dressing

(F. Chevy, PRA 2006)

the impurity

$$|\psi_{\mathbf{p}}\rangle = \phi_0 c_{\mathbf{p}\downarrow}^{\dagger} |0\rangle_{\uparrow} + \sum_{q < k_F}^{k > k_F} \phi_{\mathbf{q}\mathbf{k}} \ c_{\mathbf{p}+\mathbf{q}-\mathbf{k}\downarrow}^{\dagger} \left( c_{\mathbf{k}\uparrow}^{\dagger} c_{\mathbf{q}\uparrow} \ |0\rangle_{\uparrow} \right)$$

non-interacting Fermi sea



Particle-Hole dressing

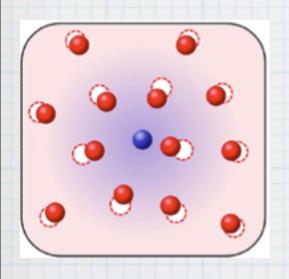
Very good agreement with QMC results for  $\mu_{\downarrow}$  and m\*

(F. Chevy, PRA 2006)

the impurity

$$|\psi_{\mathbf{p}}\rangle = \phi_0 c_{\mathbf{p}\downarrow}^{\dagger} |0\rangle_{\uparrow} + \sum_{q < k_F}^{k > k_F} \phi_{\mathbf{q}\mathbf{k}} \ c_{\mathbf{p}+\mathbf{q}-\mathbf{k}_{\downarrow}}^{\dagger} c_{\mathbf{q}\uparrow} \ |0\rangle_{\uparrow}$$

non-interacting Fermi sea



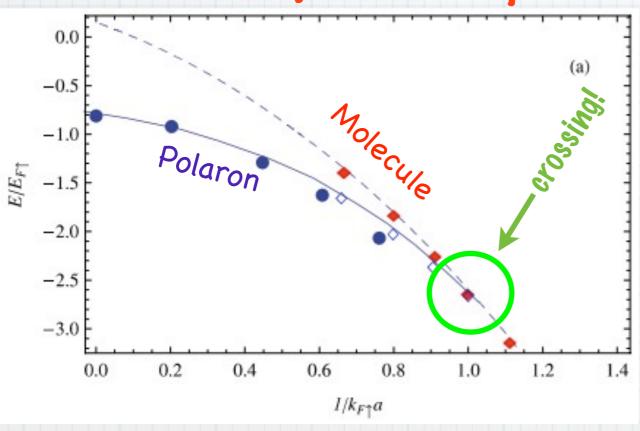
Particle-Hole dressing

Very good agreement with QMC results for  $\mu_{\downarrow}$  and m\*

This variational Ansatz has a diagrammatic equivalent: the forward scattering, or ladder, approximation.

(Combescot et al., PRL 2007)

#### Chemical potential $\mu_{\downarrow}$



--, ···: variat, diagr

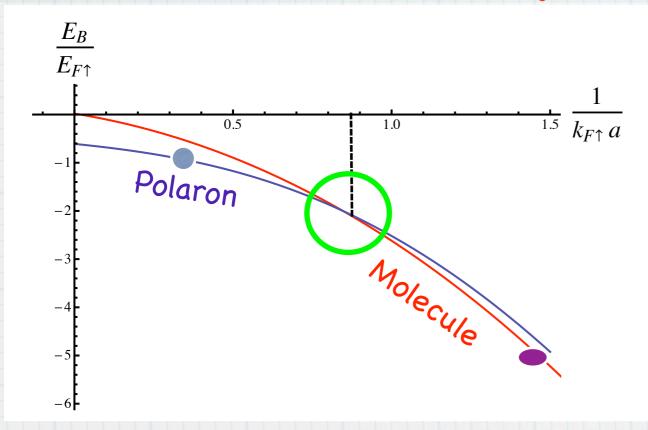
•: MIT expmt

QMC: Prokof'ev&Svistunov

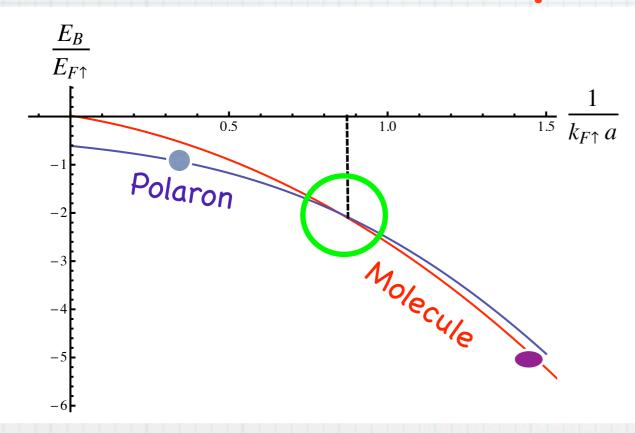
Variational and diagrammatic: Chevy, Recati, Lobo, Stringari, Combescot, Leyronas Massignan&Bruun, Zwerger, Punk, Stoof, Mora,...

Experiments: MIT, ENS

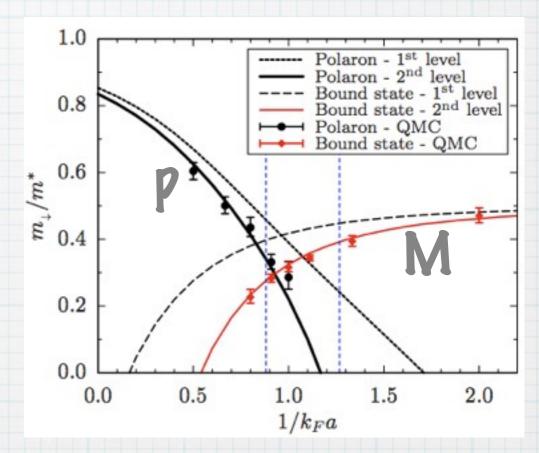
#### Chemical potential $\mu_{\downarrow}$



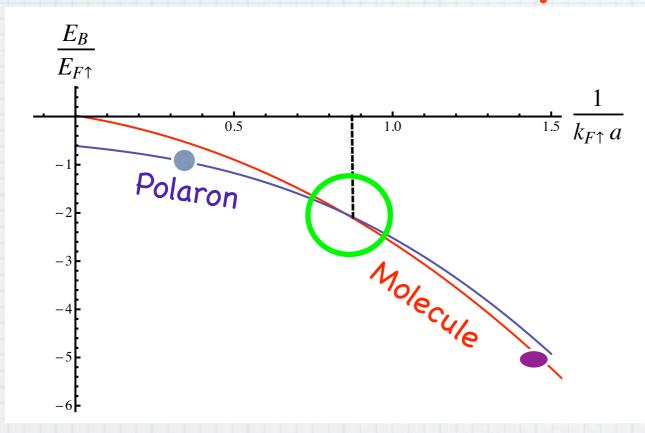
#### Chemical potential $\mu_{\downarrow}$



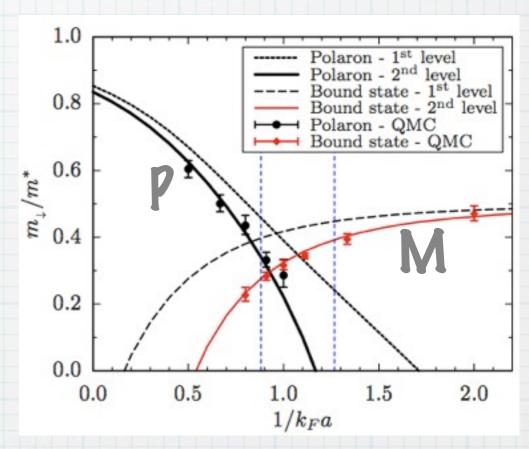
#### Effective mass m\*



#### Chemical potential $\mu_{\downarrow}$



#### Effective mass m\*



P-P Interactions: Mora&Chevy, PRL 2010

Zhenhua, Zöllner & Pethick, PRL 2010

# Equation of state of a unitary Fermi gas

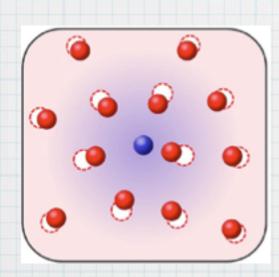
In the normal phase at T=0,

non-interacting 1

non-interacting QP

$$P = \frac{1}{15\pi^2} \left(\frac{2m_{\uparrow}}{\hbar^2}\right)^{3/2} \mu_{\uparrow}^{5/2} + \frac{1}{15\pi^2} \left(\frac{2m_{\downarrow}^*}{\hbar^2}\right)^{3/2} (\mu_{\downarrow} - A\mu_{\uparrow})^{5/2}$$

A = -0.615 $m_{\downarrow}^* = 1.2m$ 



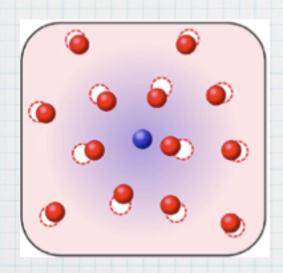
# Equation of state of a unitary Fermi gas

In the normal phase at T=0,

non-interacting 1

$$P = \frac{1}{15\pi^2} \left(\frac{2m_{\uparrow}}{\hbar^2}\right)^{3/2} \mu_{\uparrow}^{5/2} + \frac{1}{15\pi^2} \left(\frac{2m_{\downarrow}^*}{\hbar^2}\right)^{3/2} (\mu_{\downarrow} - A\mu_{\uparrow})^{5/2}$$

A = -0.615 $m_{\perp}^* = 1.2m$ 



Same thermodynamics for:

- ultracold atoms
- dilute neutron matter

# What's left?

## What's left?

- chemical potential
- ✓ renormalized mass
- shielded interactions
- ✓ lifetime

G. Bruun & PM, PRL 2010

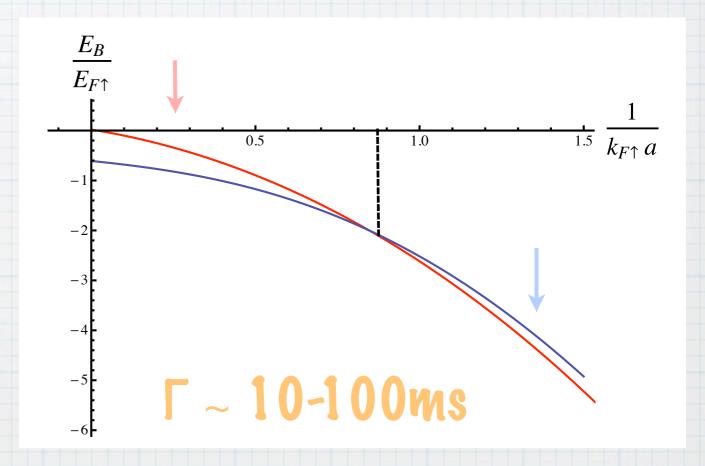
#### Very long QP lifetimes!

G. Bruun & PM, PRL 2010

$$\Gamma_P \sim Z_M \left(\Delta\omega\right)^{9/2}$$

$$\Delta\omega = \omega_P - \omega_M$$

$$\Gamma_M \sim Z_P \left(-\Delta\omega\right)^{9/2}$$



$$\Delta\omega = \omega_P - \omega_M > 0$$

Polaron: 
$$G_{\downarrow}(\mathbf{p},z)^{-1}=G_{\downarrow}^{0}(\mathbf{p},z)^{-1}-\Sigma_{P}(\mathbf{p},z)$$

**Vecay rate:** 
$$\Gamma_P = -\mathrm{Im}\Sigma_P(p=0,\omega_P)$$

$$\Delta\omega = \omega_P - \omega_M > 0$$

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Hole expansion: 
$$\Sigma_P(\mathbf{p},z) = \Sigma_P^{(1)}(\mathbf{p},z) + \Sigma_P^{(2)}(\mathbf{p},z) + \dots$$

$$\Delta\omega = \omega_P - \omega_M > 0$$

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$$\Sigma_P(\mathbf{p},z) = \Sigma_P^{(1)}(\mathbf{p},z) + \Sigma_P^{(2)}(\mathbf{p},z) + \dots$$

Ladder: 
$$\downarrow \rightarrow (\Sigma_P^{(1)}) \rightarrow = T_2$$

Ladder:

$$T_2$$
 $T_2$ 
 $T_2$ 

in the ladder approx.

$$\Delta\omega = \omega_P - \omega_M > 0$$

Polaron: 
$$G_{\downarrow}(\mathbf{p},z)^{-1}=G_{\downarrow}^{0}(\mathbf{p},z)^{-1}-\Sigma_{P}(\mathbf{p},z)$$

**Pecay rate:**  $\Gamma_P = -\mathrm{Im}\Sigma_P(p=0,\omega_P)$ 

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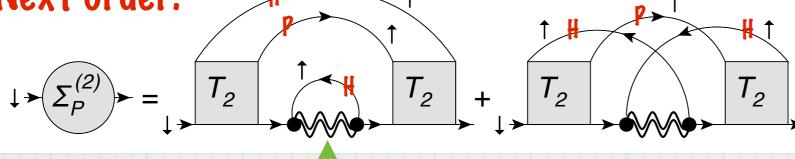
#### Ladder:

$$\downarrow \rightarrow \left( \sum_{P}^{(1)} \right) \rightarrow = \left( \begin{array}{c} \uparrow \\ \uparrow \\ \downarrow \end{array} \right)$$

$$\downarrow \to \left( \sum_{P}^{(1)} \right) = \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) = \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) + \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) = \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) + \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) = \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) + \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) = \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) + \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) = \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) + \left( \begin{array}{c} \uparrow \\ \downarrow \\ \downarrow \end{array} \right) = \left( \begin{array}{c} \uparrow \\ 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no damping

#### Next order:



dressed molecule

$$\Delta\omega = \omega_P - \omega_M > 0$$

Polaron: 
$$G_{\downarrow}(\mathbf{p},z)^{-1}=G_{\downarrow}^{0}(\mathbf{p},z)^{-1}-\Sigma_{P}(\mathbf{p},z)$$

**Pecay rate:**  $\Gamma_P = -\mathrm{Im}\Sigma_P(p=0,\omega_P)$ 

Hole expansion:  $\Sigma_P(\mathbf{p},z) = \Sigma_P^{(1)}(\mathbf{p},z) + \Sigma_P^{(2)}(\mathbf{p},z) + \dots$ 

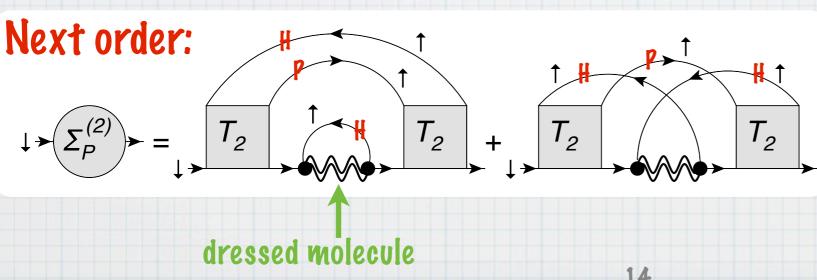
#### Ladder:

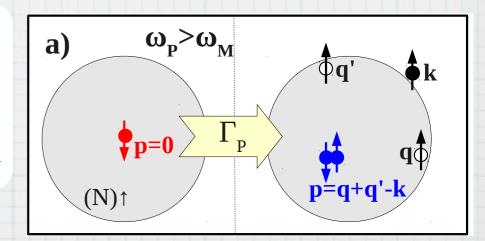
$$\downarrow \rightarrow \left( \sum_{P}^{(1)} \right) \rightarrow = T_2$$

$$T_2 = + T_2$$
in the ladder approx.

no damping

#### 3-body process





## Fermi's Golden rule

atom-molecule coupling

matrix element~Δω

density of final states  $\sim \Delta \omega^{7/2}$ 

## Fermi's Golden rule

atom-molecule coupling

$$\Gamma_P = \frac{g^2 Z_M}{2} \int \frac{d^3 k \ d^3 q \ d^3 q'}{(2\pi)^9} \left[ F(\mathbf{q}, \mathbf{k}, \omega_P) - F(\mathbf{q'}, \mathbf{k}, \omega_P) \right]^2 \delta \left( \Delta \omega + \xi_{q\uparrow} + \xi_{q'\uparrow} - \xi_{k\uparrow} - \frac{(\mathbf{q} + \mathbf{q'} - \mathbf{k})^2}{2m_M^*} \right)$$

matrix element~Δω

density of final states  $\sim \Delta \omega^{7/2}$ 

$$q, q' < k_F , k > k_F$$

$$F(\mathbf{q}, \mathbf{k}, \omega) = T_2(\mathbf{q}, \omega + \xi_{q\uparrow}) G^0_{\downarrow}(\mathbf{q} - \mathbf{k}, \omega + \xi_{q\uparrow} - \xi_{k\uparrow})$$

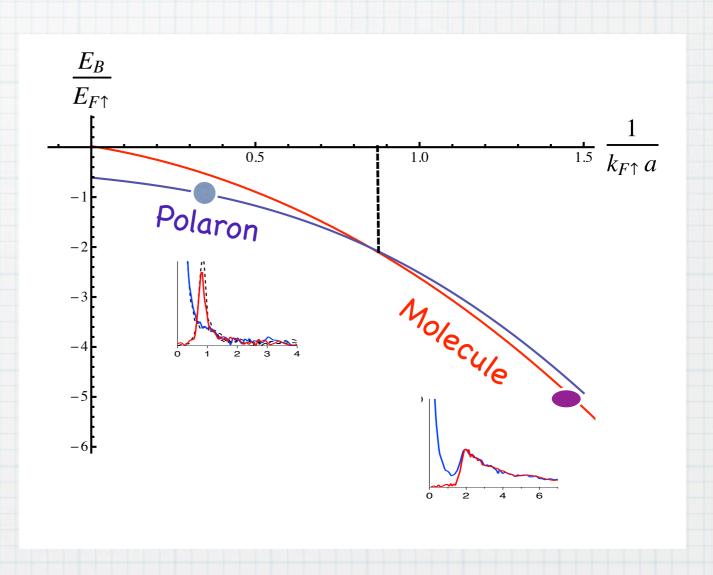
## Experimental observation

#### Methods:

- RF spectra
- Collective modes to measure m\* vs. time
- Density profiles in the trap

#### Issues:

- \* Phase separation?
  - \* stabilized by finite T
  - \* work with m ↓ ≠ m ↑
  - \* use bosonic impurities
- \* No decay to deeply bound molecular states



# ...is there more?

Yes..

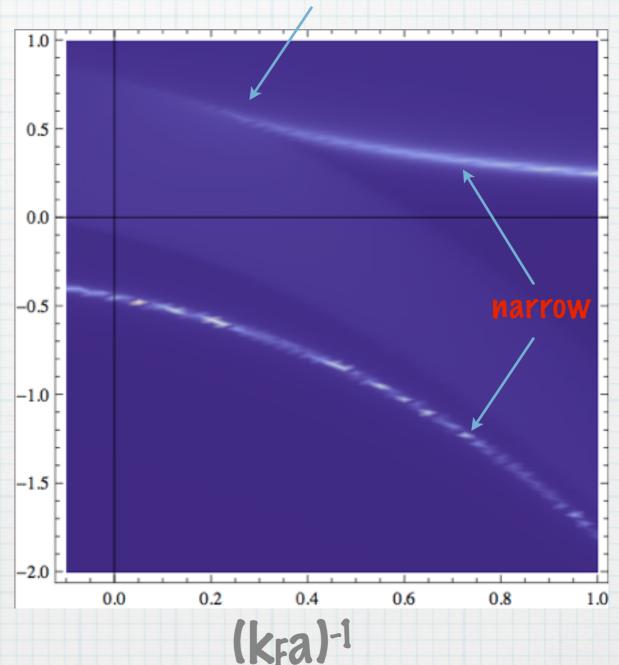
## ...is there more?

Yes..

spectral function

 $A_{\downarrow}(\omega)=-Im[G_{\downarrow}(k=0,\omega+i0+)]$ 

energy



broadening

<sup>40</sup>K impurity in a Fermi sea of <sup>6</sup>Li

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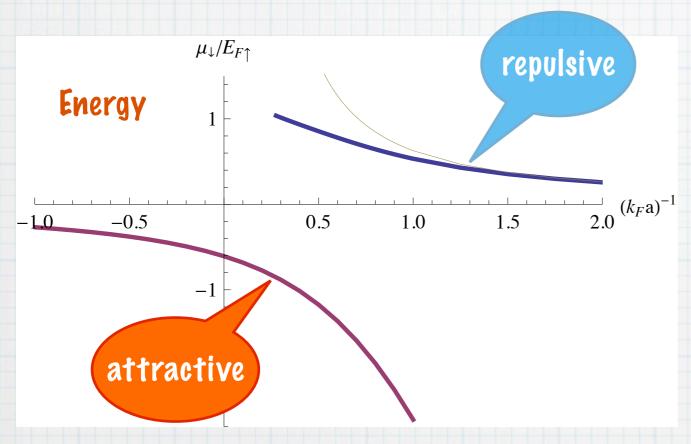


0.5 0.0 0.4 0.0 0.2 0.6 0.8 1.0 (kfa)-1

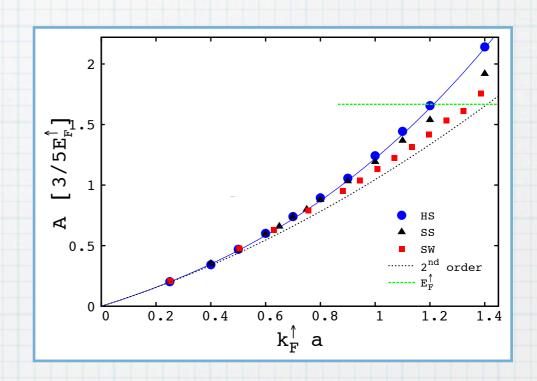
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#### Polaron energies

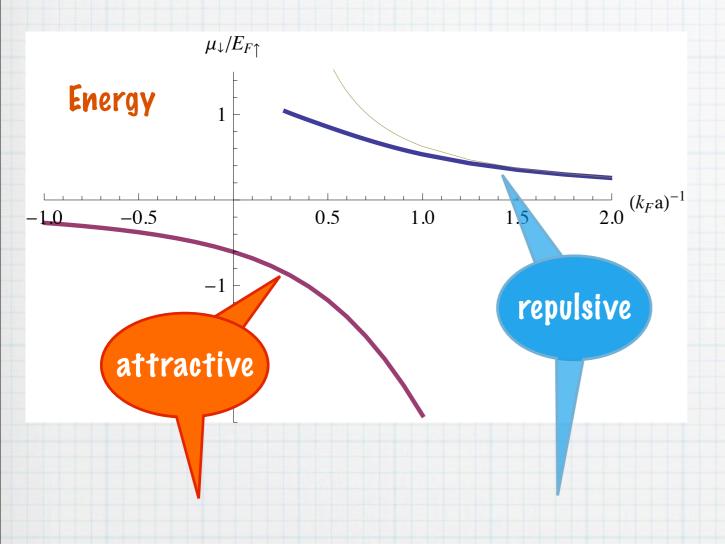


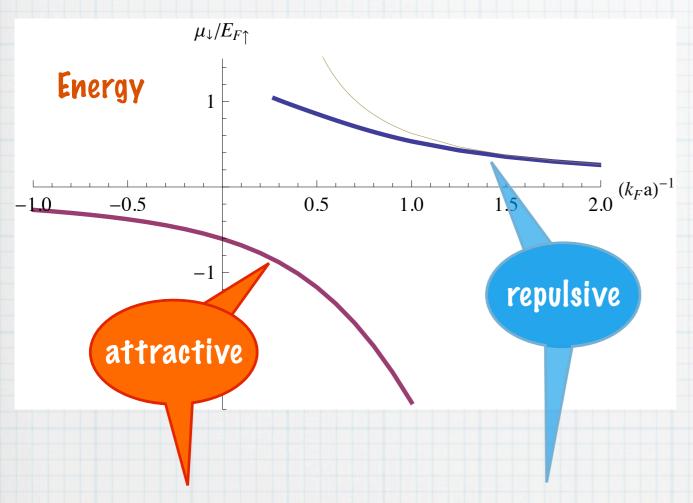
Massignan & Bruun (in preparation)

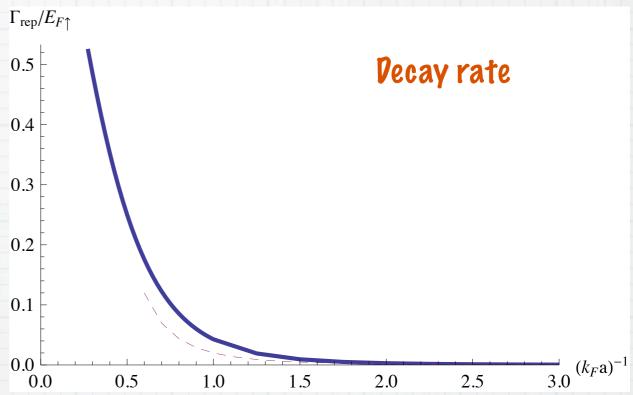


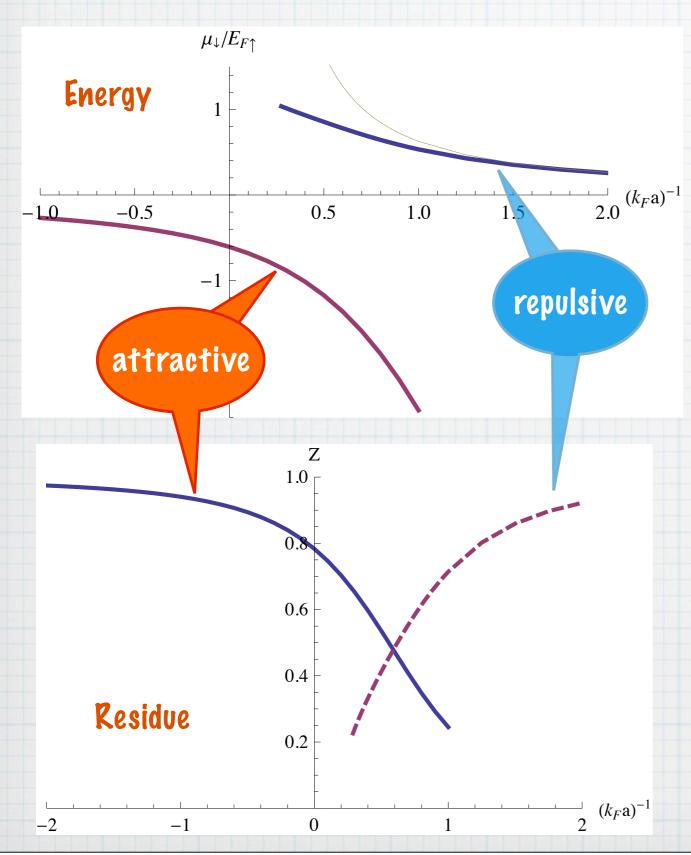
QMC by Pilati et al., PRL 2010

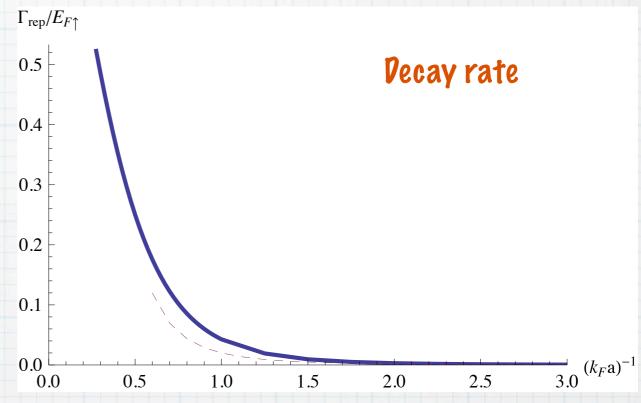
weak coupling: 
$$\frac{\mu_{\downarrow}}{E_{F\uparrow}} = \frac{4}{3\pi}(k_F a) + \frac{2}{\pi^2}(k_F a)^2 + \dots$$

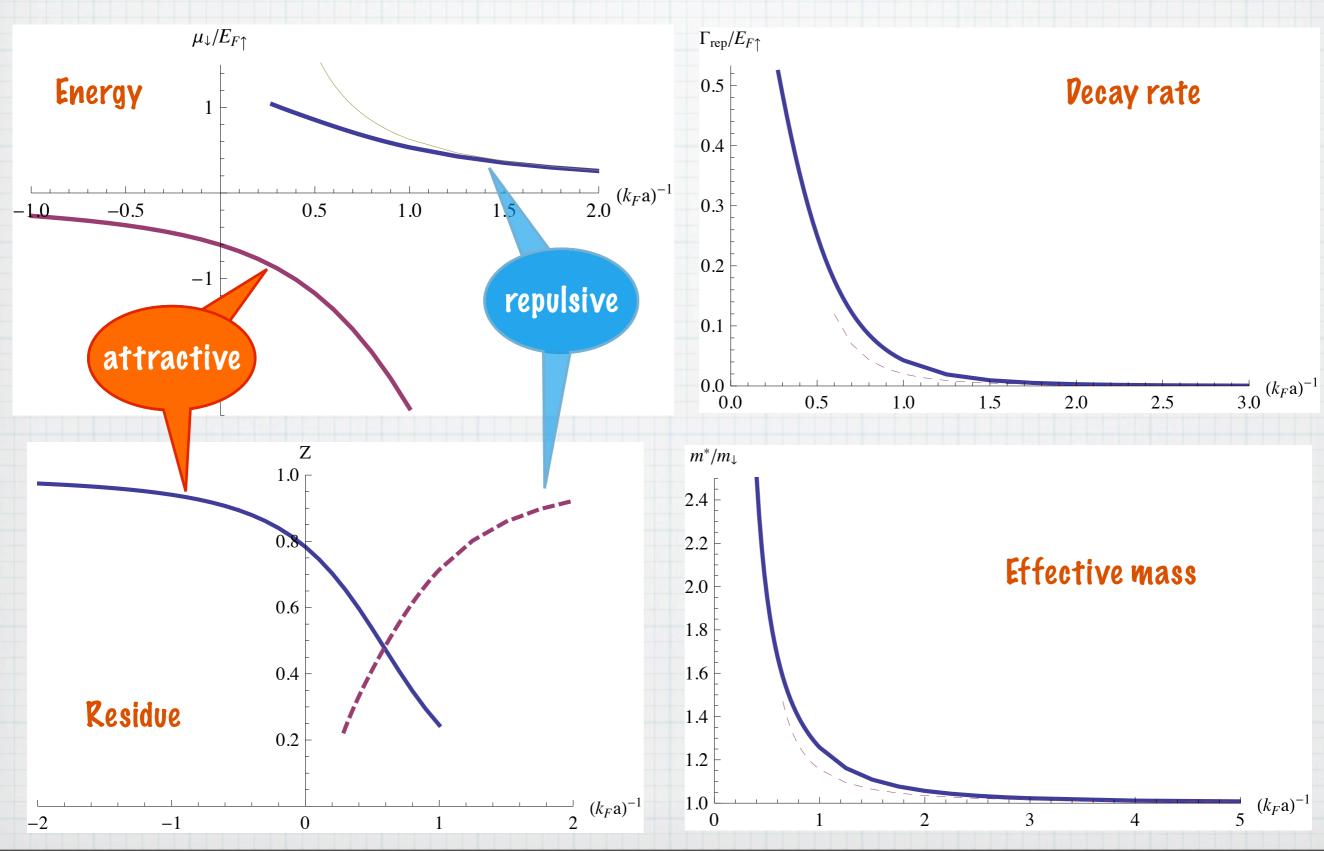






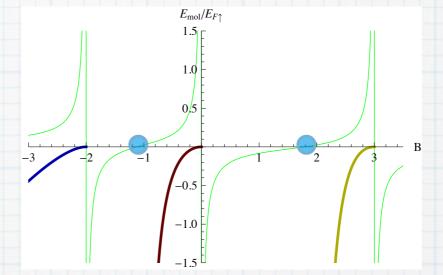


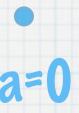




## a toy model with 3 FR

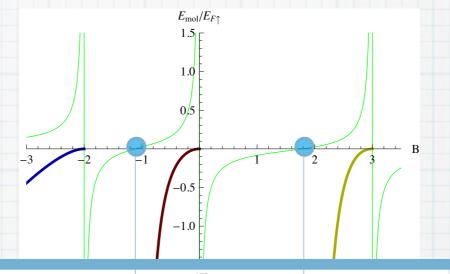
2-body bound states:





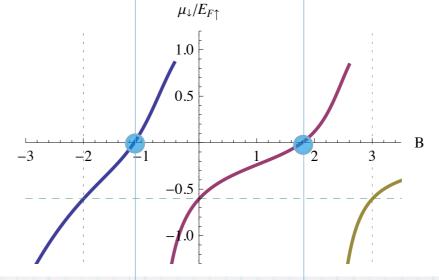
## atoy model with 3 FR

2-body bound states:





Polaronic states:

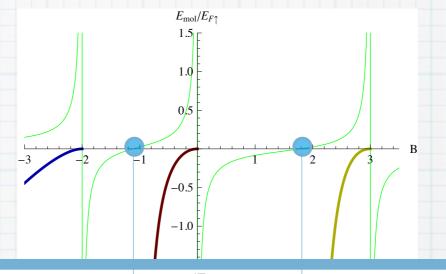


weak coupling:

E∝a

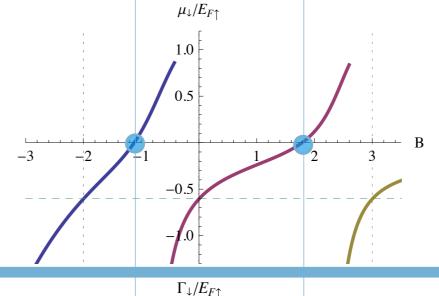
## atoy model with 3 FR

2-body bound states:





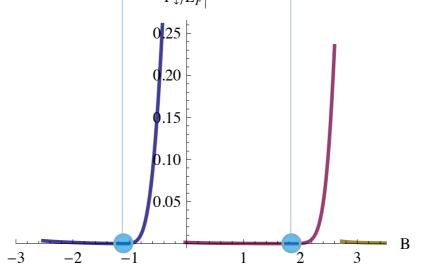
Polaronic states:



weak coupling:

E∝a

Decay rates:



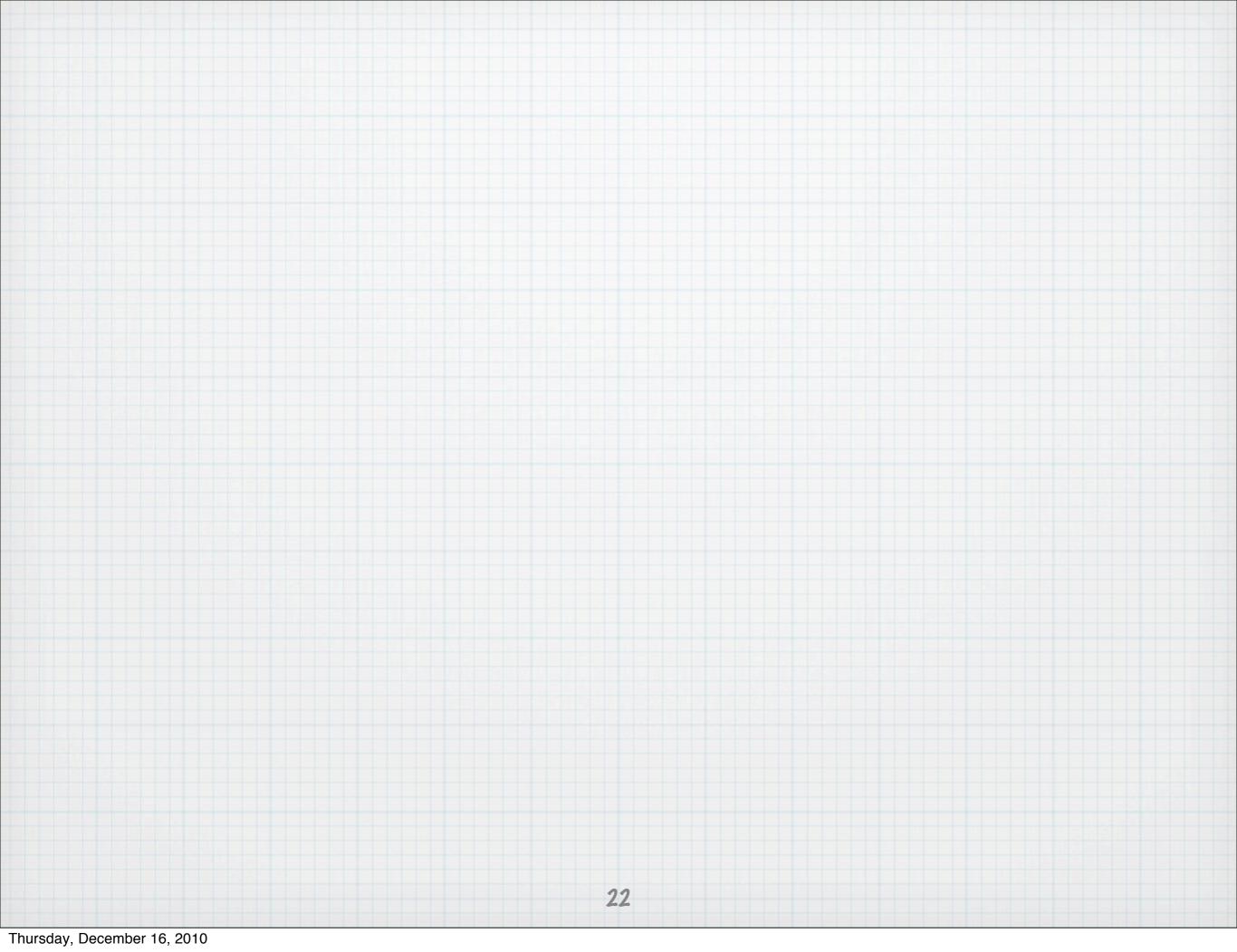
 $\Gamma \propto \theta(a)$ 

20

## Conclusions

- Quasi-particles properties fix completely the equation of state
- At small momenta, the process coupling molecules and polarons requires at least 3-bodies
- Strongly suppressed P-M decay due to a combination of small final density of states and Fermi statistics
- Expected lifetimes ~ 10-100ms
- Complete characterization of the repulsive branch

G. Bruun and PM, Phys. Rev. Lett. 105, 020403 (2010)
K. Sadeghzadeh, G. Bruun, C. Lobo, PM, and A Recati, arXiv:1012.0484
PM and G. Bruun, coming soon



$$\phi_q = rac{\sqrt{8\pi a^3}}{1+q^2a^2}$$
 or  $\phi_r \propto rac{\mathrm{e}^{-r/a}}{r}$ 

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dressed molecule:

$$D(\mathbf{p},\omega) \simeq rac{Z_M}{\omega - \omega_M - p^2/2m_M^*}.$$

atom-molecule coupling:

$$\bullet = \frac{1}{g(\mathbf{p}, z)} = \int \frac{d^3q}{(2\pi)^3} \phi_q \frac{1 - f(\xi_{\mathbf{p} - \mathbf{q}\uparrow})}{z - \xi_{\mathbf{p} - \mathbf{q}\uparrow} - \xi_{q\downarrow}} \sim -\sqrt{\frac{m_r^2 a}{2\pi}}$$

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(Bruun&Pethick, PRL 2004)

$$\Gamma_P = \frac{g^2 Z_M}{2} \int \frac{d^3 k \ d^3 q \ d^3 q'}{(2\pi)^9} \left[ F(\mathbf{q}, \mathbf{k}, \omega_P) - F(\mathbf{q'}, \mathbf{k}, \omega_P) \right]^2 \delta \left( \Delta \omega + \xi_{q\uparrow} + \xi_{q'\uparrow} - \xi_{k\uparrow} - \frac{(\mathbf{q} + \mathbf{q'} - \mathbf{k})^2}{2m_M^*} \right)$$

$$q, q' < k_F$$
,  $k > k_F$ 

$$F(\mathbf{q}, \mathbf{k}, \omega) = T_2(\mathbf{q}, \omega + \xi_{q\uparrow}) G^0_{\downarrow}(\mathbf{q} - \mathbf{k}, \omega + \xi_{q\uparrow} - \xi_{k\uparrow})$$

#### In the neighborhood of the P-M crossing,

$$\int \frac{d^3k \ d^3q \ d^3q'}{(2\pi)^9} \delta(\ldots) \sim (m_M^*)^{3/2} (\Delta\omega)^{7/2}$$

$$\Delta\omega\ll\epsilon_F$$
 $q\simeq k\simeq k'\simeq k_F$ 

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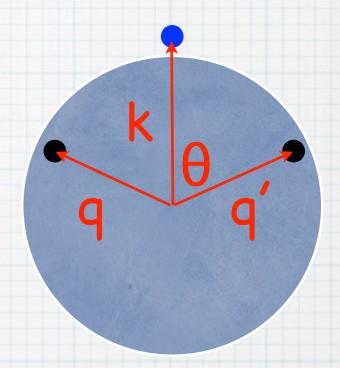
The P+H+H form an equilateral triangle, since  $q+q'-k\sim 0$ 

At the crossing, Fermi antisymmetry yields a vanishing of the matrix element:

$$F(\mathbf{q}, \mathbf{k}, \omega_P) - F(\mathbf{q}', \mathbf{k}, \omega_P)$$

Expand the difference to get an extra factor of  $\Delta\omega$ :

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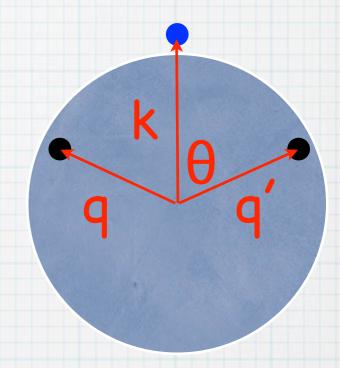
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the angular dependence of F is only on  $\theta$ 

Expand the difference to get an extra factor of  $\Delta\omega$ :

$$\Gamma_P \sim Z_M(k_F a) \left(m_M^*\right)^{3/2} \left(\Delta \omega\right)^{9/2}$$

1st order transition between the P&M states (no coupling at the crossing)

## Mol-Pol decay

$$\Delta\omega = \omega_P - \omega_M < 0$$

**Molecule:** 
$$D(\mathbf{p}, z)^{-1} = D_0(\mathbf{p}, z)^{-1} - \Sigma_M(\mathbf{p}, z)$$

**Pecay rate:** 
$$\Gamma_M = -\mathrm{Im}\Sigma_M(p=0,\omega_M)$$

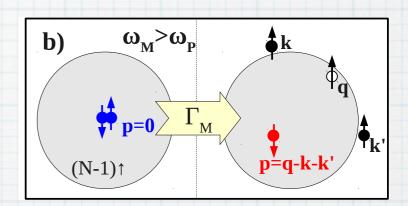
$$\textbf{Vacuum:} \quad D_0(\mathbf{p},z) = \int d^3\check{q}\phi_q^2 \frac{1-f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z-\xi_{\mathbf{p}-\mathbf{q}\uparrow}-\xi_{q\downarrow}} + \frac{T_2(\mathbf{p},z)}{g(\mathbf{p},z)^2}$$

## Mol-Pol decay

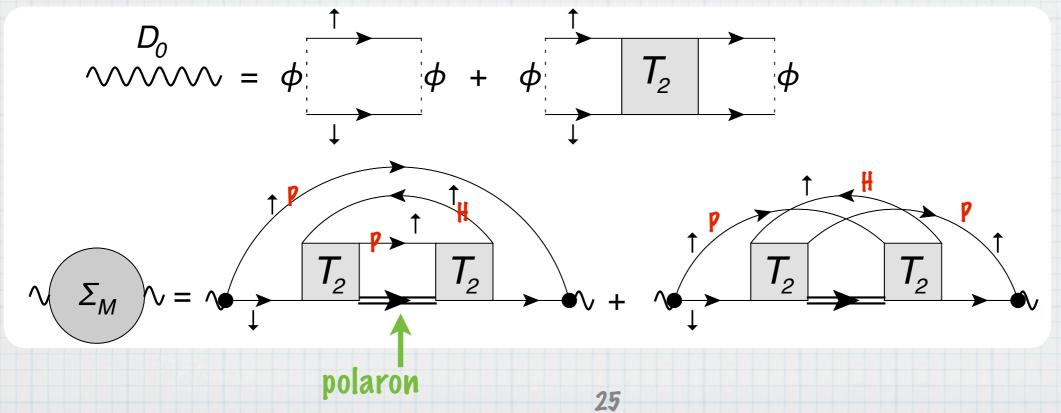
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3-body process

$$\Gamma_{M} = \frac{g^{2}Z_{P}}{2} \int \frac{d^{3}k \ d^{3}k' \ d^{3}q}{(2\pi)^{9}} \left[ C(\mathbf{q}, \mathbf{k}, \omega_{M}) - C(\mathbf{q}, \mathbf{k'}, \omega_{M}) \right]^{2} \delta \left( |\Delta\omega| + \xi_{q\uparrow} - \xi_{k\uparrow} - \xi_{k'\uparrow} - \frac{(\mathbf{q} - \mathbf{k} - \mathbf{k'})^{2}}{2m_{P}^{*}} \right)$$

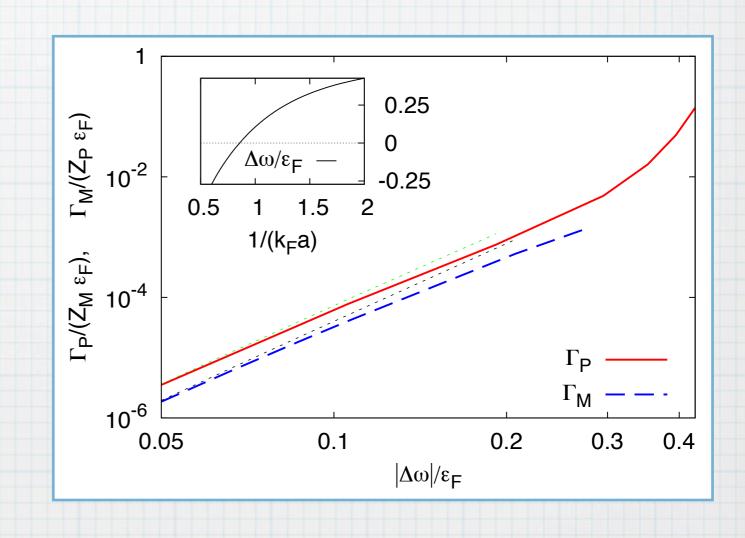
In the neighborhood of the M-P crossing,  $\Gamma_M \sim Z_P(k_F a) \, (m_P^*)^{3/2} \, (-\Delta \omega)^{9/2}$ 

In the numerics:

$$\omega_{M} = -\frac{\hbar^{2}}{2m_{r}a^{2}} - \epsilon_{F} + g_{3}n_{\uparrow}$$

$$a_{3} = 1.18a$$

$$T_{2}(\mathbf{p}, \omega) = \frac{2\pi a/m_{r}}{1 - \sqrt{2m_{r}a^{2}\left(\frac{p^{2}}{2m_{M}} - \omega - \epsilon_{F} + g_{3}n_{\uparrow}\right)}}$$



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In the neighborhood of the M-P crossing,  $\Gamma_M \sim Z_P(k_F a) \, (m_P^*)^{3/2} \, (-\Delta \omega)^{9/2}$ 

For both decay processes, very long lifetimes are ensured by:

- limited phase-space
- Fermi antisymmetry

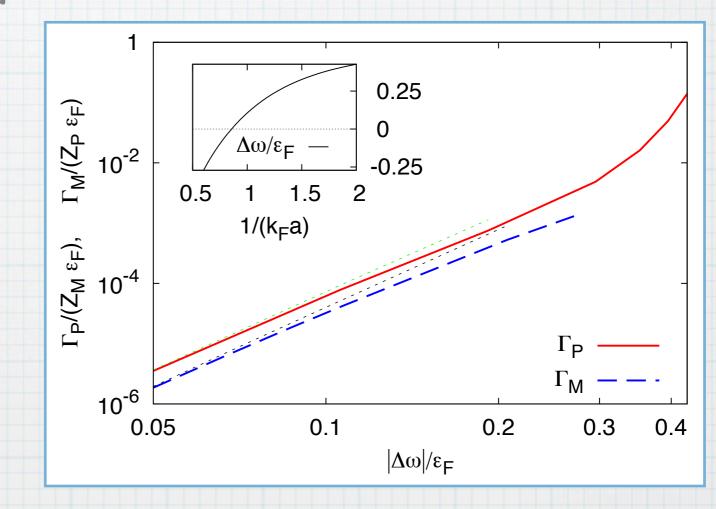
much longer than usual Fermi liquids

In the numerics:

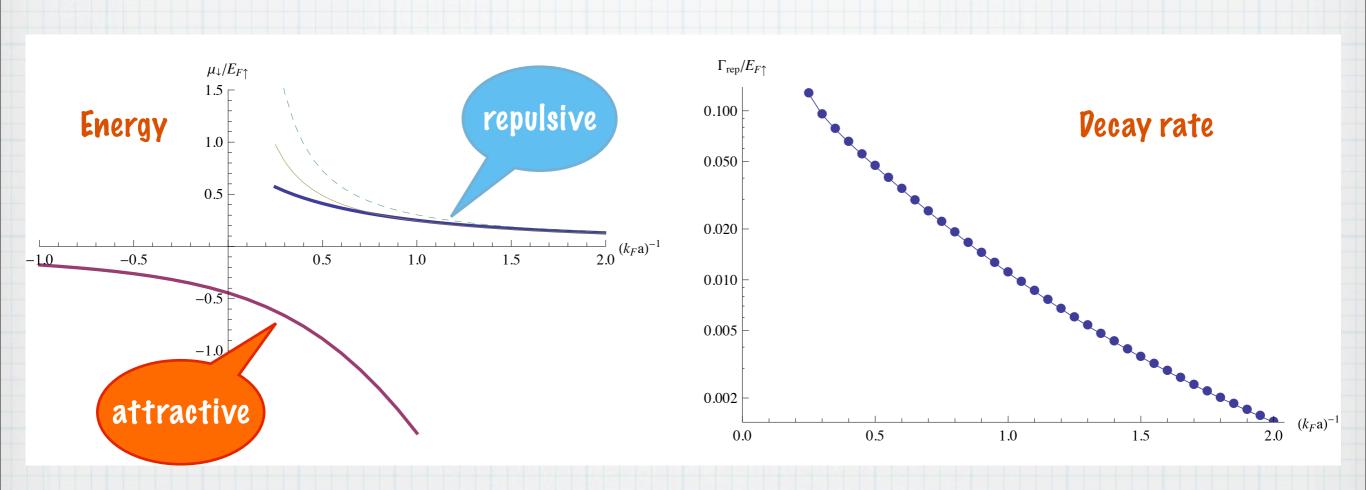
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## Repulsive polaron



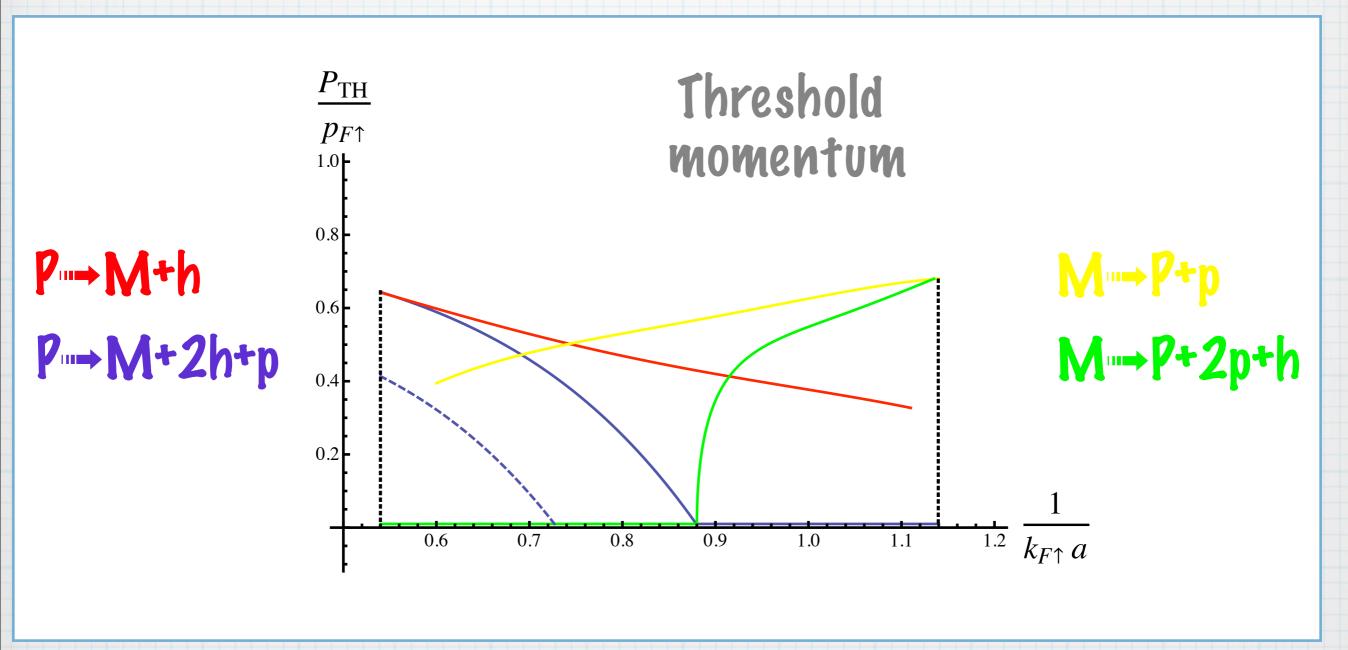
A 40K impurity in a Fermi sea of 6Li

atom-molecule coupling:

$$\bullet = \frac{1}{g(\mathbf{p}, z)} = \int \frac{d^3q}{(2\pi)^3} \phi_q \frac{1 - f(\xi_{\mathbf{p} - \mathbf{q}\uparrow})}{z - \xi_{\mathbf{p} - \mathbf{q}\uparrow} - \xi_{q\downarrow}} \sim -\sqrt{\frac{m_r^2 a}{2\pi}}$$

$$\textbf{Vacuum:} \quad D_0(\mathbf{p},z) = \int d^3\check{q}\phi_q^2 \frac{1-f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z-\xi_{\mathbf{p}-\mathbf{q}\uparrow}-\xi_{q\downarrow}} + \frac{T_2(\mathbf{p},z)}{g(\mathbf{p},z)^2}$$

# Pecay of p=0 QP



(preliminary)

The many-body physics discussed here can be

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defined

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calculated

The many-body physics discussed here can be

defined

calculated

and measured!