Efimov states close to a Feshbach resonance

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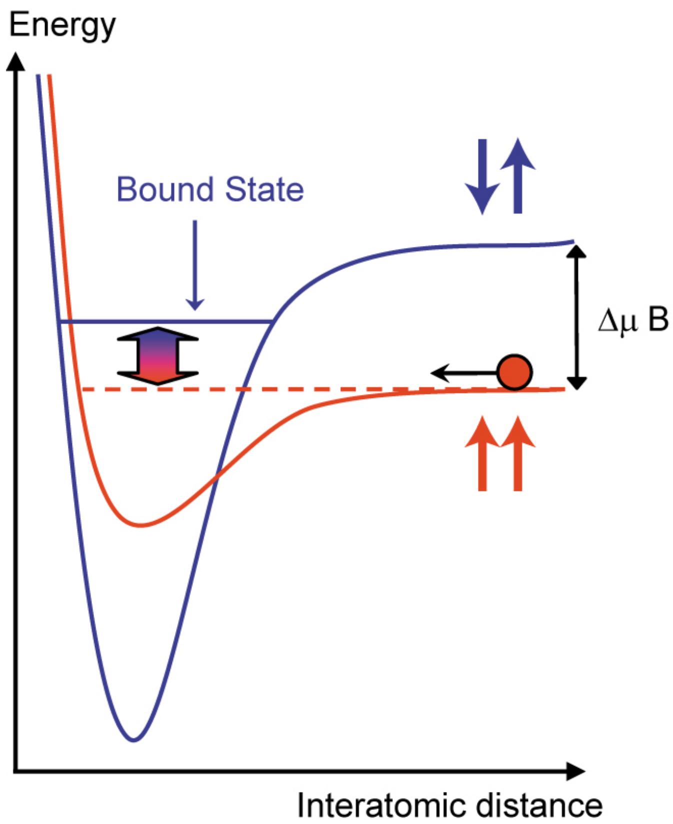
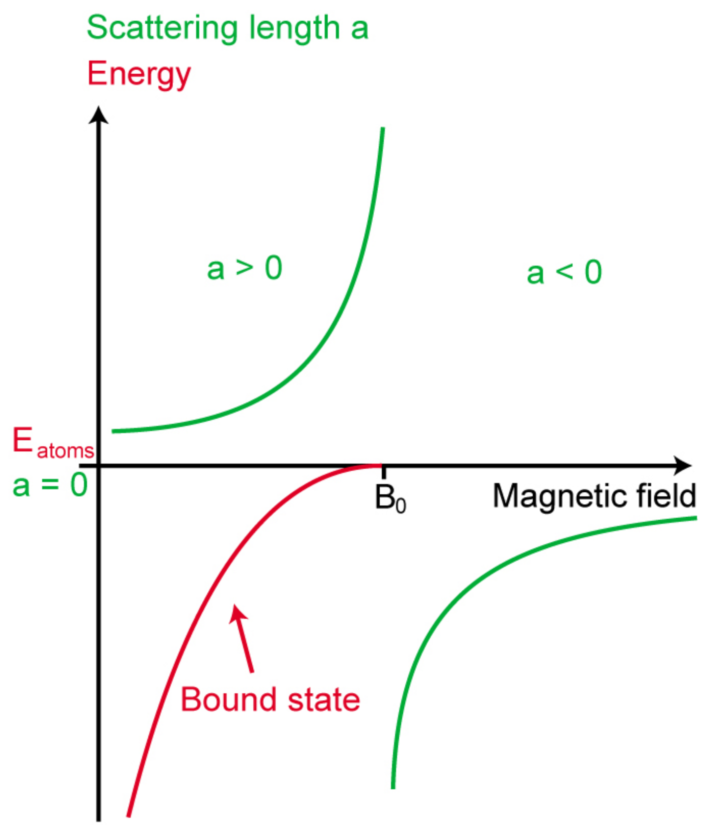
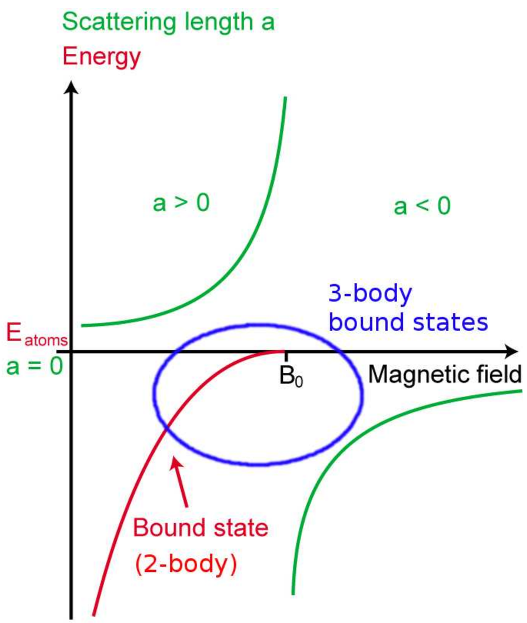
Feshbach resonances in ultracold gases





Efimov (70s): as $|a| \gg r_0$ there's more...



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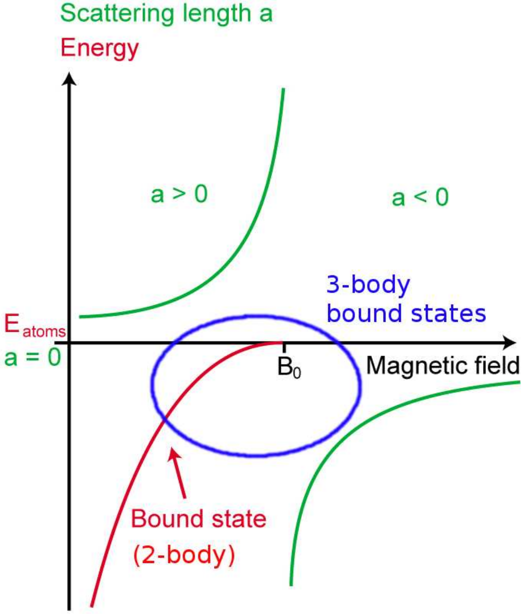
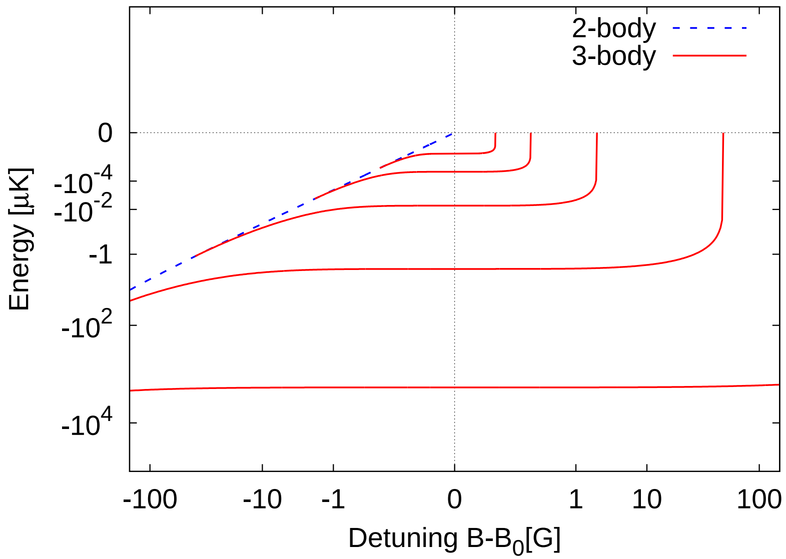


rescaled units: $E^{-1/10}$ vs $(B - B_0)^{1/5}$



Discrete scaling: $\frac{a_{n+1}}{a_n} = 22.7, \qquad \frac{E_{n+1}}{E_n} = 22.7^{-2}$

Review: Braaten and Hammer (2006).

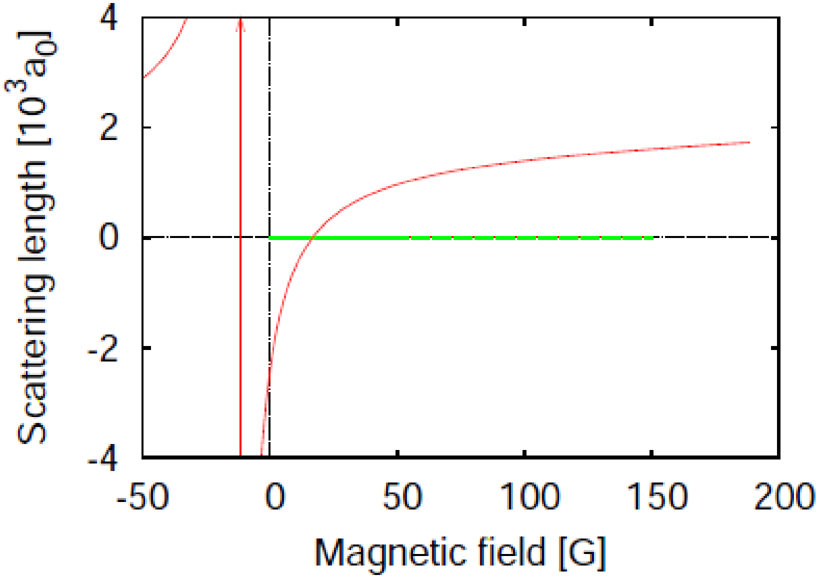
Universality: physics depends on a and an additional (3-body or low-energy) parameter.

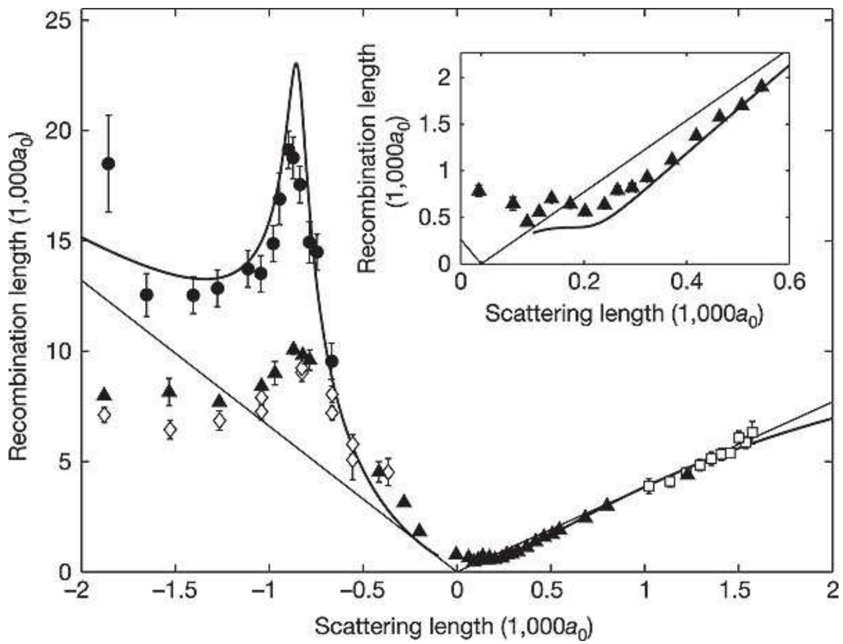
Innsbruck experiment (Nature 2006)

First experimental evidence for Efimov physics in cold gases.

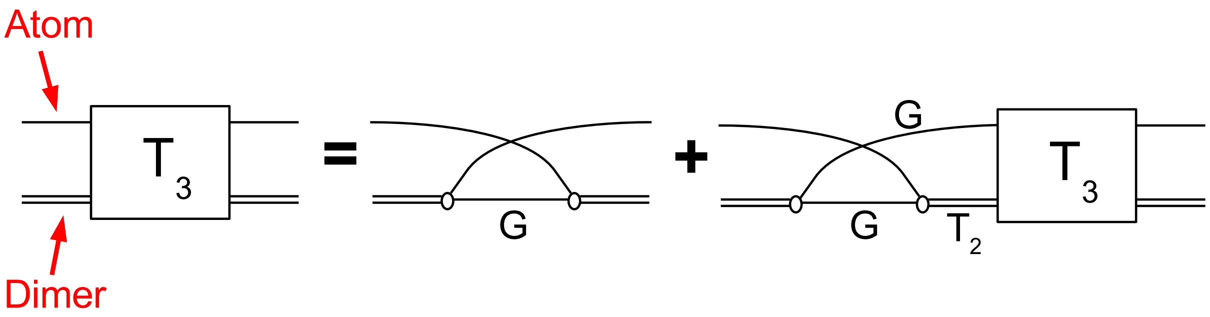
Measurement of the recombination rate for A+A+A⇌A+D (A:atom, D:dimer) in a thermal gas of bosonic caesium.

$$\mathrm{d}n/\mathrm{d}t \propto -n^3 \rho_{\mathrm{rec}}^4$$





T_3: T-matrix for atom-dimer scattering



2-body s-wave interaction $\longrightarrow$ **STM** eq. (Skorniakov&Ter-Martirosian, 1956):

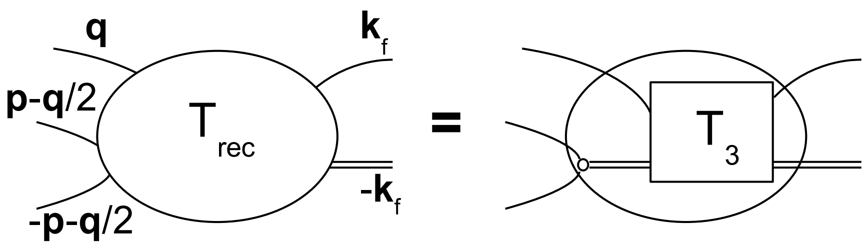
$$T_3(k, k'; E) = G(\ldots) + \int_0^\infty \mathrm{d}\mathbf{k}'' G(\ldots)G(\ldots)T_2(\ldots)T_3(\ldots).$$

G: atom propagator, T_2 : dimer propagator (... more to come)

Scattering length: $a_{\mathrm{AD}} \propto T_3(0, 0; E_b)$.

Brodsky, Kagan, Klaptsov, Combescot & Leyronas (2006); Levinsen & Gurarie (2006).

3-body recombination rate: AAA⇌AD



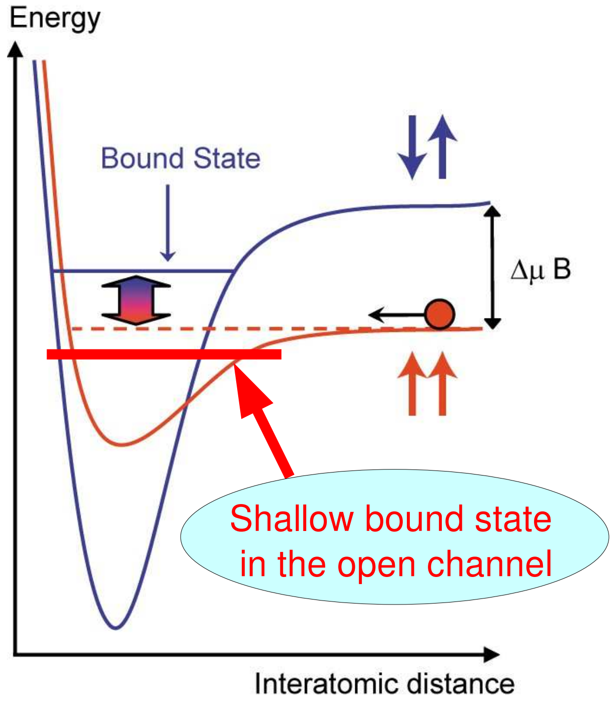
$$T_{\rm rec}(p, q; E_b) \propto T_2(\ldots)\; T_3(\ldots)$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\frac{n^3}{3!}\int_\Omega \frac{\mathrm{d}\Omega}{6\pi^2} k_f\, |T_{\rm rec}|^2 = -\left(\frac{\sqrt{3}\hbar}{2m}\right) n^3 \rho_{\rm rec}^4$$

⋮

and the recombination length $\rho_{\rm rec}$ scales as a

Accurate model of Feshbach resonances with large $a_{\rm bg}$



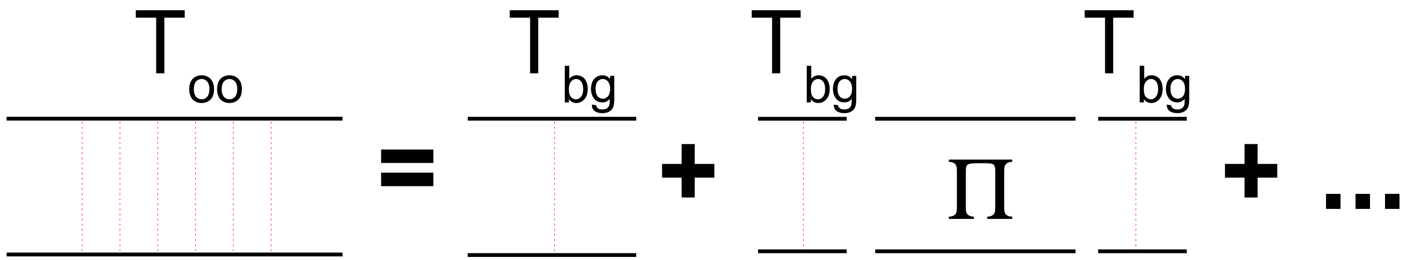
Scattering length: $a = a_{\rm bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$

avoided crossing between the two bound states

$$T_2 = T_{\rm oo} + T_{\rm oc}$$

Bruun, Jackson, and Kolomeitsev (2005); Duine and Stoof (2004).

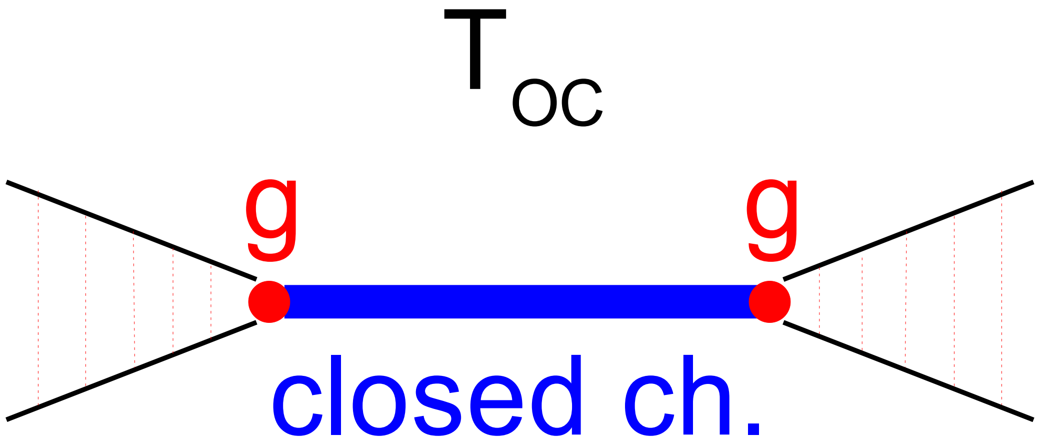
Background scattering



$$T_{\mathrm{OO}} = \frac{T_{\mathrm{bg}}}{1 - T_{\mathrm{bg}}\Pi(E)} \qquad \text{with } T_{\mathrm{bg}} = \frac{4\pi\hbar^2 a_{\mathrm{bg}}}{m}$$

$$\text{pair propagator: } \Pi(E) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} f^2(k) \left(\frac{1}{E - k^2} + \frac{1}{k^2} \right) \propto \sqrt{-E} + r_{\mathrm{bg}} E + \ldots$$

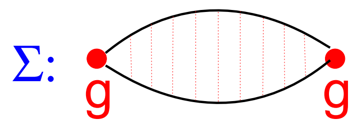
$f(k)$: cut-off function ($r_{\mathrm{bg}} = 0$ if $f(k)$ =const.)



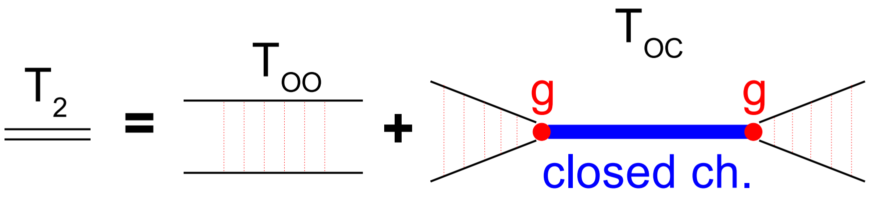
$$T_{OC}(E) = \left(\frac{g}{1 - T_{\text{bg}}\Pi(E)}\right)^2 \frac{1}{E - \Delta\mu(B - B_0) - \hbar\Sigma(E)}$$

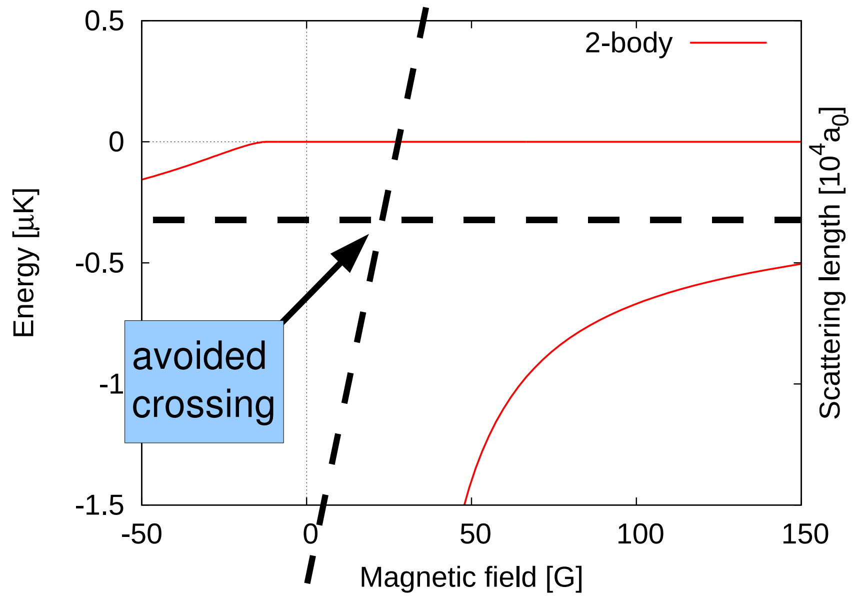
bare coupling between open and closed channels: g $\qquad$ $g^2 = \frac{4\pi\hbar^2 a_{\text{bg}} \Delta B \Delta\mu}{m}$

self-energy of the closed ch. molecule: $\hbar\Sigma(E) = g \frac{\Pi(E)}{1 - T_{\text{bg}}\Pi(E)} g$



Pole structure of T_2





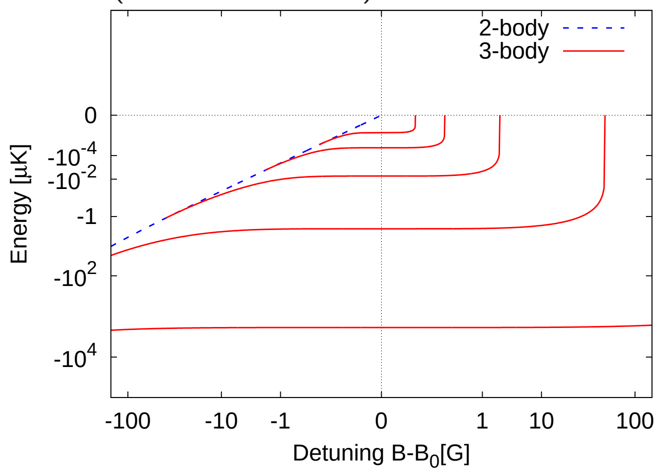
with $r_{\rm bg} \leq 0$, T_2 has 1 or 2 bound states: correct low-energy spectrum

with $r_{\rm bg} > 0$, T_2 has an additional deep pole (with negative residue...)

Finite-range correction for T_2?

Yes!, since T_3 with $T_2 \sim \frac{1}{a^{-1}+ik}$ is cut-off dependent (Danilov 1961)

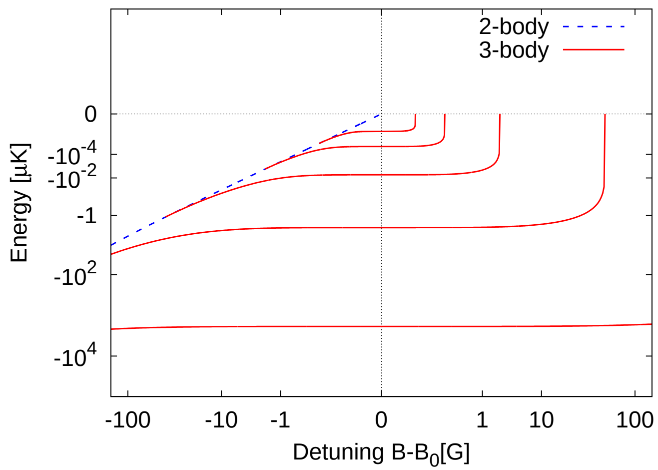
$$T_3(\ldots) = \ldots + \int_0^\Lambda \mathrm{d}q \ldots T_2(\ldots) T_3(\ldots).$$



Finite-range correction for T_2? With negative effective range?

Yes!, since T_3 with $T_2 \sim \frac{1}{a^{-1}+ik}$ is cut-off dependent (Danilov 1961)

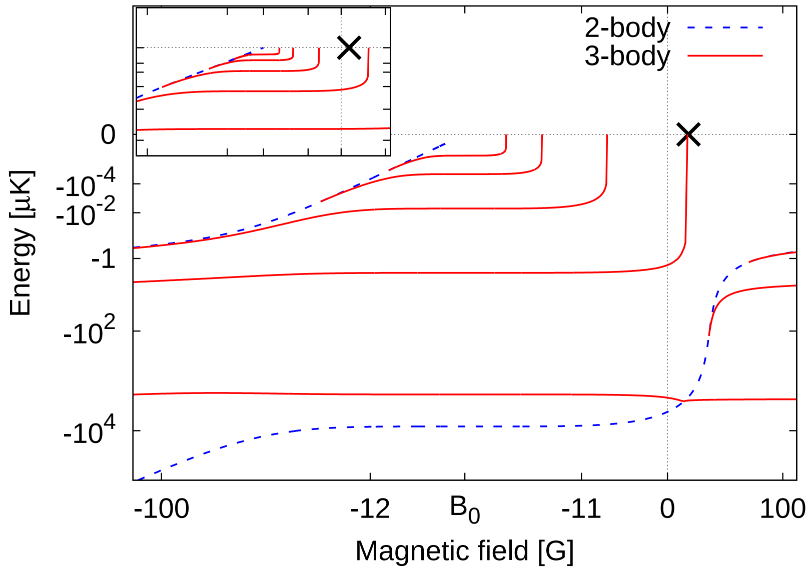
$$T_3(\ldots) = \ldots + \int_0^\Lambda \mathrm{d}q \ldots T_2(\ldots) T_3(\ldots).$$

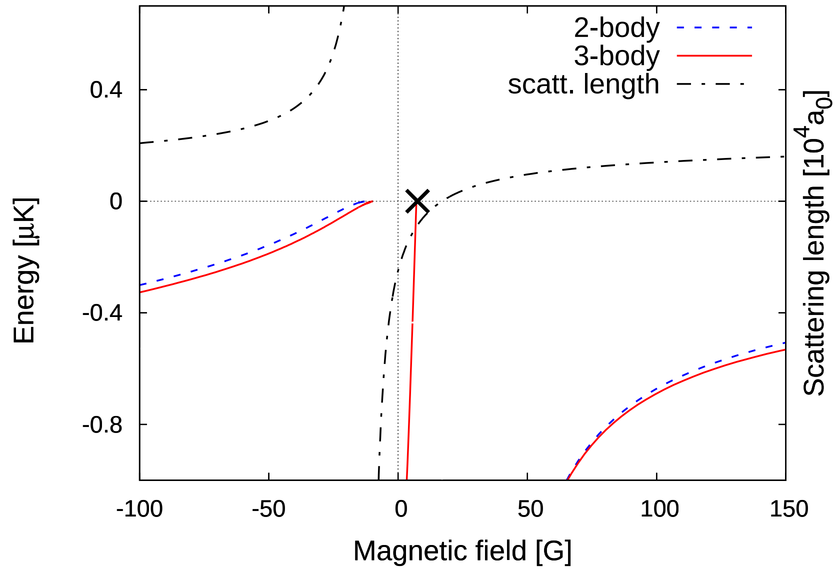


Why not! E.g., for narrow resonances (see Landau, Petrov),

$$f(E) \approx -\frac{\gamma}{E - E_{\text{res}} + i\gamma\sqrt{E}} \approx -\frac{1}{\frac{1}{a} + ik - \frac{1}{2} r_{\mathrm{e}} k^2} \qquad \text{with } r_{\mathrm{e}}(\gamma) < 0.$$

Results for ^{133}Cs: 2- and 3-body energy levels

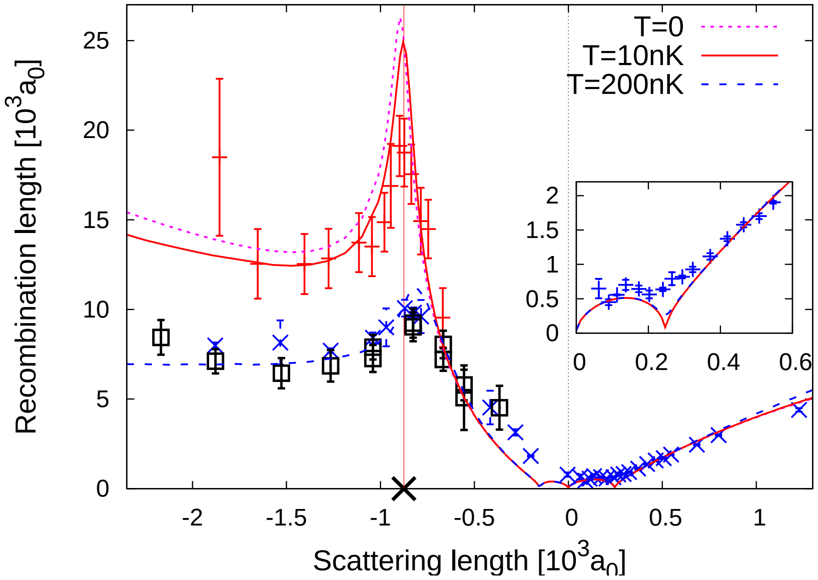




(rescaled units)

(normal units)

Results for ^{133}Cs: recombination length



✔ positions of min/max with a single free parameter ($r_{\rm bg}$) instead of the 4 needed in the original paper

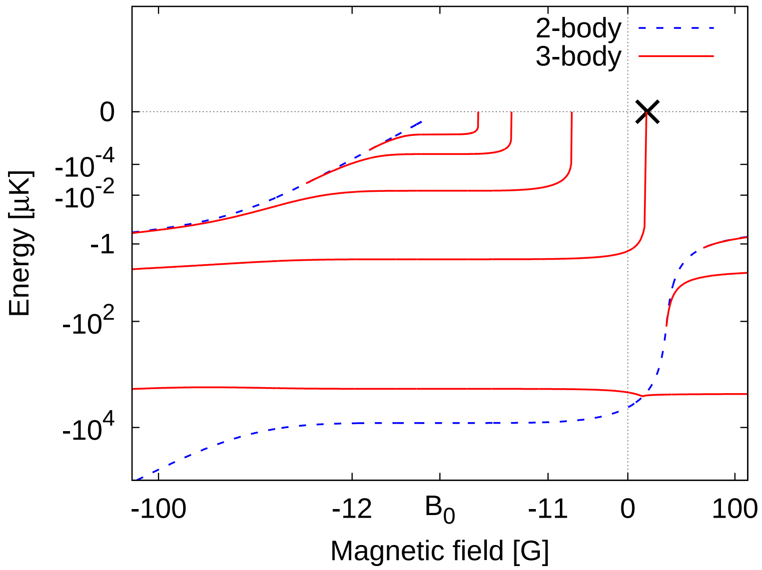
✔ correct temperature dependence

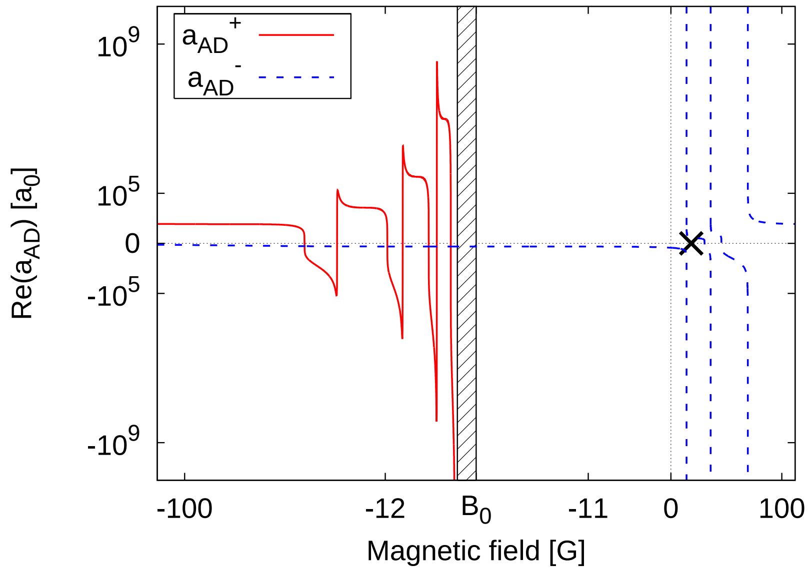
- **Additional prediction:** a minimum at $a \simeq -200 a_0$!

(see also: Jonsell; Yamashita, Frederico and Tomio; Braaten and Hammer; Lee, Köhler and Julienne)

Results for ^{133}Cs: atom-dimer scattering lengths

$$\text{Scattering lengths: } \frac{3\pi\hbar^2 a^{\pm}_{\rm AD}}{m} = Z T_3(0,0;E_b^{\pm}).$$





In conclusion

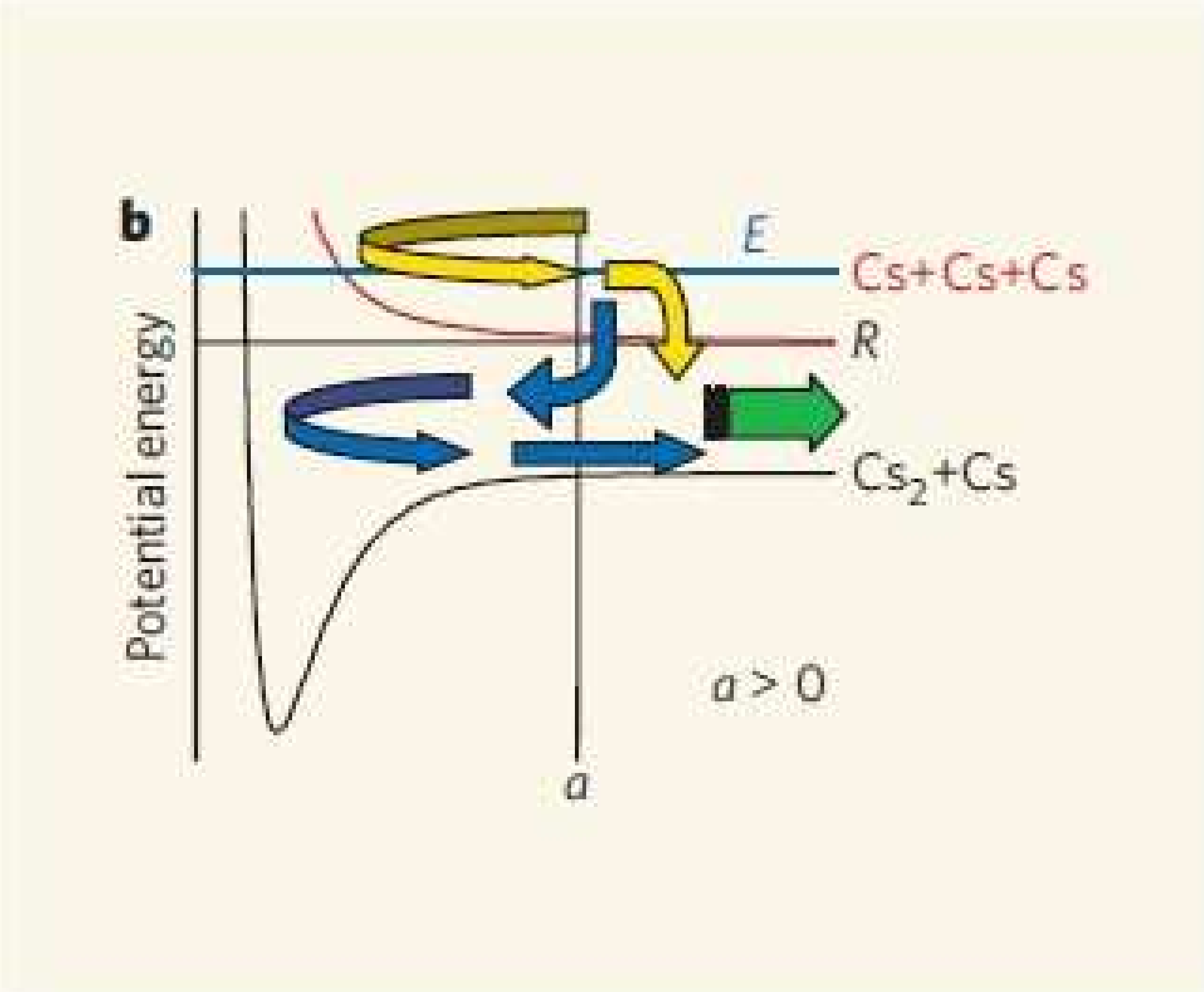
We have:

- found the energy dependence of the relevant 2-body and 3-body bound states
- recovered the temperature dependence of the experimental recombination
- predicted atom-dimer scattering lengths

In progress:

- why the numerical value of our $r_{\rm bg}$ coincides with $r_{\rm Petrov} = -\frac{2\hbar^2}{m a_{\rm bg}\Delta\mu\Delta_B} \sim -0.27 a_0$ (that is valid only if $|r_{\rm Petrov}| \gg r_0$)?
- calculation of atom-dimer loss rates by inclusion of deeper bound states
- can 3-body processes stabilize a gas with attractive 2-body interactions?
- 4-body problem: evaluate the influence of a large $a_{\rm bg}$ on the result $a_{\rm d} = 0.6 a_{\rm aa}$

Paper available as: Pietro Massignan and Henk T. C. Stoof, cond-mat/0702462.


b
Potential energy
E
Cs+Cs+Cs
R
Cs2+Cs
a > 0
a