TOPOLOGICAL SUPERFLUIDS IN OPTICAL LATTICES

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QuaGATUA
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in collaboration with

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- Landau: most phases of matter may be classified by the symmetries they break
  - translational (solids)
  - rotational (magnets)
  - gauge (superfluids)
- BUT: some materials possess distinguishable phases with no broken symmetries

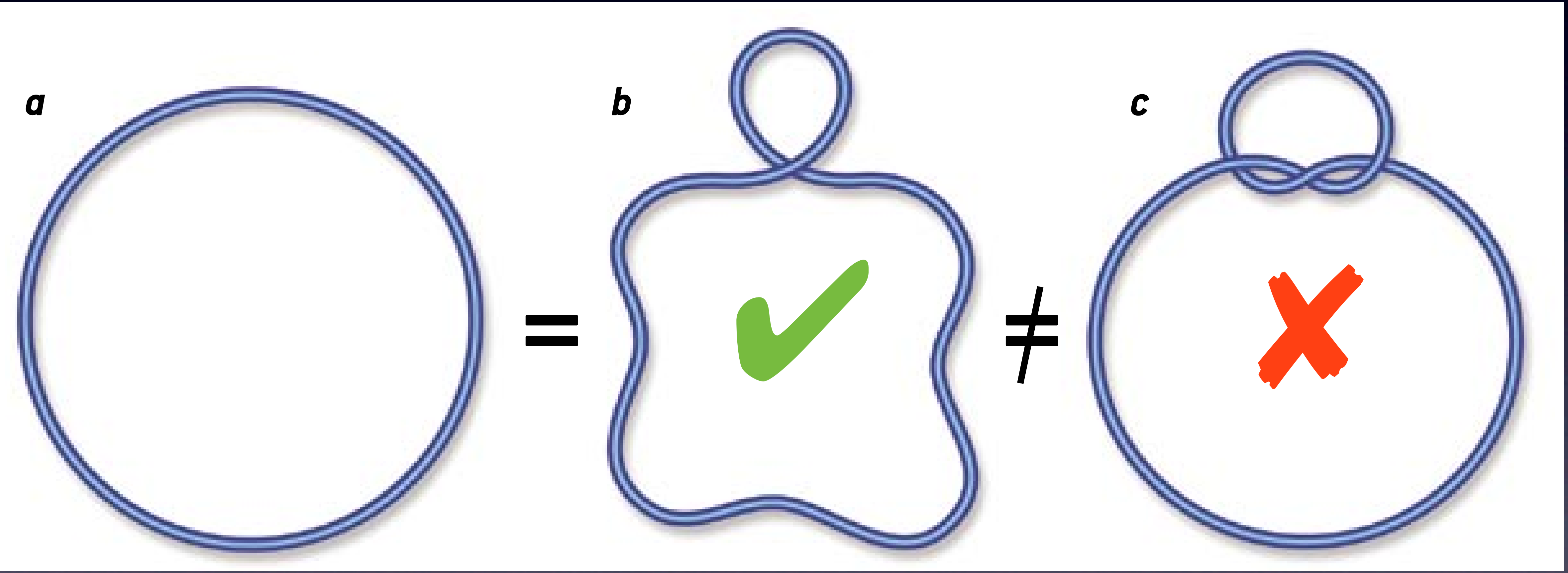
(QH and QSH effect)

Topological phase transitions!

Topological properties

✔: stretching, bending

✘: cutting, joining



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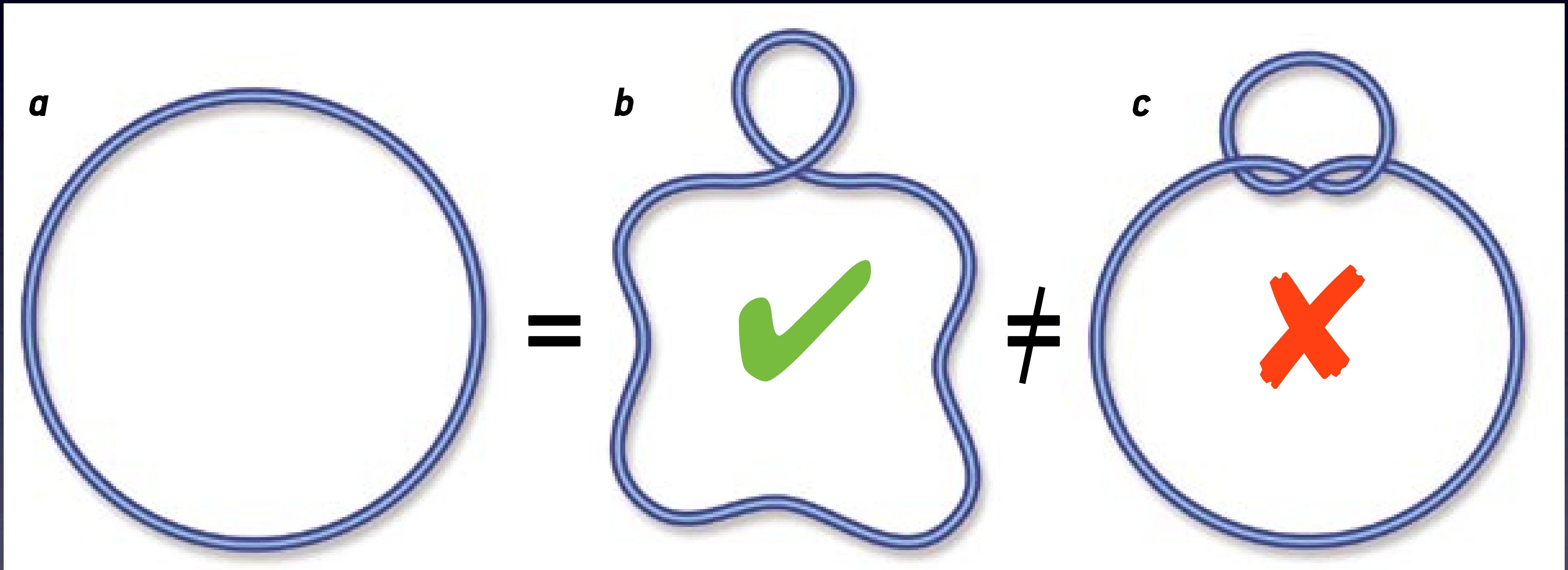


Concern the whole system (non-local)

Characterized by integer numbers

Robust

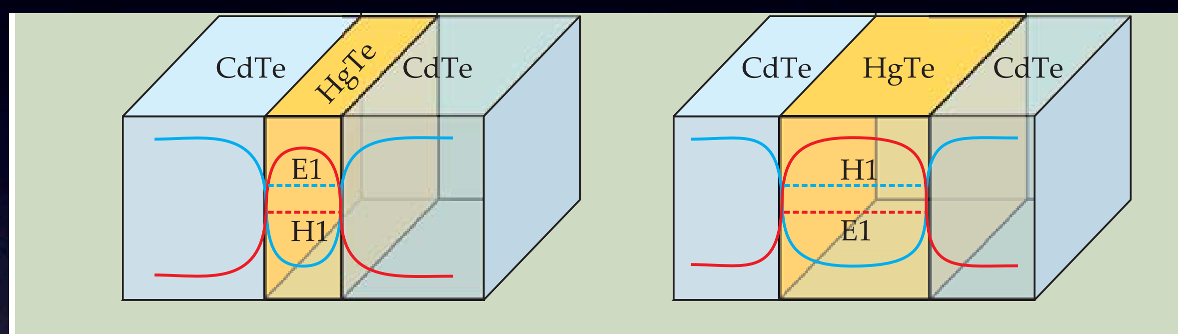
A topological insulator

Hg-Te quantum well

Hg: Mercury
Te: Telluride

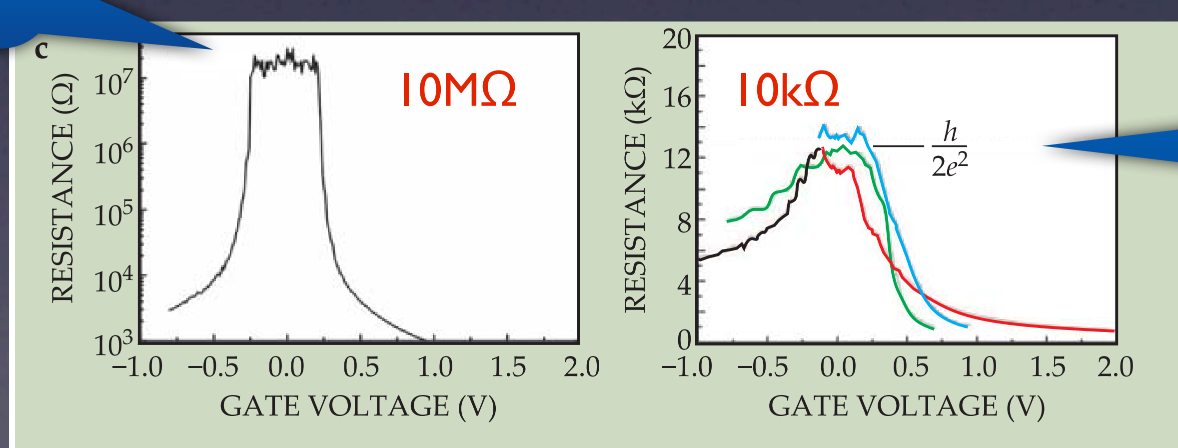
normal
insulator

d



Phase transition at d=d_{crit}:
normal-to-topological insulator

very large resistance



2 quanta of conductance
(independent of d, when d>d_{crit})

Qi & Zhang, Physics Today 2010

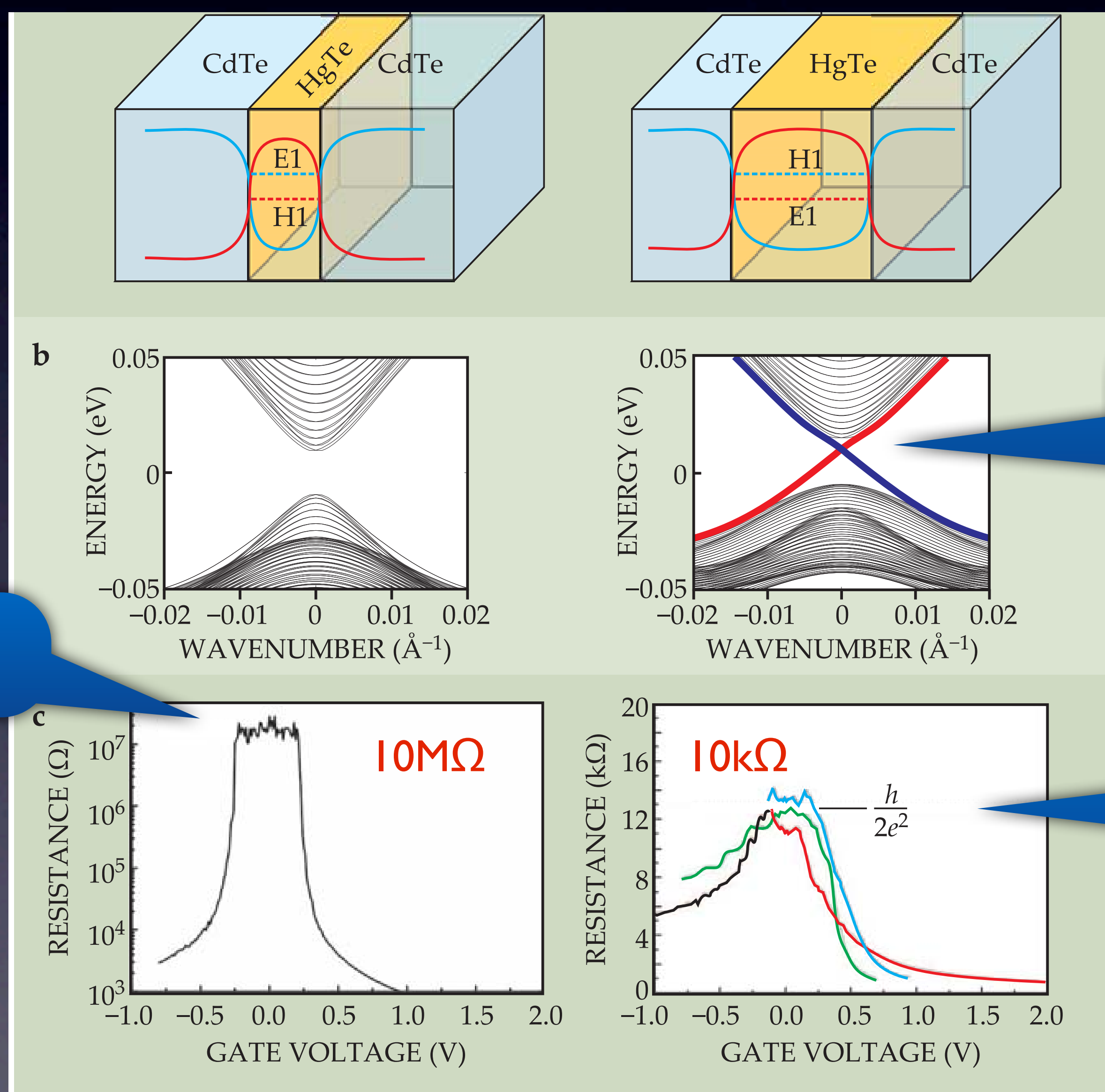
A topological insulator

Hg-Te quantum well

Hg: Mercury
Te: Telluride

normal
insulator

d



$d > d_{crit}$: topological insulator

Edge states
(Dirac cone)

Hg-Te has strong
spin-orbit coupling

very large
resistance

2 quanta of conductance
(independent of d, when $d > d_{crit}$)

Qi & Zhang, Physics Today 2010

interesting..., but where?

- talks by Mudry, Taylor, Morais-Smith, Le Hur, Jackiw, Macrí, Egger, ...
- cond.mat. topological insulators
  (quantum wells, bismuth antimony alloys, Bi_2Se_3 crystals, ...)
- ν=5/2 FQH state (Pfaffian)
- 2D p-wave SF of identical ↑ fermions
  Read&Green, PRB 2000
- 2D s-wave SF of imbalanced ↑↓ fermions with spin-orbit coupling
  Sato, Takahashi & Fujimoto, PRL 2009
  Sau Jay, Lutchyn, Tewari and Das Sarma, PRL 2010
- ... ...

Outlook of the talk

↑ 2D p-wave ferm. SF

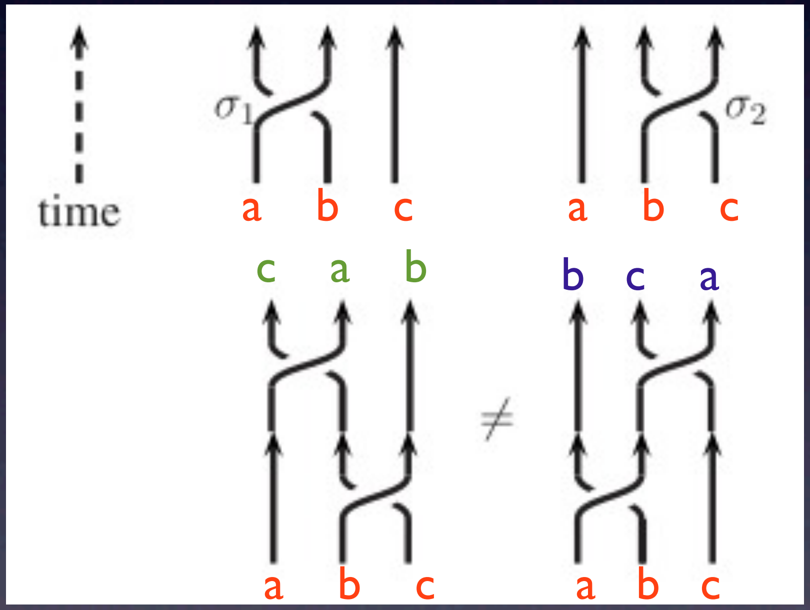
↑↓ 2D s-wave ferm. SF
with $n_\uparrow \neq n_\downarrow$
and spin-orbit coupling

Why 2D?

In 2D particles need not to be either bosons/fermions,
but may have anyonic statistics
(anyons: any phase under exchange of two particles)

In particular, the statistics can be **non-Abelian**, i.e.,
the exchange of two particles must be described by a matrix

BRAIDING

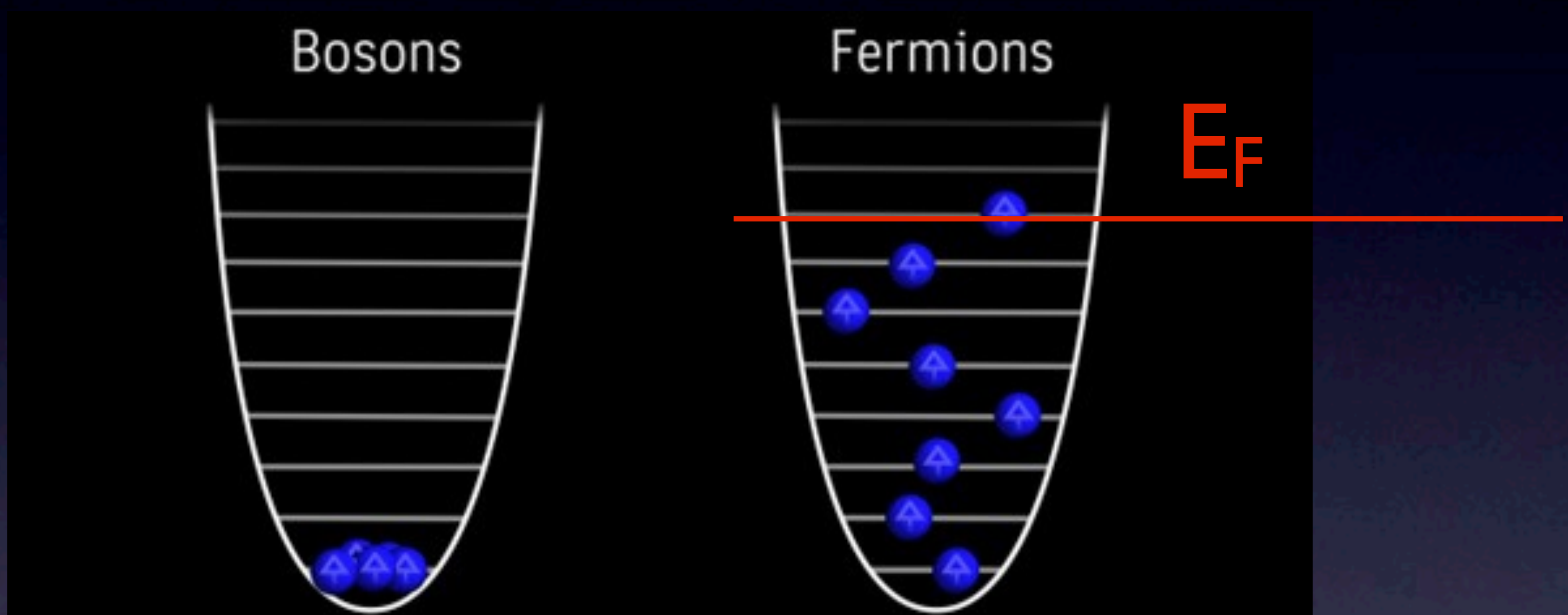


Non-Abelian anyons are a necessary ingredient for topological quantum computation

Nayak, Simon, Stern, Freedman, and Das Sarma, RMP 2008

Why fermions?

Bosons are not so good,
as they condense in the lowest available energy state.



On the contrary, due to Pauli principle fermions have to occupy distinguishable momentum states.

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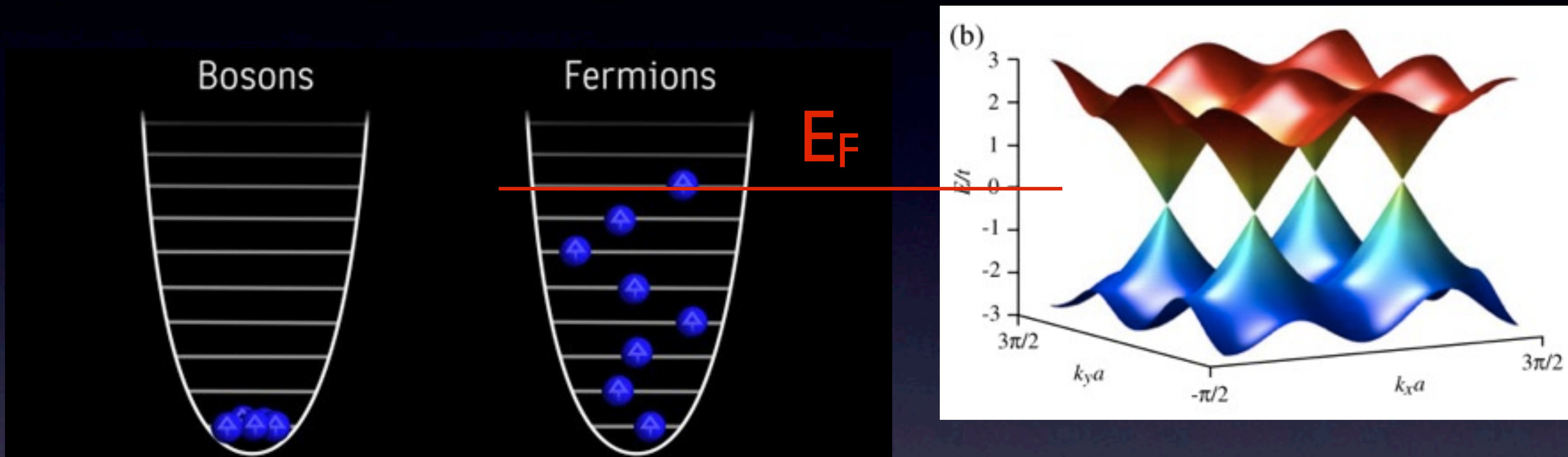


On the contrary, due to Pauli principle fermions have to occupy distinguishable momentum states.

By changing the number of particles, we are able to investigate the interesting excitations, and the system becomes sensitive to the global (topological) properties of the band structure.

↑ 2D p-wave SF

PHYSICAL REVIEW B VOLUME 61, NUMBER 15 15 APRIL 2000-I

Paired states of fermions in two dimensions with breaking of parity and time-reversal symmetries and the fractional quantum Hall effect

N. Read and Dmitry Green
Departments of Physics and Applied Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120
(Received 30 June 1999)

We analyze pairing of fermions in two dimensions for fully gapped cases with broken parity (P) and time reversal (T), especially cases in which the gap function is an orbital angular momentum (l) eigenstate, in particular $l=-1$ (p wave, spinless, or spin triplet) and $l=-2$ (d wave, spin singlet). For $l\neq 0$, these fall into two phases, weak and strong pairing, which may be distinguished topologically. In the cases with conserved spin, we derive explicitly the Hall conductivity for spin as the corresponding topological invariant. For the spinless p-wave case, the weak-pairing phase has a pair wave function that is asympototically the same as that in the Moore-Read (Pfaffian) quantum Hall state, and we argue that its other properties (edge states, quasihole, and toroidal ground states) are also the same, indicating that nonabelian statistics is a *generic* property of such a paired phase. The strong-pairing phase is an abelian state, and the transition between the two phases involves a bulk Majorana fermion, the mass of which changes sign at the transition. For the d-wave case, we argue that

A **stable** p-wave SF?

3-body losses at a p-wave Feshbach resonance

A stable p-wave SF?

3-body losses at a p-wave Feshbach resonance

Ultracold proposals:

- "dissipation-induced stability" in optical lattices (1,2)
  (i.e., how to get no losses from large losses)
- time-dependent lattices (3,4,5)
- RF dressing of 2D fermionic polar molecules leads to long-range interactions ($\propto r^{-3}$) and high T_C (6)
- super-exchange interactions in Bose-Fermi mixtures (7,8)

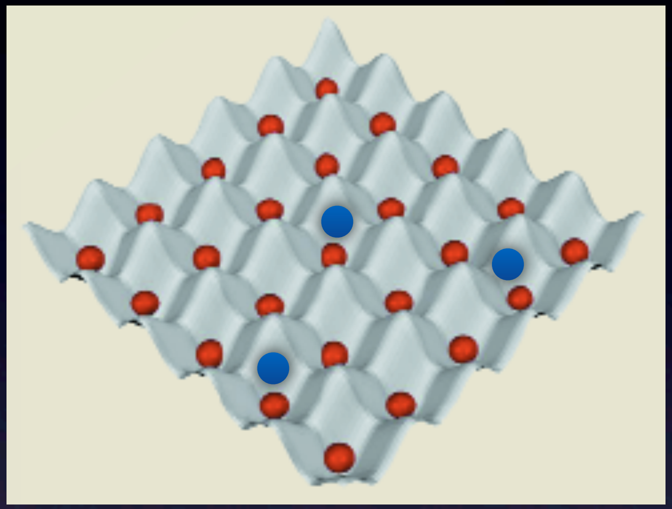
1:Han, Chan, Yi, Daley, Diehl, Zoller & Duan, PRL 2009
2:Roncaglia, Rizzi & Cirac, PRL 2009
3:Lim, Lazarides, Hemmerich & Morais-Smith, EPL 2009
4:Pekker, Sensarma & Demler, arXiv:0906.0931
5:Dutta & Lewenstein, arXiv:0906.2115 & PRA 2010
6:Cooper & Shlyapnikov, PRL 2009
7:Lewenstein, Santos, Baranov & Fehrmann, PRL 2004
8:Massignan, Sanpera & Lewenstein, PRA 2010

Bose-Fermi mixture

1) $U_{BB} > 0$
2) Strong coupling:
$t_B\ ,\ t_F \ll U_{BB}\ ,\ |U_{BF}|$
(bosons in n=1 Mott state)

Lewenstein, Santos, Baranov & Fehrmann, PRL 2004

Bose-Fermi mixture



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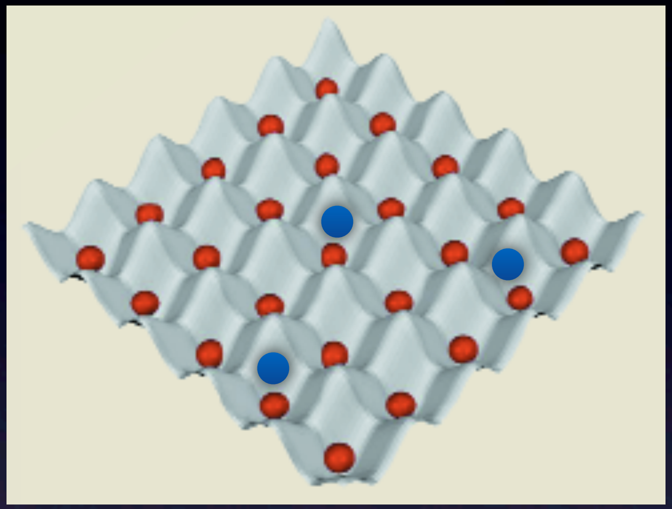

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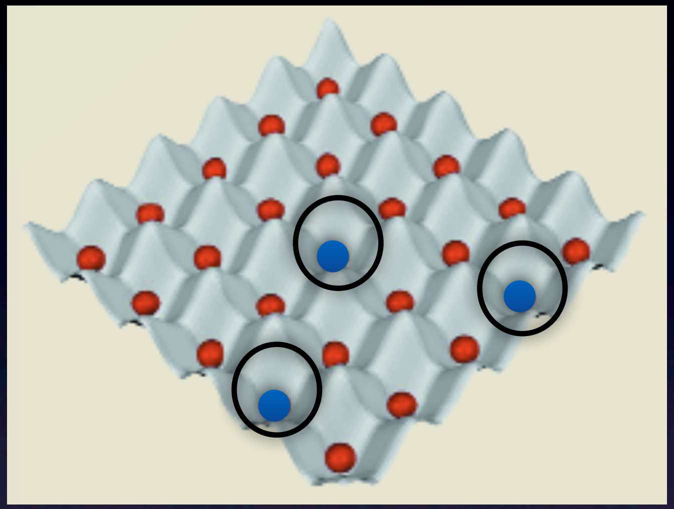

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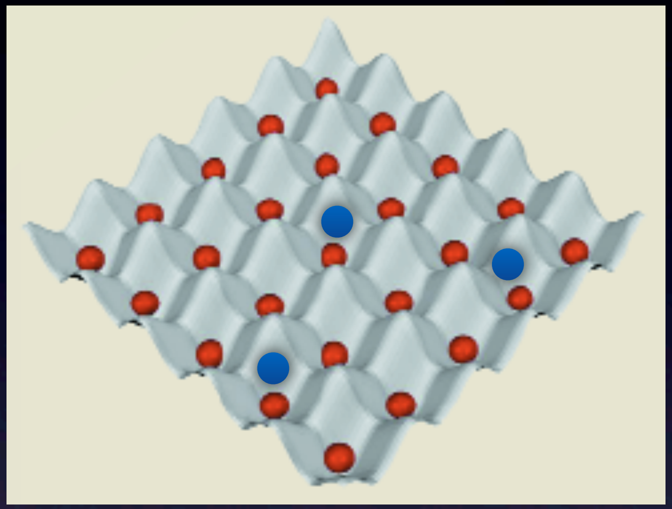

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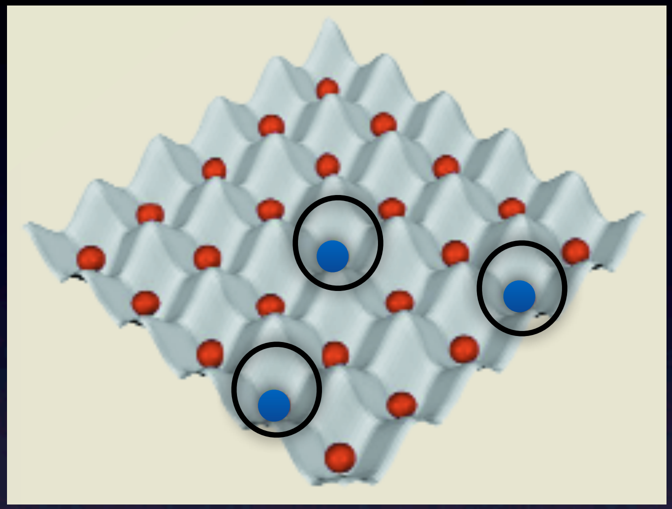

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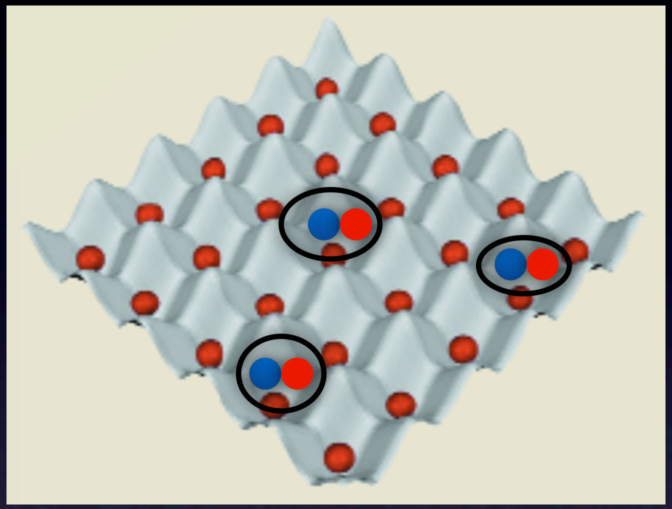

$U_{BF} < 0$

composite fermions

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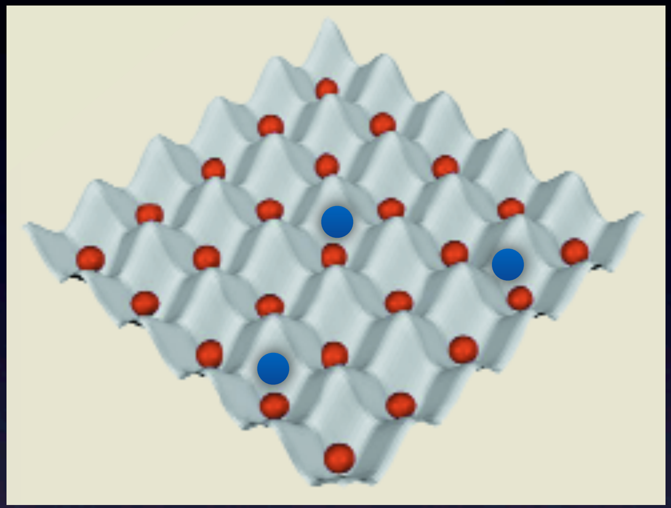

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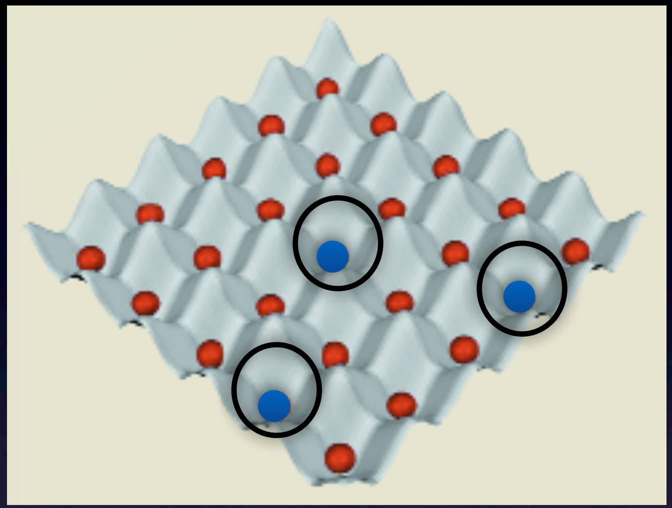

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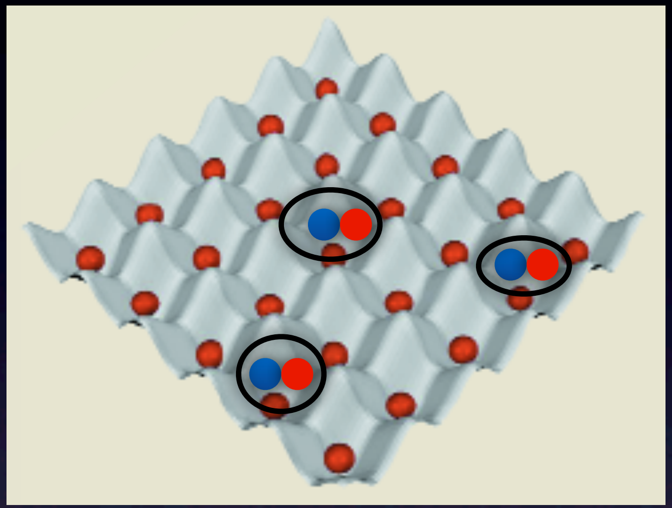

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composite fermions

1) $U_{BB} > 0$

2) Strong coupling:

$t_B\ ,\ t_F \ll U_{BB}\ ,\ |U_{BF}|$

(bosons in n=1 Mott state)

Attractive interaction when $U_{BF} > U_{BB}$

Lewenstein, Santos, Baranov & Fehrmann, PRL 2004

Effective Fermi-Hubbard model

nearest-neighbor interaction (super-exchange)

$$H = -t \sum_{<i,j>} c_i^\dagger c_j - \frac{U}{2} \sum_{<i,j>} n_i n_j - \mu \sum_i n_i$$

$t \sim (t_B t_F)/U_{BF}$

$U>0$

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$U>0$

- BCS approach: introduce BdG operators

$$\gamma_n = \sum_i u_n(i) c_i + v_n(i) c_i^\dagger$$

- Self-consistent "p-wave gap equation"

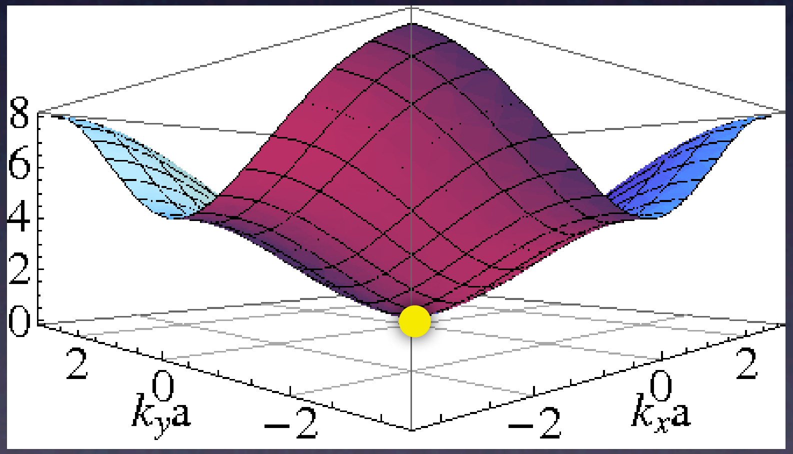
$$\Delta_{ij} = U\langle c_i c_j \rangle = U \sum_{E_n>0} u_n^*(i) v_n(j) \tanh\left(\frac{E_n}{2k_B T}\right)$$

Spectrum on a lattice
(homogeneous system)

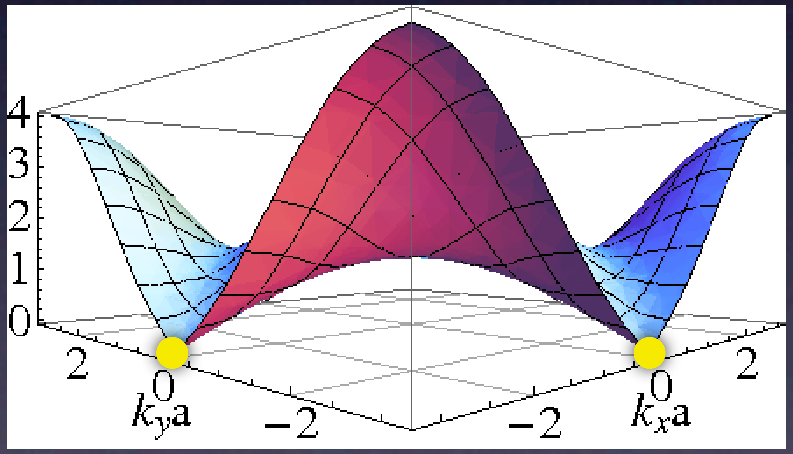
2D chiral ($p_x \pm i p_y$) SF: $$E(\mathbf{k}) = \sqrt{\xi(\mathbf{k})^2 + \Delta_\mathbf{h}(\mathbf{k})^2}$$

with $\xi = -2t[\cos(k_x a) + \cos(k_y a)] - \mu$ and $\Delta_h^2 = \Delta_0[\sin^2(k_x a) + \sin^2(k_y a)]$

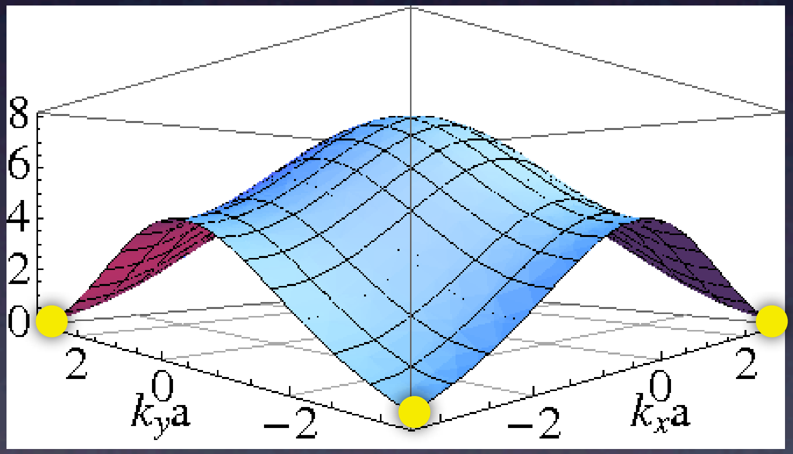
Linear dispersion at the Dirac cones



μ=-4t



μ=0



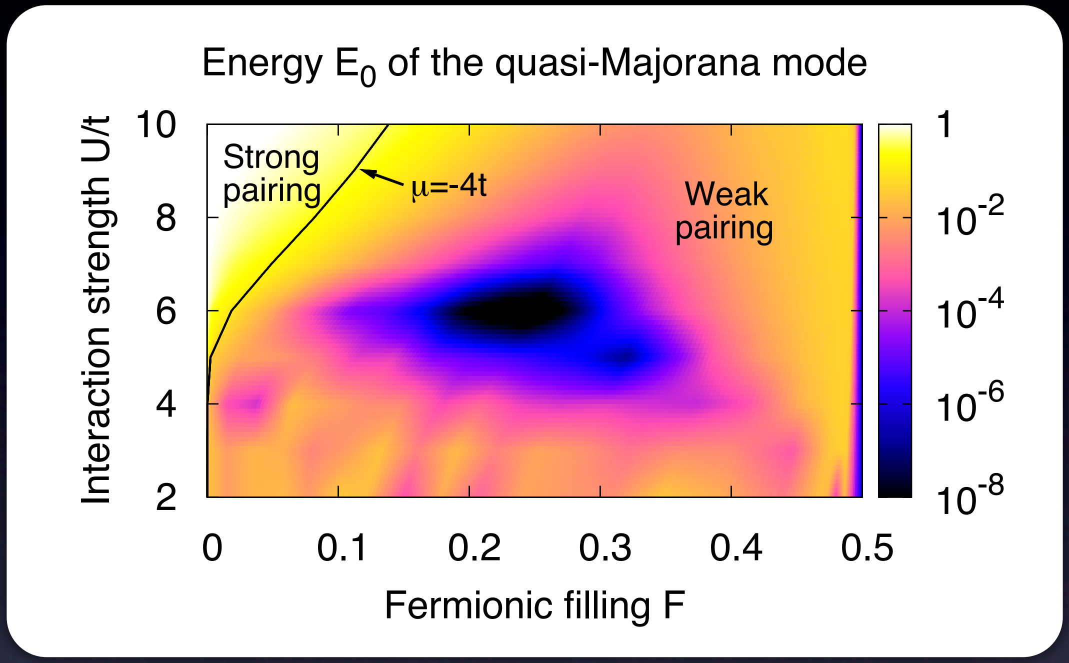
μ=4t

Two distinguishable topological phases for filling F<1/2 and F>1/2

Spectrum with vortex

$$\text{Ansatz}: \Delta_{ij} = \chi_{ij} f_i \mathrm{e}^{\mathrm{i}w\theta_i}$$

$\chi_{ij} = \{1, \mathrm{i}, -1, -\mathrm{i}\}$: chirality
$w = \pm 1$: vortex direction of rotation
f_i : vortex amplitude at site i
θ_i : polar angle of site i



Δ_0~t~10nK (super-exch.)

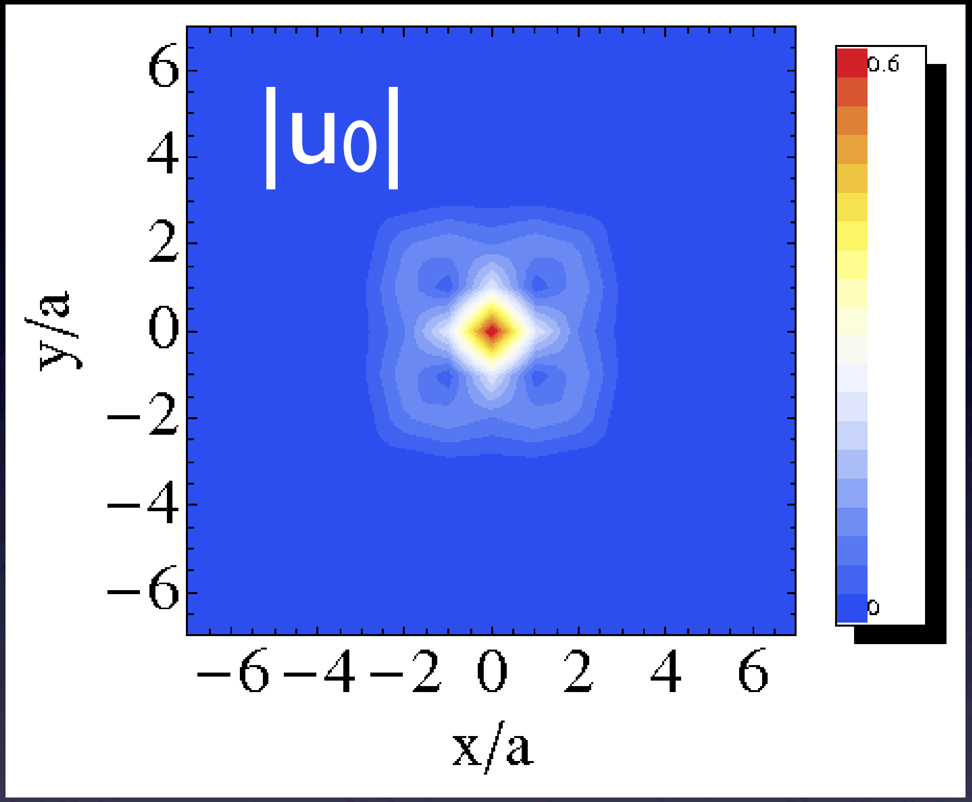
Low-lying spectrum: $E_n \approx n\omega_0$ (n=0,1,2,...)

The eigenstate with $E_0 \ll \Delta_0$ is a Majorana fermion.

Particle-hole symmetry of the BdG eqs.: $\{E_n, \psi_n\} \leftrightarrow \{-E_n, \sigma_1\psi_n^*\}$. Then, if $E_0 = 0,\ u_0 = v_0^*$

P.M., A. Sanpera & M. Lewenstein, PRA 2010

E=0 wavefunction



w=-1

U=5t

Oscillating wavefunction with exponentially decaying envelope

u_0 has a maximum in the core for w=-1, a node for w=1

↑↓ 2D s-wave SF with $n_\uparrow \neq n_\downarrow$ and spin-orbit coupling

Synthetic gauge fields for neutral atoms

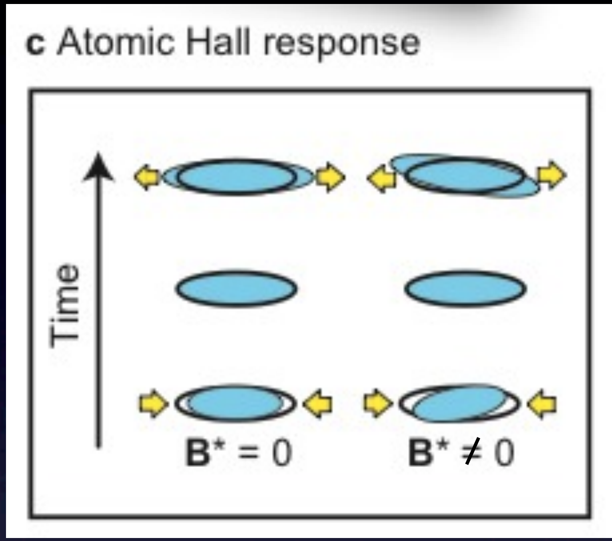
Theory: Jaksch&Zoller, NJP 2003
Osterloh et al., PRL 2005
Gerbier&Dalibard, NJP 2010
Bermudez et al., PRL 2010 (TRI Top. Ins.)

- adiabatic Raman passage
- adiabatic control of superpositions of degenerate dark states
- spatially varying Raman coupling
- Raman-induced transitions to auxiliary states in optical lattices

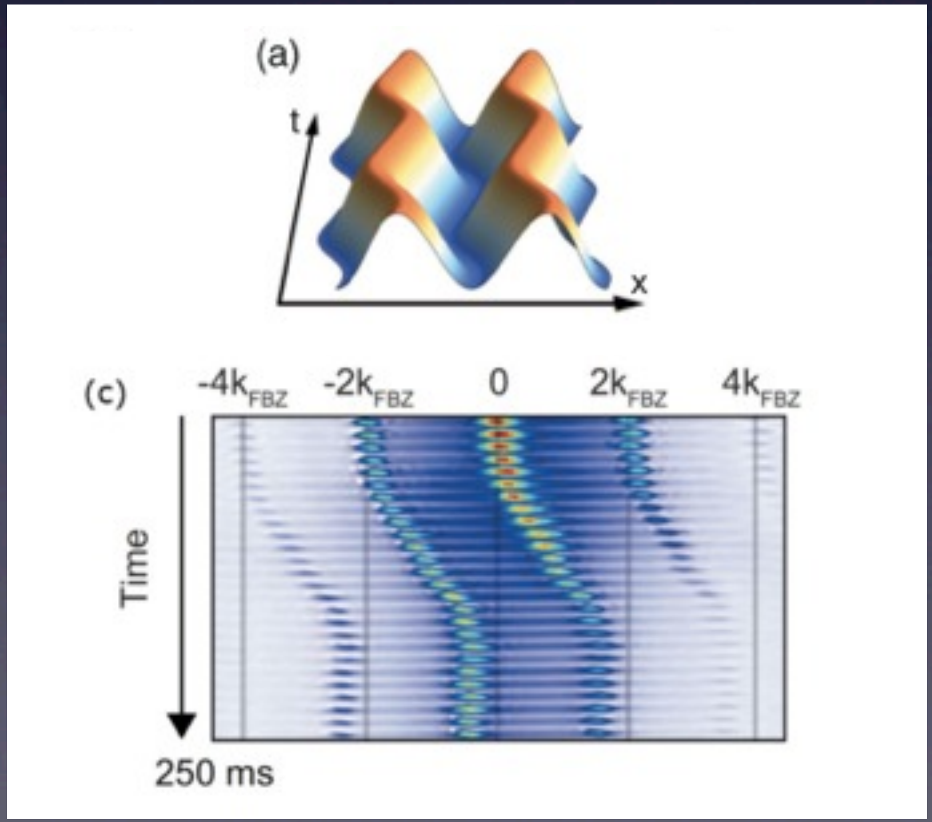
REVIEW: *Artificial gauge potentials for neutral atoms*
J. Dalibard, F. Gerbier, G. Juzeliūnas, and P. Öhberg, RMP 2011

a field moving fast..

NIST: *Synthetic magnetic fields for ultracold neutral atoms*, Nature (2009)
A synthetic electric force acting on neutral atoms, Nature Phys. (2011)
Spin-orbit-coupled Bose-Einstein condensates, Nature (2011)
Observation of a superfluid Hall effect, PNAS (2012)
Peierls Substitution in an Engineered Lattice Potential, PRL (2012)
(theory) *Chern numbers hiding in time-of-flight images*, PRA (2011)



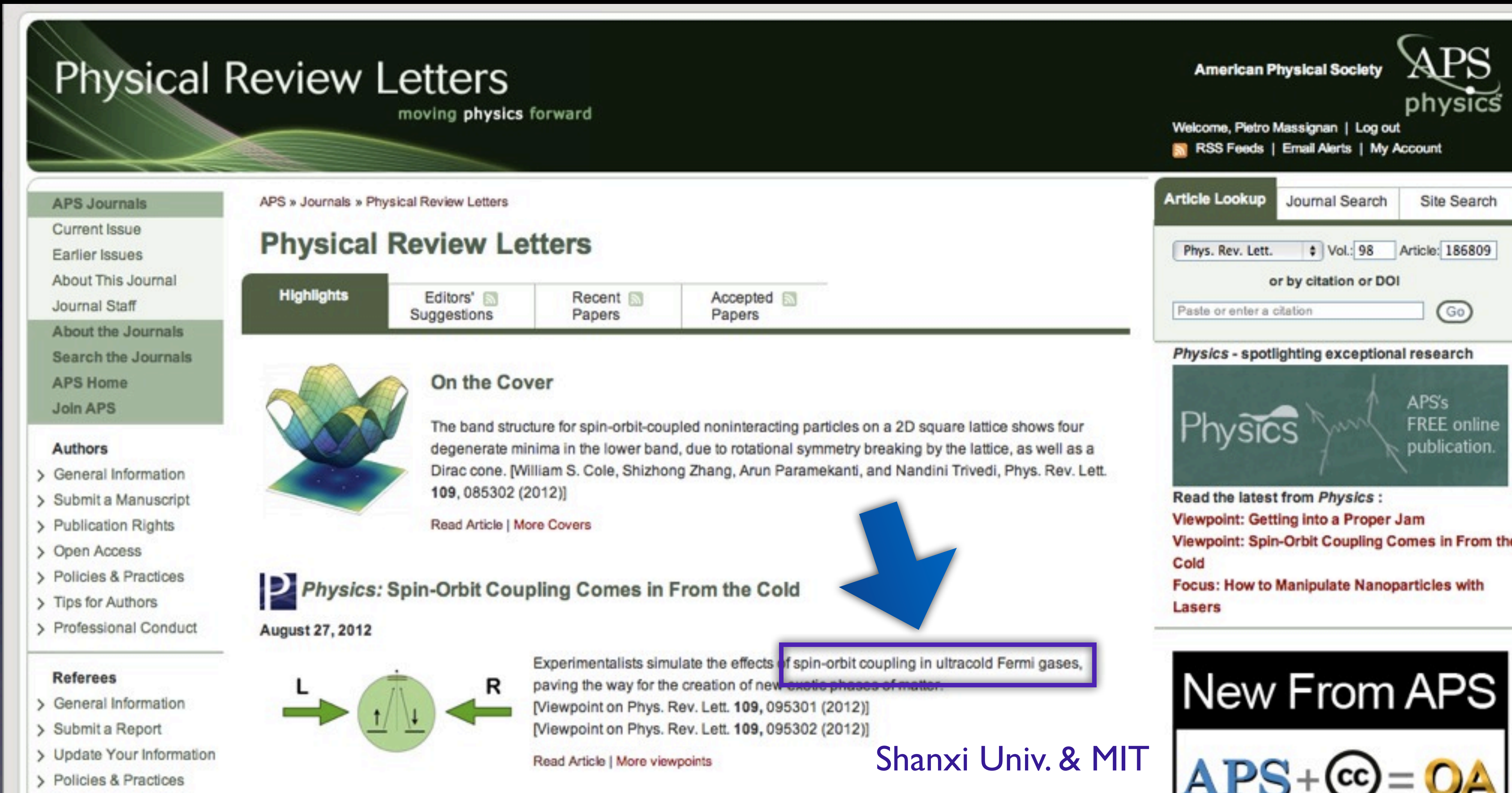
ICFO & Hamburg & Dresden:
Tunable Gauge Potential for Neutral Spinless Particles in Driven Optical Lattices, PRL (2012)
(method independent of the internal structure of the atoms!!)



Munich: *Experimental realization of strong effective magnetic fields in an optical lattice*, PRL (2011)

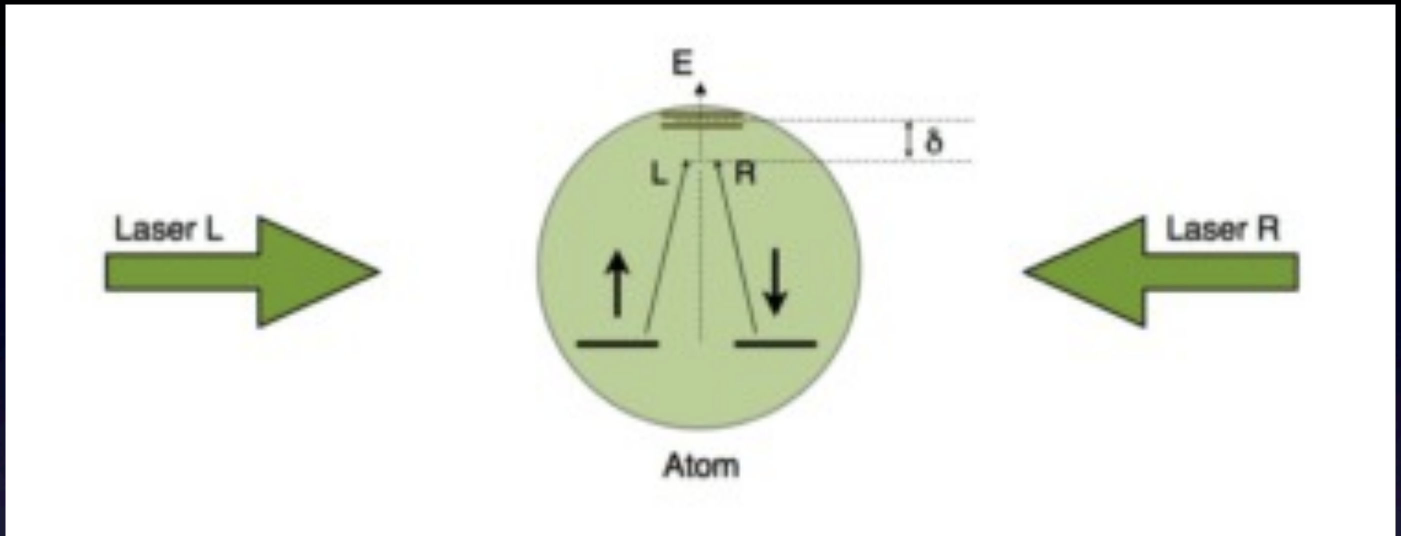
… … … …

PRL this week



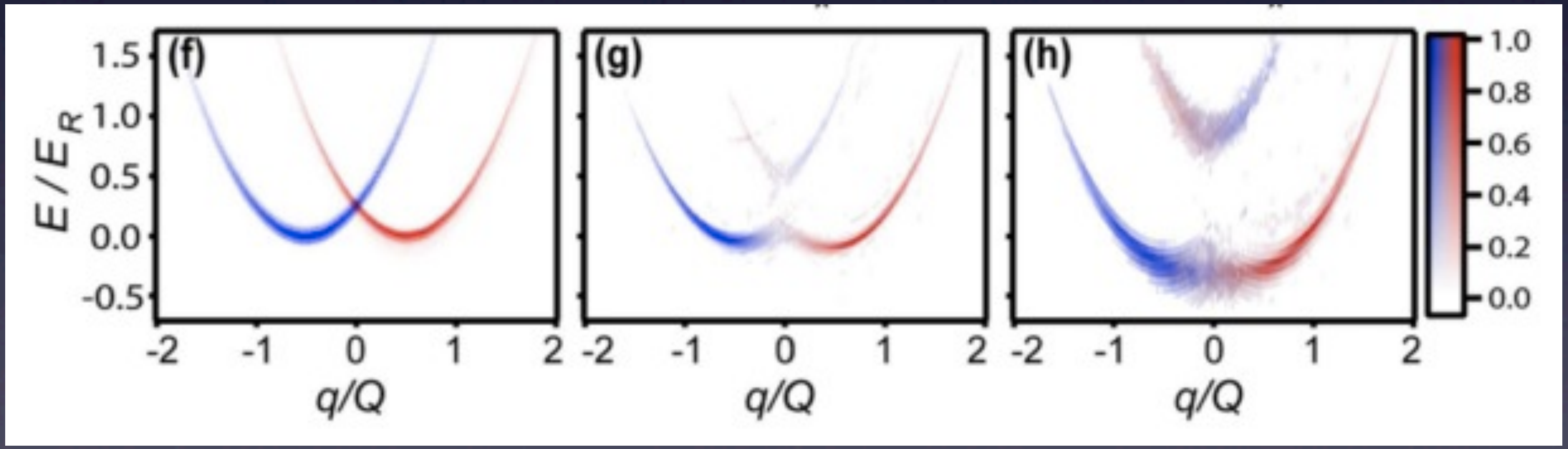
Shanxi Univ. & MIT

Synthetic gauge fields for neutral atoms



$|\uparrow, q=k_x-Q/2\rangle$

$|\downarrow, q=k_x+Q/2\rangle$



spin-orbit gap

increasing intensity of Raman lasers

spin flip ↔ momentum kick,
i.e., spin-orbit coupling

↑↓ fermions in synthetic gauge fields

a fictitious magnetic field yields
Peierl's phases = complex hoppings

$$\mathcal{H}_0 = -\mathrm{t} \sum_{\mathrm{i}} \left[\mathbf{c}^{\dagger}_{\mathrm{i}+\hat{x}} e^{i\sigma_y \alpha} \mathbf{c}_{\mathrm{i}} + \mathbf{c}^{\dagger}_{\mathrm{i}+\hat{y}} e^{i\sigma_x \beta} \mathbf{c}_{\mathrm{i}} + \mathrm{h.c.} \right]$$

$$\mathbf{c}^{\dagger}_{\mathrm{i}} = (c^{\dagger}_{\mathrm{i}\uparrow}, c^{\dagger}_{\mathrm{i}\downarrow})$$

External non-Abelian gauge fields yield a fictitious spin-orbit coupling

Add attractive interactions

$\downarrow$

BCS superfluid

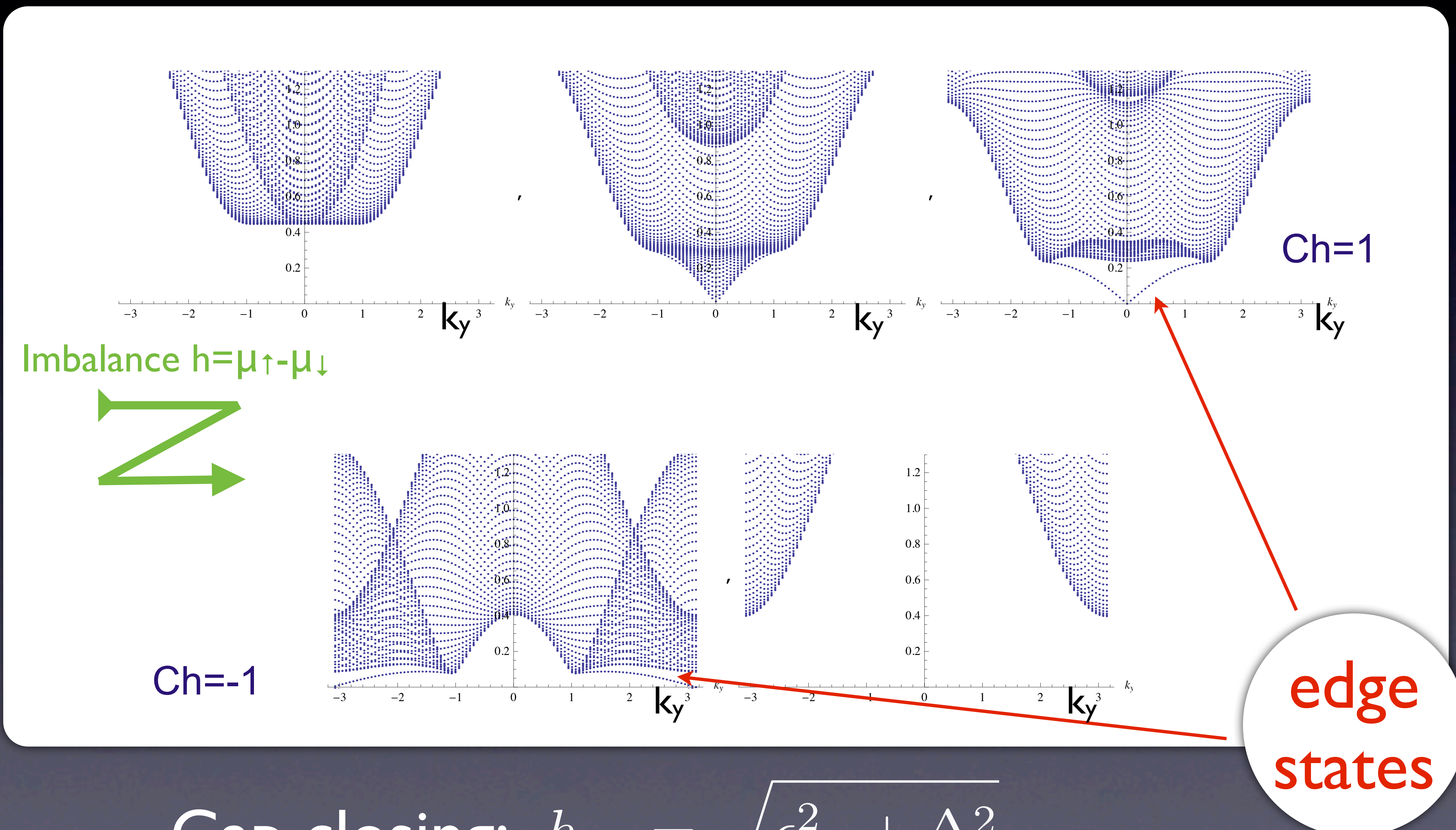
$\downarrow$

strong imbalance $\Rightarrow$ topological states

Time-reversal and spin-rotation invariances are destroyed by the Zeeman and SO terms
as a consequence our BCS Hamiltonian belongs to the most general symmetry class "D"
(Altland&Zirnbauer, PRB 1997)
its topological phases are indexed in terms of an integer number

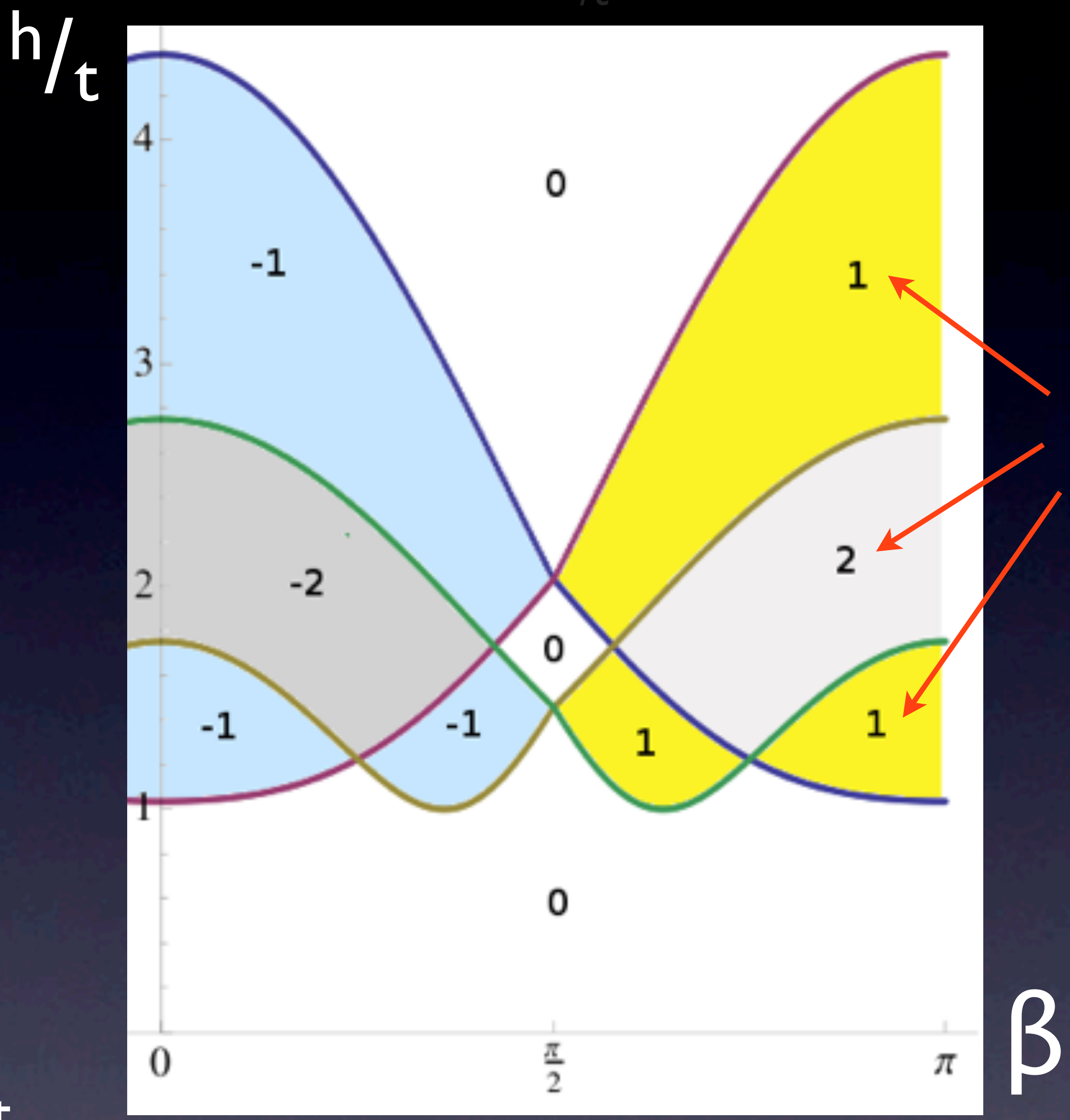
Spectrum on a cylinder

(open b.c. along x)



Gap closing: $h_{\mathbf{k}_0} = \sqrt{\epsilon_{\mathbf{k}_0}^2 + \Delta^2}$

Topological phases



$h=\mu_{\uparrow}-\mu_{\downarrow}$

Chern numbers

easy to calculate!
(see J. Bellissard, condmat/9504030)

Gap closing at $(\mathbf{k}_0, \tilde{h})$:

$$\mathcal{H}_{\text{eff}}(\mathbf{k},h) = E(\mathbf{k},h) + \vec{\sigma}\cdot\vec{f}(\mathbf{k},h)$$
$$\Delta\text{CN}(\tilde{h}) = \text{sign}\{\det[J_{\vec{f}}(\mathbf{k}_0,\tilde{h})]\}.$$

$\Delta=t$
$\alpha=\pi/4$
$\mu=-0.5t[|\cos(\alpha)|+|\cos(\beta)|]$

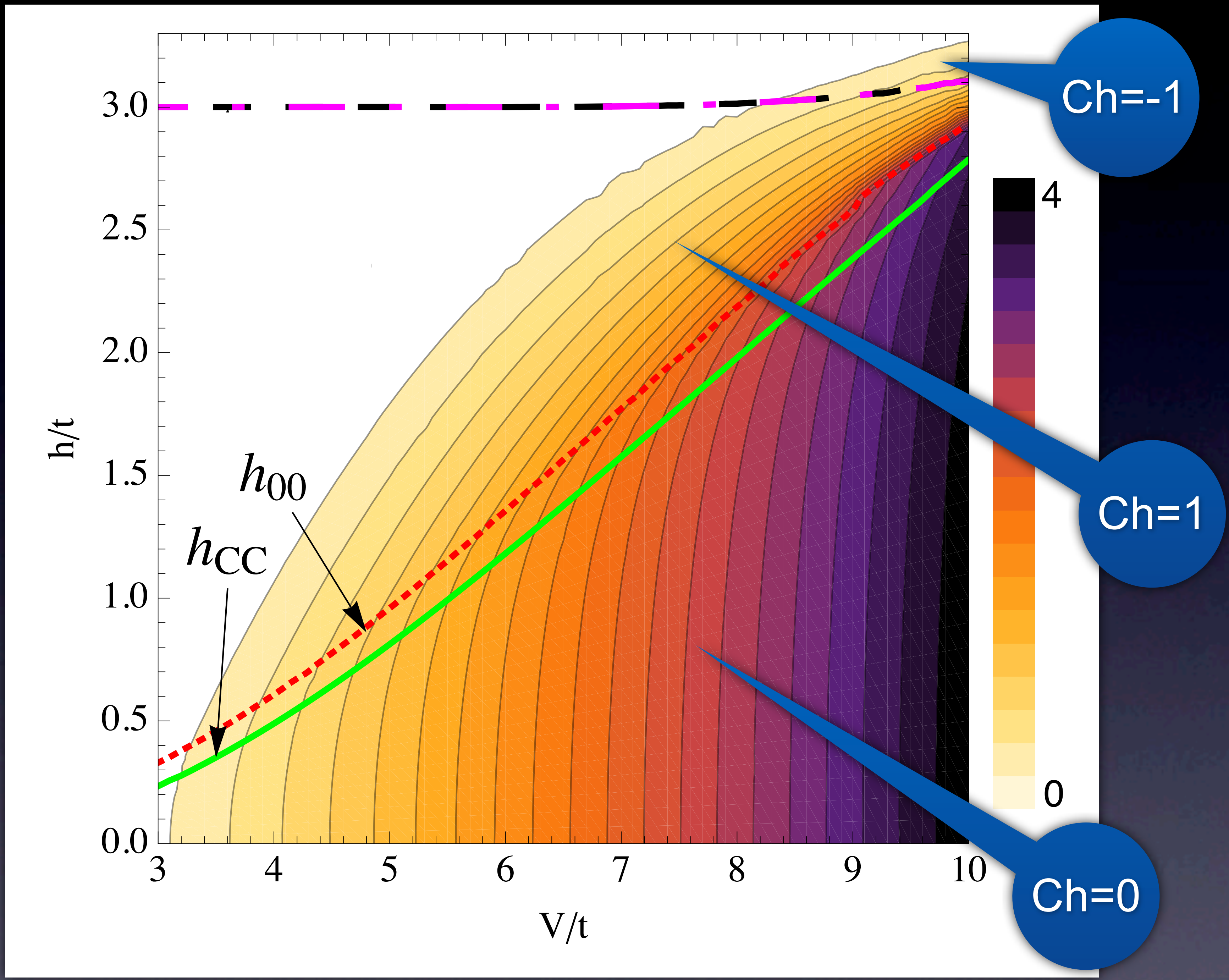
A. Kubasiak, P.M. & M. Lewenstein, EPL 2010

Spin imbalance vs. pair breaking

without SO coupling:
analytic CC limit
($h_{CC} = \Delta_0/\sqrt{2}$)

with SO coupling:
self-consistent calculation of Δ
from the BCS gap equation

$\alpha=\beta=\pi/4$ $\mu=-3t$



A. Kubasiak, P.M. & M. Lewenstein, EPL 2010

Conclusions

- Ultracold SF fermions possess
  <u>*non-trivial topological phases*</u>
- Optical lattices stabilize p-wave SF ➢ FQH
- ↑↓ fermions in non-Abelian gauge fields

P. M., A. Sanpera & M. Lewenstein, PRA(R) 2010
A. Kubasiak, P. M. & M. Lewenstein, EPL 2010

- Applications to:
  - ➡ relativistic QED
  - ➡ lattice gauge theories
  - ➡ topological quantum computation