Polarons and molecules close to narrow Feshbach resonances

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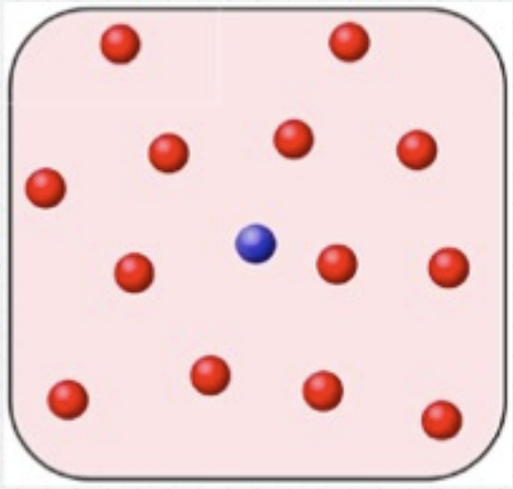
$N \gg 1$



normal Fermi liquid

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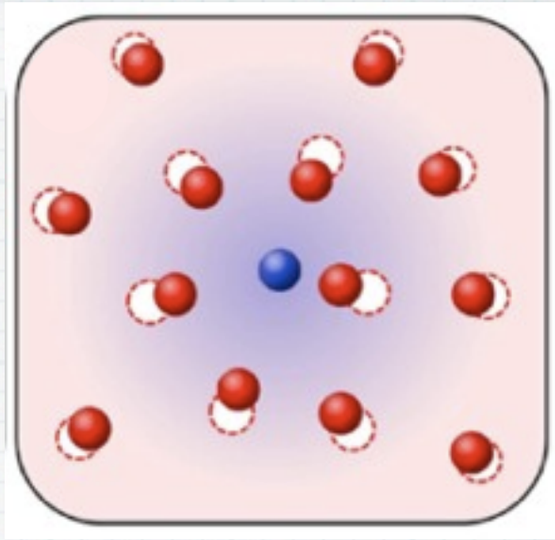
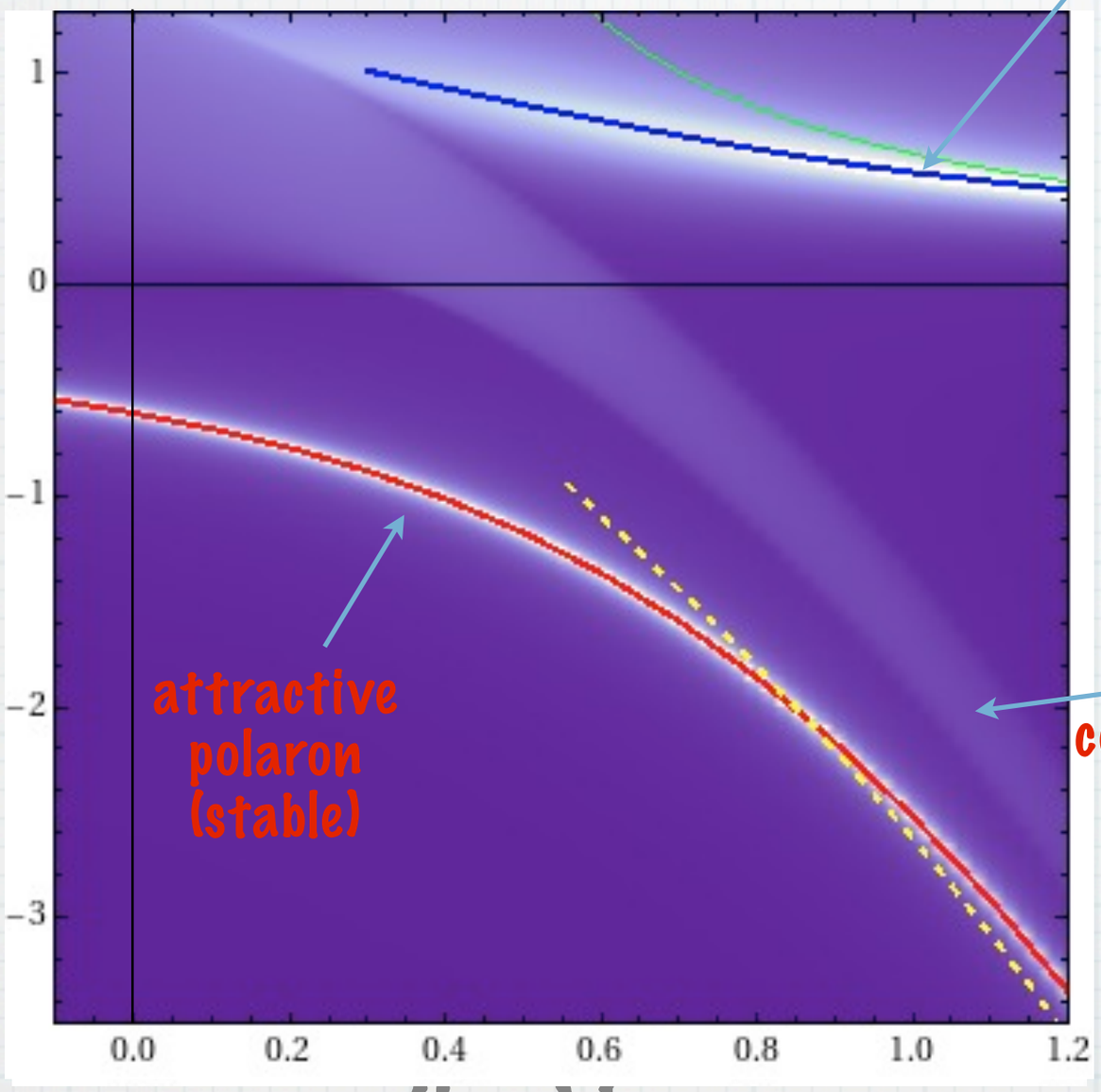
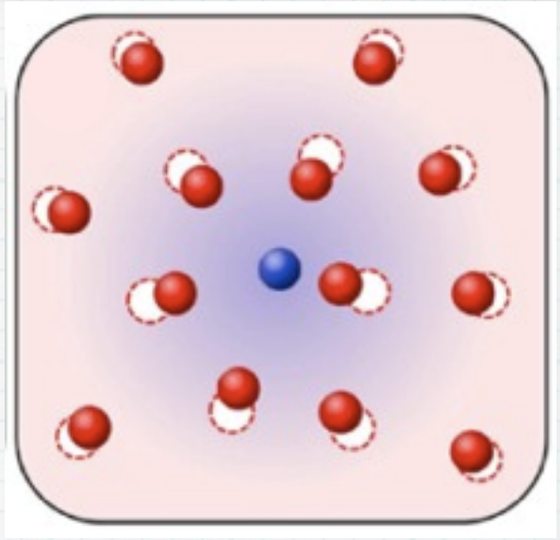
Quasi-Particles

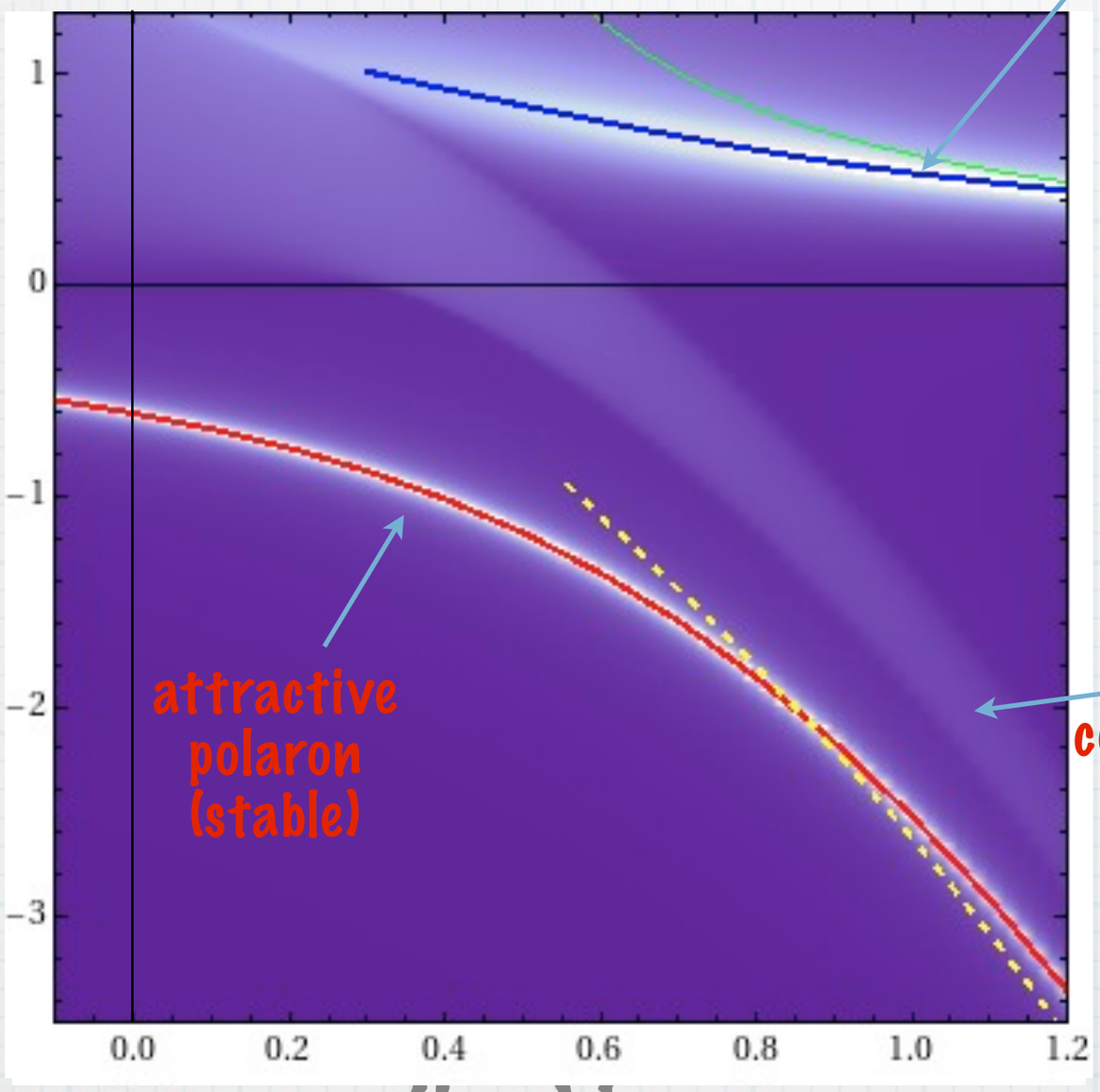




$$E_M = -\frac{\hbar^2}{2m_r a^2} - \epsilon_F + \frac{2\pi\hbar^2 a_3}{m_3} n_\uparrow$$

Quasi-Particles





QP properties:

@ renormalized mass

@ chemical potential

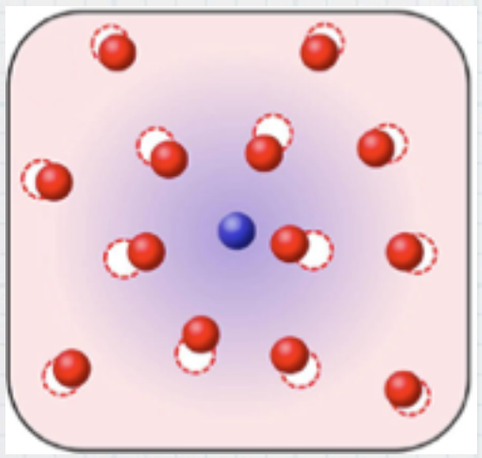
@ shielded interactions

@ q. numbers (charge, spin, ...)

@ lifetime

$$E_M = -\frac{\hbar^2}{2m_r a^2} - \epsilon_F + \frac{2\pi\hbar^2 a_3}{m_3} n_\uparrow$$

Polaron: variational Ansatz



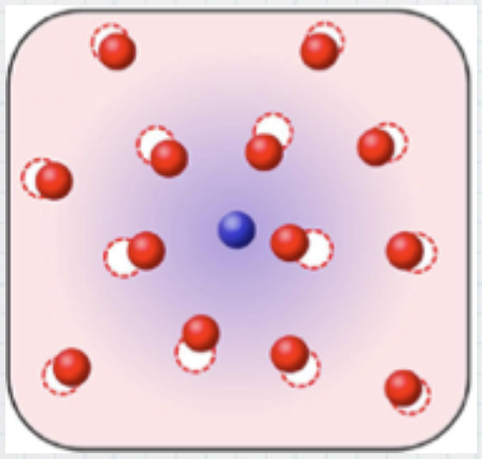
Polaron: variational Ansatz

the ↓ impurity

$$|\psi_{\mathbf{p}}\rangle = \phi_0 c^\dagger_{\mathbf{p}\downarrow}|0\rangle_\uparrow + \sum_{\substack{q<k_F \\ k>k_F}} \phi_{\mathbf{qk}}\, c^\dagger_{\mathbf{p}+\mathbf{q}-\mathbf{k}\downarrow}\, c^\dagger_{\mathbf{k}\uparrow}\, c_{\mathbf{q}\uparrow}\, |0\rangle_\uparrow$$

non-interacting ↑ Fermi sea

Particle-Hole dressing



Narrow Feshbach Resonances

Scattering amplitude: $f = -\left[a^{-1} + ik + R^* k^2 + \ldots\right]^{-1}$

$$R^* = -\frac{r_e}{2} = \frac{\hbar^2}{2m_r a_{\rm bg} \Delta B \delta\mu}$$

FR narrow if $R^* \gg R_{VdW}$

$k_F R^* \gg 1$

No broad heteronuclear FR found yet.

$$E_M = -\frac{\hbar^2}{2m_r a^*} \qquad \text{with} \quad a^* = \frac{2R^*}{\sqrt{1 + 4R^*/a} - 1}$$

$$a^*\left(\frac{R^*}{a} \ll 1\right) \sim a$$

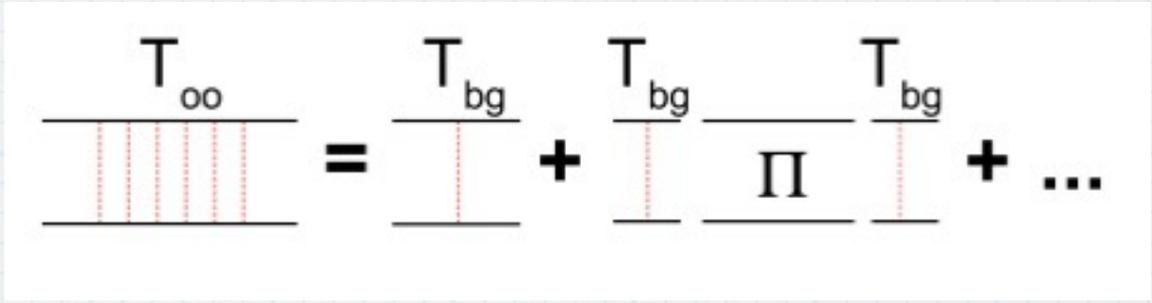
$$a^*\left(\frac{R^*}{a} \ll 1\right) \sim \sqrt{R^* a}$$

Many-body description of narrow FR

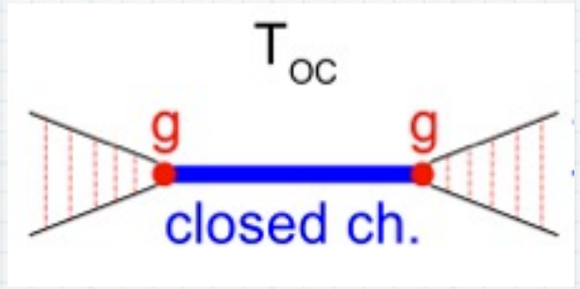
Bruun, Jackson & Kolomeitsev, PRA 2005
Massignan & Stoof, PRA 2008

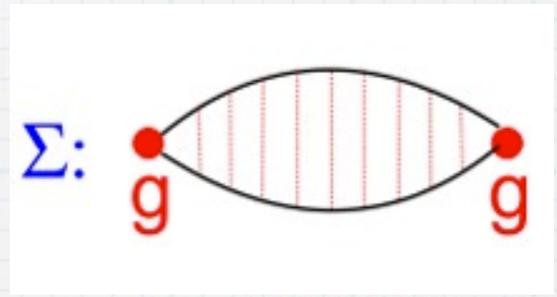
$$T = T_{\mathrm{OO}} + T_{\mathrm{OC}}$$

OO: open channel only



OC: involves coupling between open and closed channels





$$T_{OC} = \left(\frac{g}{1 - T_{bg}\Pi(E)}\right)^2 \frac{1}{E - \Delta\mu(B - B_0) - \Sigma(E)}$$

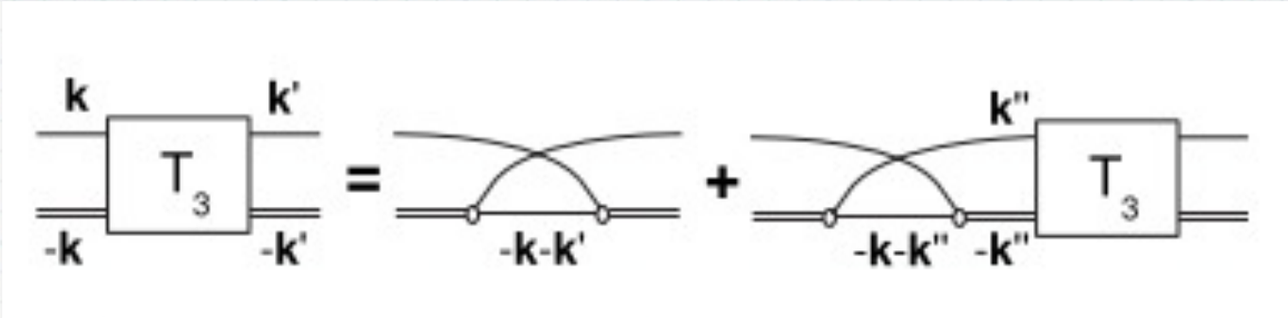
$$T = -\frac{2\pi\hbar^2}{m_r} f \quad \text{with} \quad f = -\left\{\left[a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0 - E_{\mathrm{CM}}/\delta\mu}\right)\right]^{-1} - \frac{2\pi\hbar^2}{m_r}\Pi(\mathbf{p}, E_{\mathrm{CM}})\right\}^{-1}$$

low energy expansion:

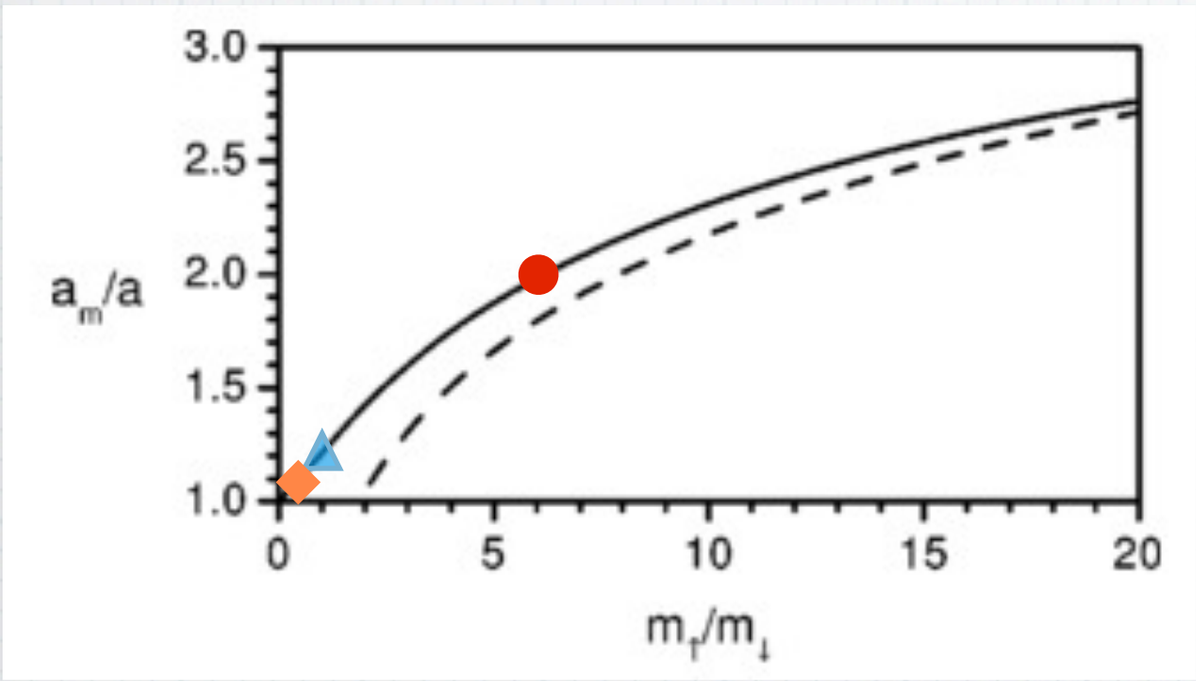
$$a^*(B) = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

$$R^*(B) = \frac{\hbar^2 \Delta B}{2 m_r a_{\mathrm{bg}}(B - B_0 - \Delta B)^2 \delta\mu}$$

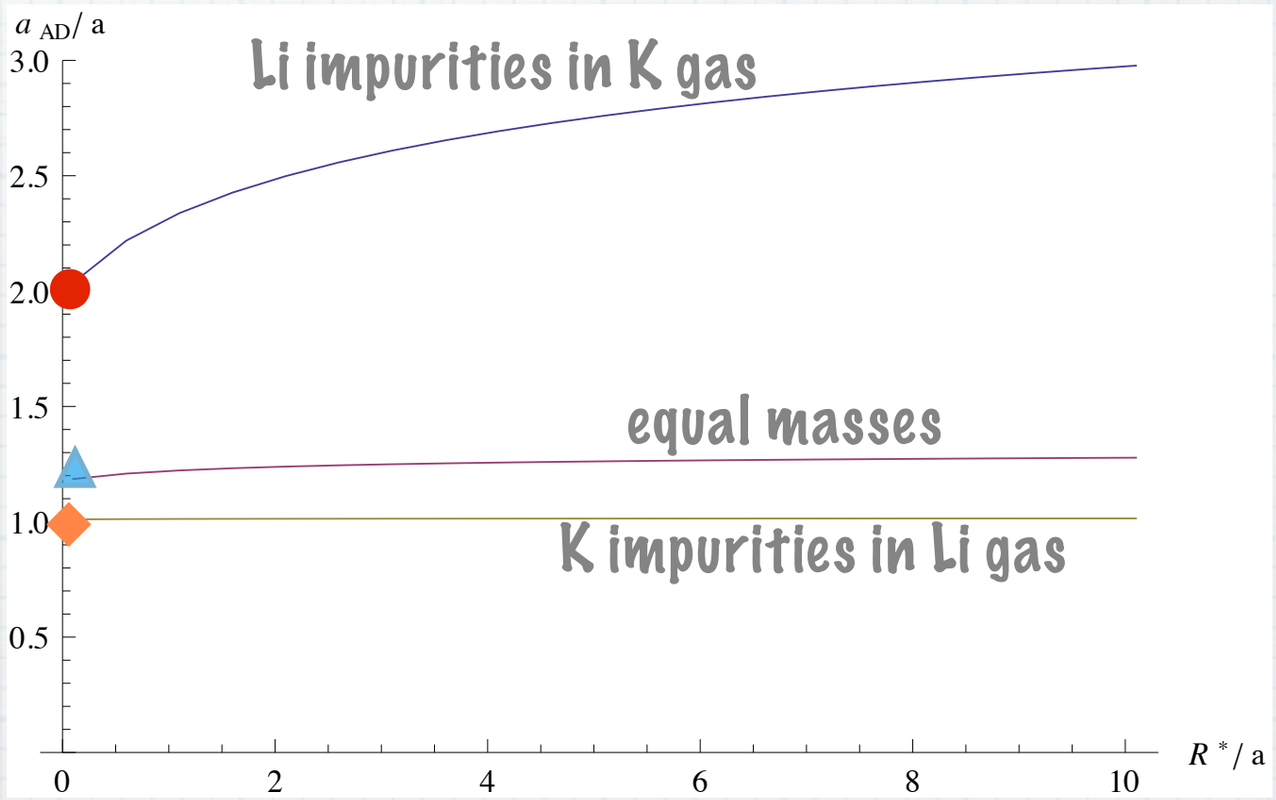
Atom-Dimer scattering



Broad FR



Narrow FR

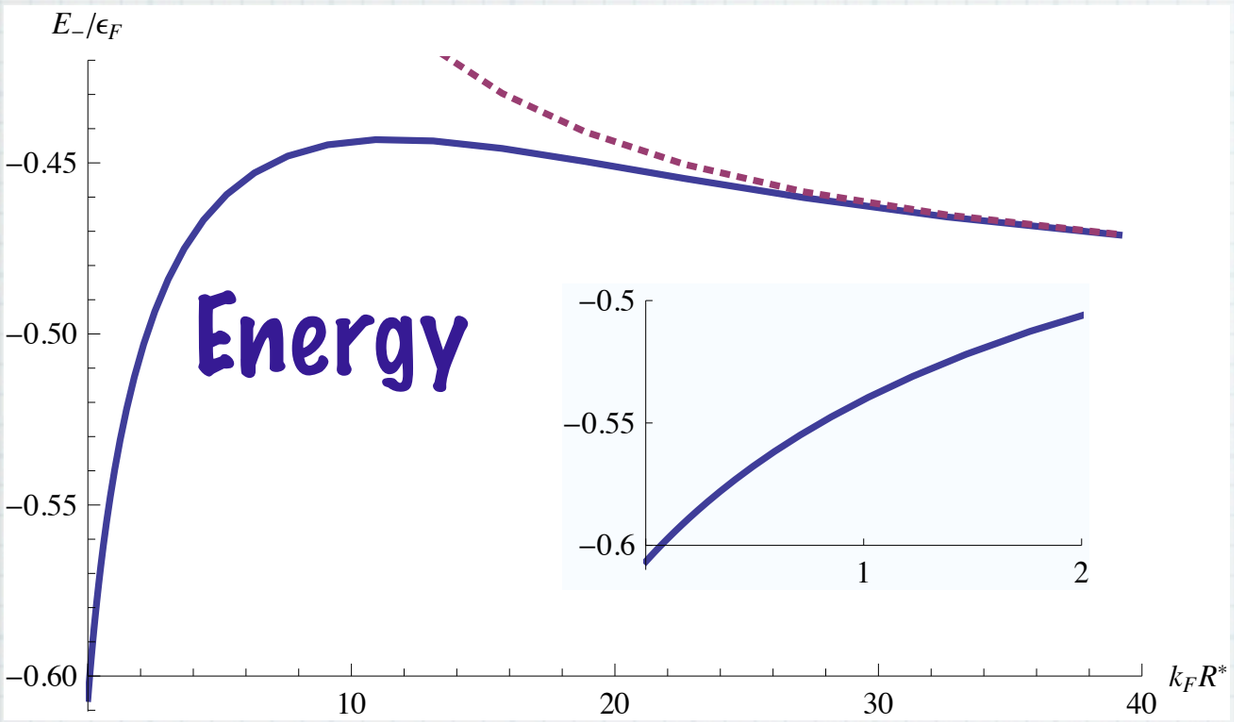


agrees with real-space calculation (Petrov, PRA 2003; Petrov&Levinsen, arXiv: 1101.5979)

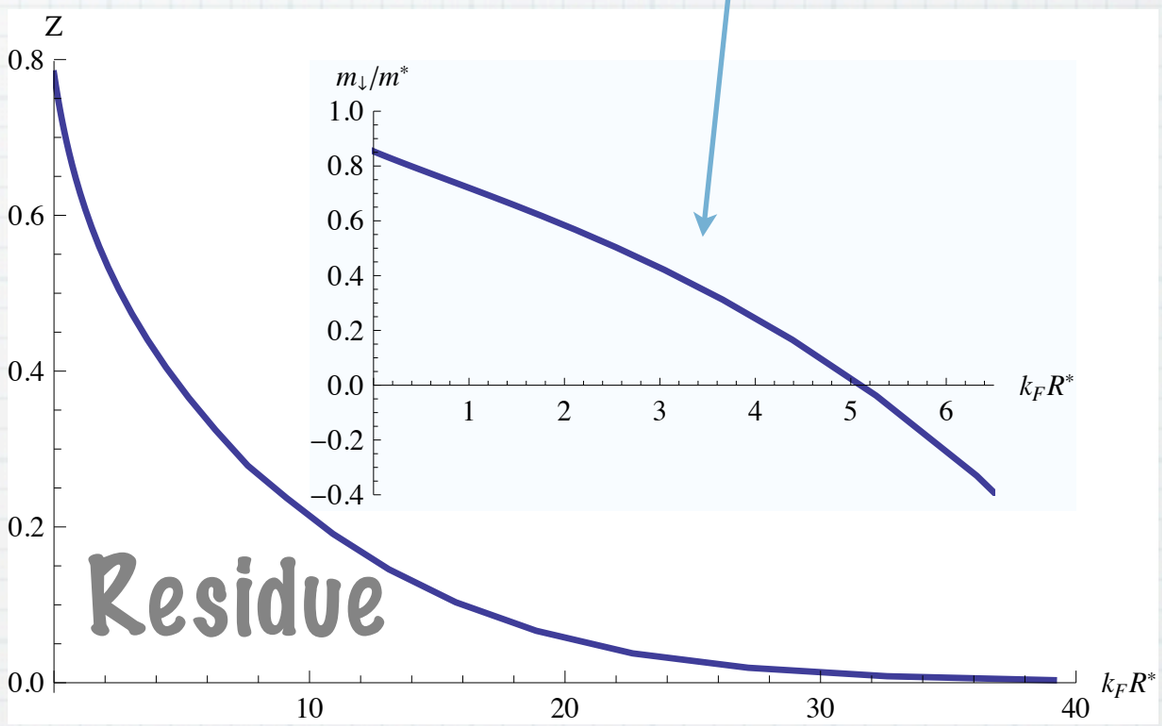
Attractive "narrow" polaron

at resonance ($a^{-1}=0$)

equal masses



(inverse) effective mass



Repulsive "narrow" polaron

at resonance ($a^{-1}=0$)

equal masses

Residue

Energy

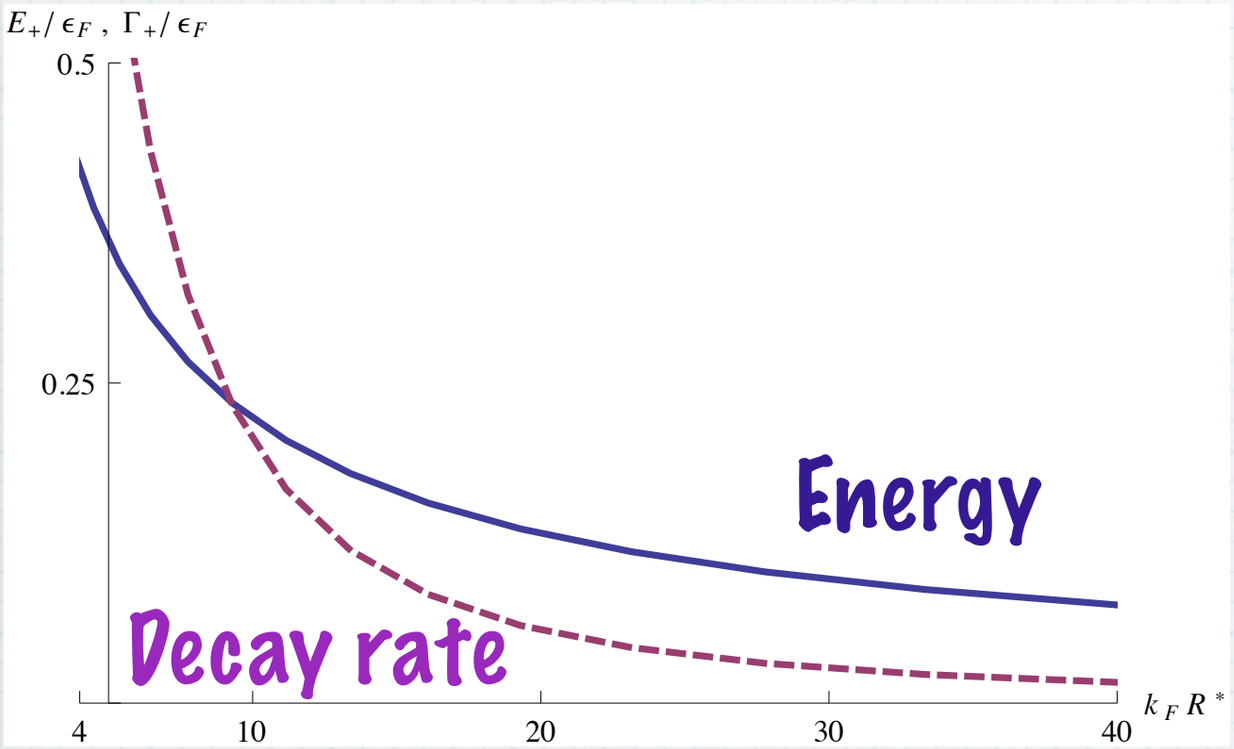
Decay rate

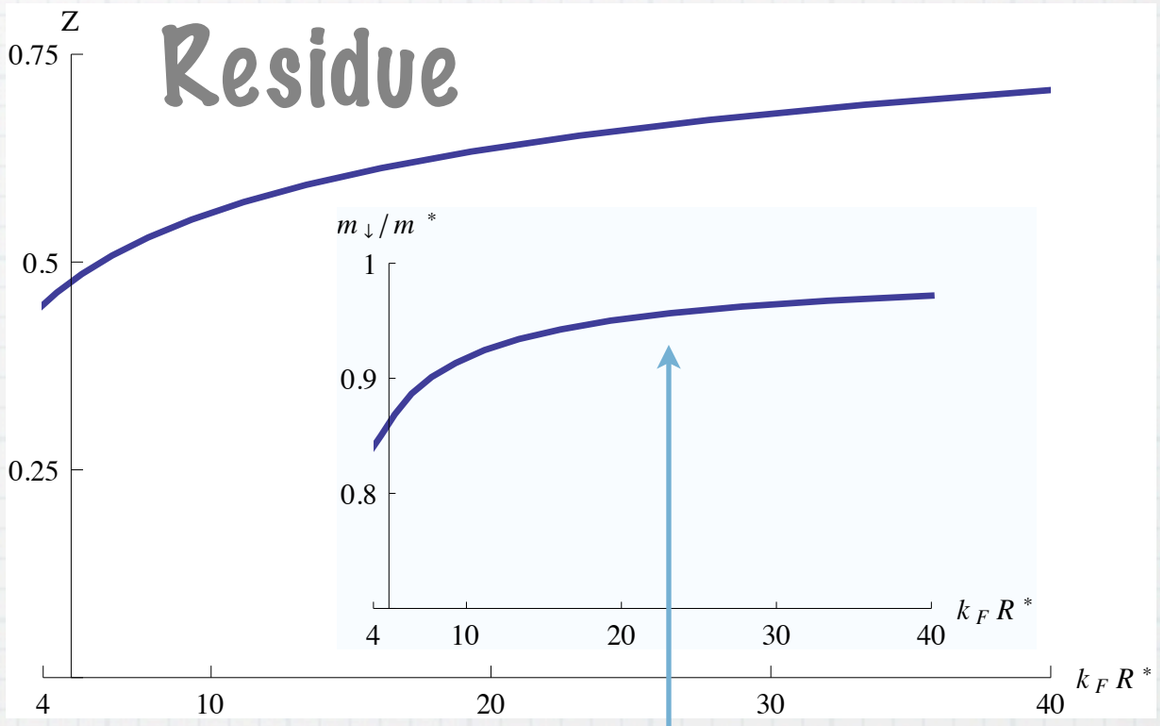
(inverse) effective mass

Repulsive "narrow" polaron

at resonance ($a^{-1}=0$)

equal masses





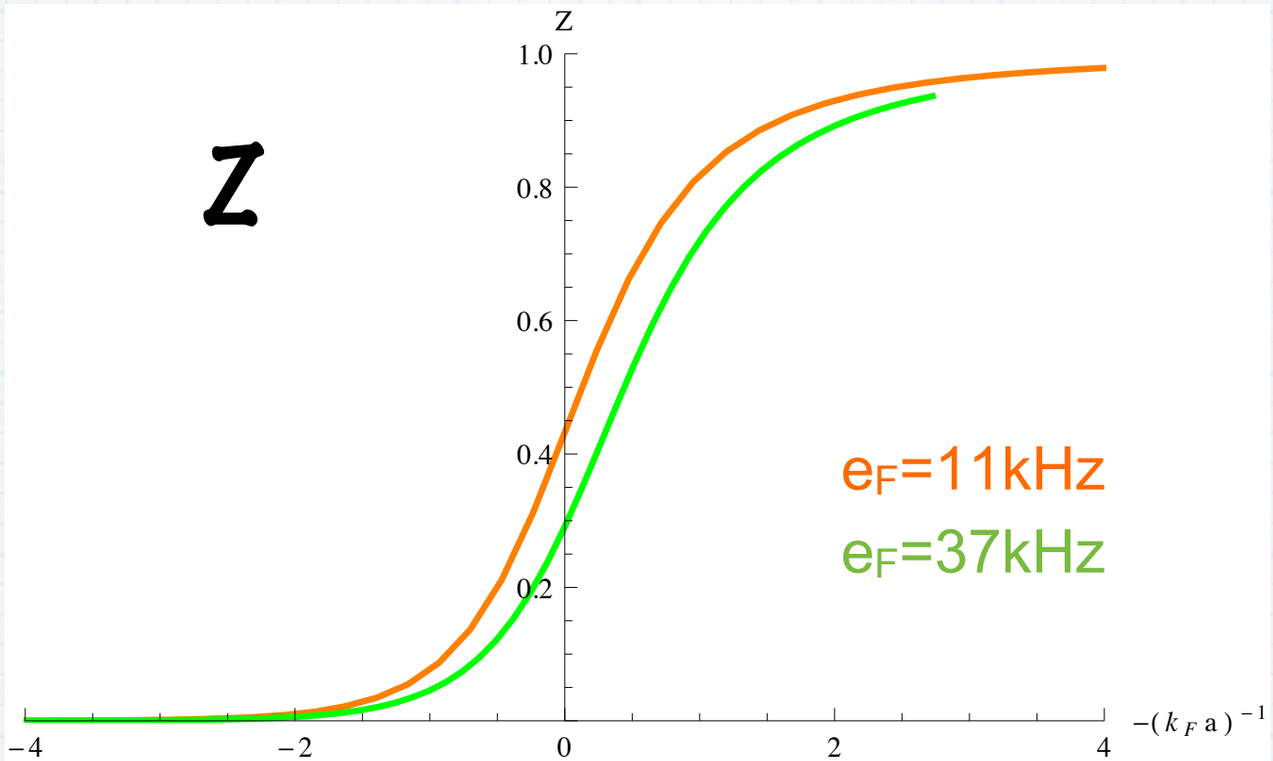
(inverse) effective mass

K impurities in a Li gas

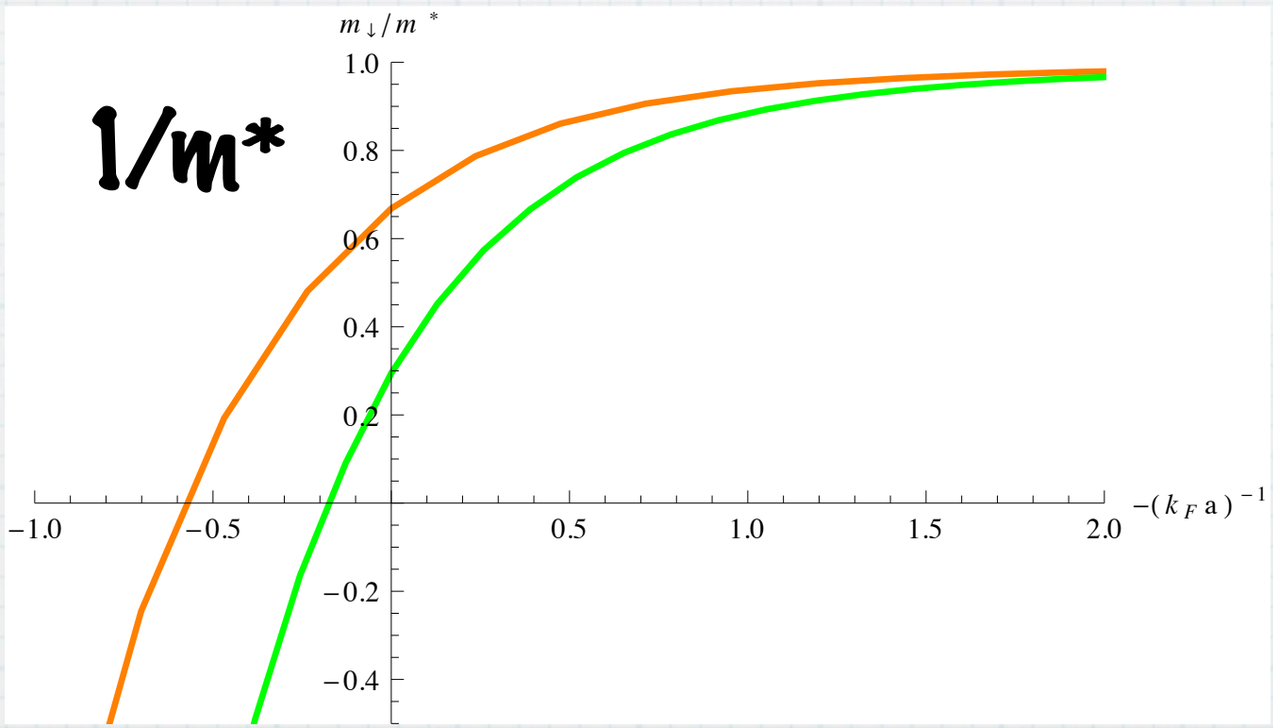
BEC

BCS

Z



1/m*



Dressed Molecules

$$|\Phi_{\mathbf{p}=0}\rangle = \left(\beta_0^{(0)} b_0^\dagger + \sum_k \beta_{\mathbf{k}}^{(1)} d_{-\mathbf{k}}^\dagger u_{\mathbf{k}}^\dagger + \sum_{\mathbf{k},\mathbf{q}} \beta_{\mathbf{k},\mathbf{q}}^{(2)} b_{\mathbf{q}-\mathbf{k}}^\dagger u_{\mathbf{k}}^\dagger u_{\mathbf{q}} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \beta_{\mathbf{k},\mathbf{k}',\mathbf{q}}^{(3)} d_{\mathbf{q}-\mathbf{k}-\mathbf{k}'}^\dagger u_{\mathbf{k}'}^\dagger u_{\mathbf{k}}^\dagger u_{\mathbf{q}}\right) |FS_{N-1}\rangle.$$

$$H = \sum_{\mathbf{p}} \left[\xi_{\mathbf{p},\uparrow} u_{\mathbf{p}}^\dagger u_{\mathbf{p}} + \xi_{\mathbf{p},\downarrow} d_{\mathbf{p}}^\dagger d_{\mathbf{p}} + (\xi_{\mathbf{p},M} + \nu_0) b_{\mathbf{p}}^\dagger b_{\mathbf{p}}\right] + \frac{g_0}{V} \sum_{\mathbf{p},\mathbf{p}'} \left(b_{\mathbf{p}}^\dagger u_{\mathbf{p}'} d_{\mathbf{p}-\mathbf{p}'} + h.c.\right)$$

renormalization conditions:

$$\frac{m_r}{2\pi a} = -\frac{\nu_0}{g_0^2} + \frac{1}{V}\sum_{\mathbf{p}} \frac{1}{2\xi_{\mathbf{p}}}$$

$$r_0 = -\frac{2\pi}{g_0^2 m_r^2}$$

minimize $\langle\Phi|\hat{H} - E|\Phi\rangle$

Pol/Mol crossing at a narrow FR

2-Body molecule in vacuum

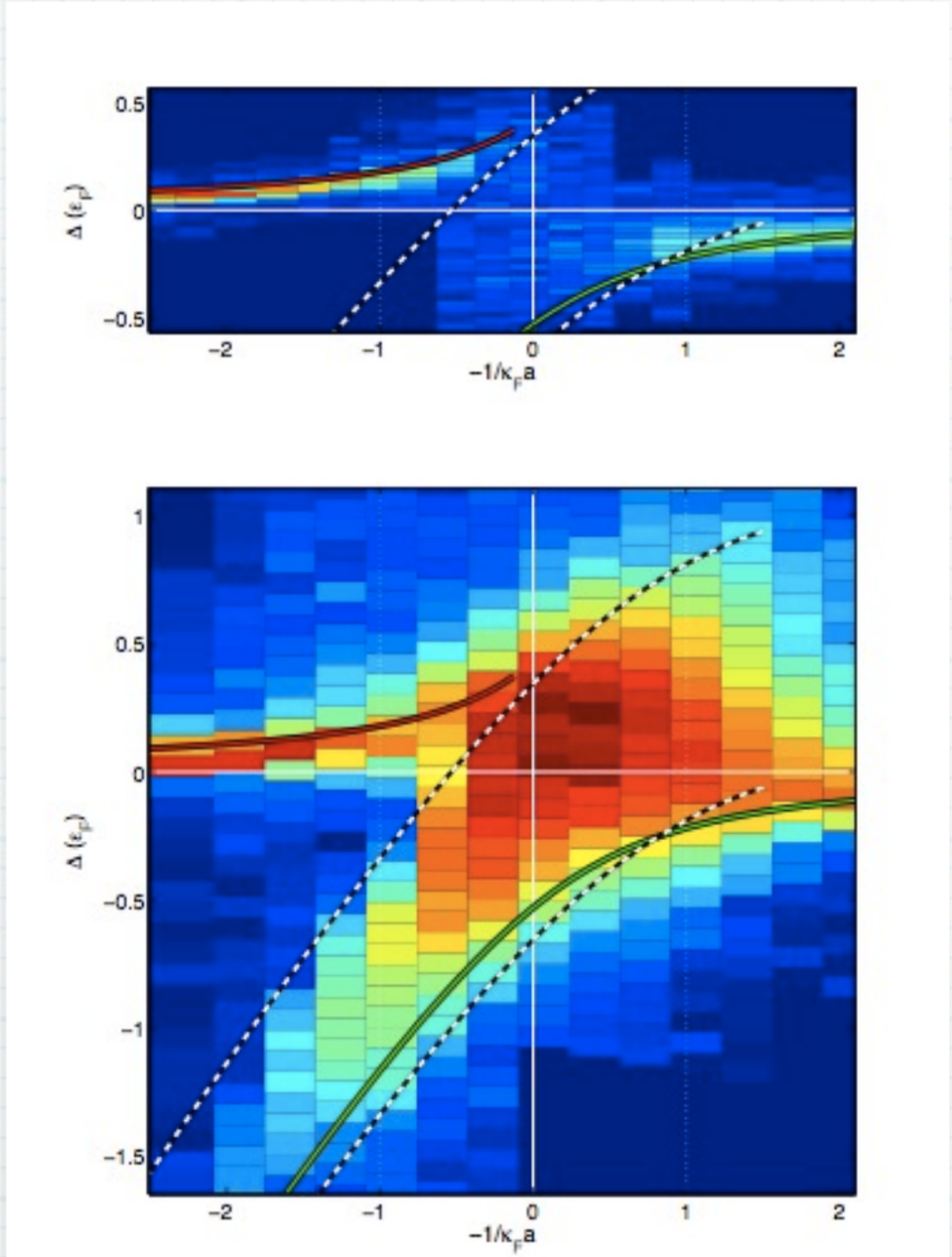
Polaron

$|k_F a| > 1$

Dressed molecule

K impurities in a Li gas
Li: E_F=37kHz

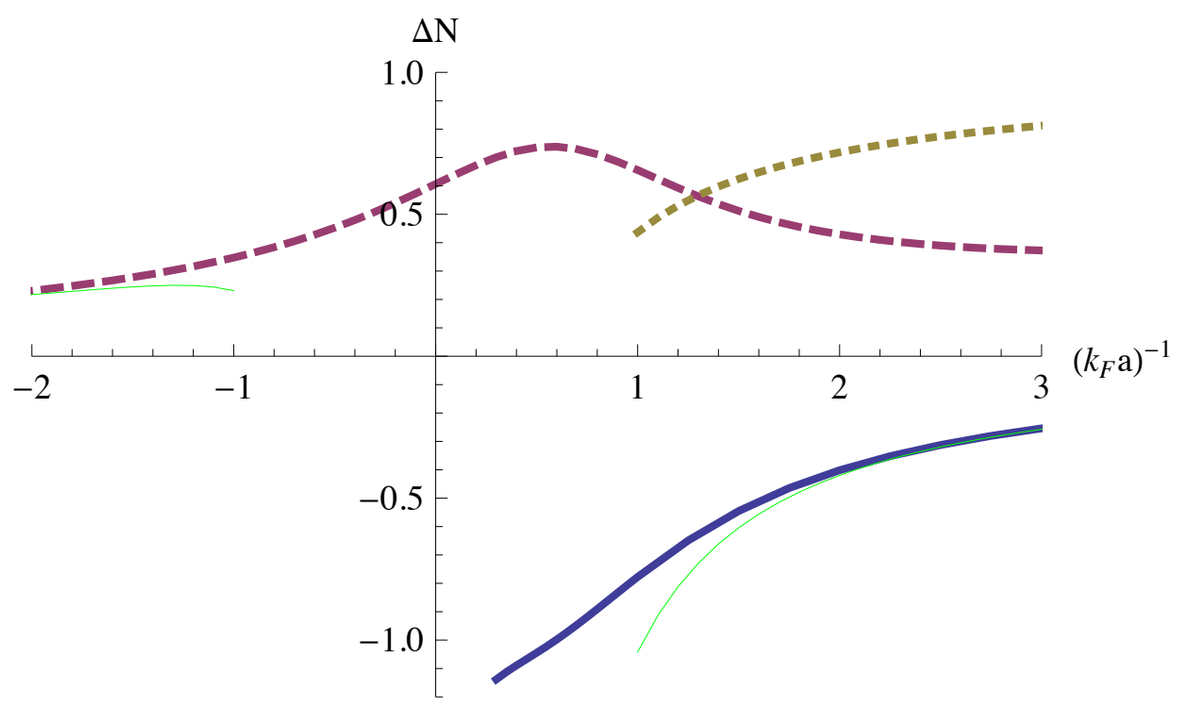
Comparison with experiment



of particles in the dressing cloud

$$\delta\mu_\uparrow = \frac{\partial^2\varepsilon}{\partial n_\uparrow \partial n_\downarrow} + \frac{\partial^2\varepsilon}{(\partial n_\uparrow)^2}\Delta N = 0$$

$$\Delta N = -\left(\frac{\partial\mu_\downarrow}{\partial n_\uparrow}\right)_{n_\downarrow} \Big/ \left(\frac{\partial\mu_\uparrow}{\partial n_\uparrow}\right)_{n_\downarrow} \approx -\left(\frac{\partial\mu_\downarrow}{\partial\epsilon_F}\right)_{n_\downarrow}$$



weak coupling: $\Delta N = -\frac{2}{\pi}k_F a - \frac{4}{\pi^2}(k_F a)^2 + \ldots$

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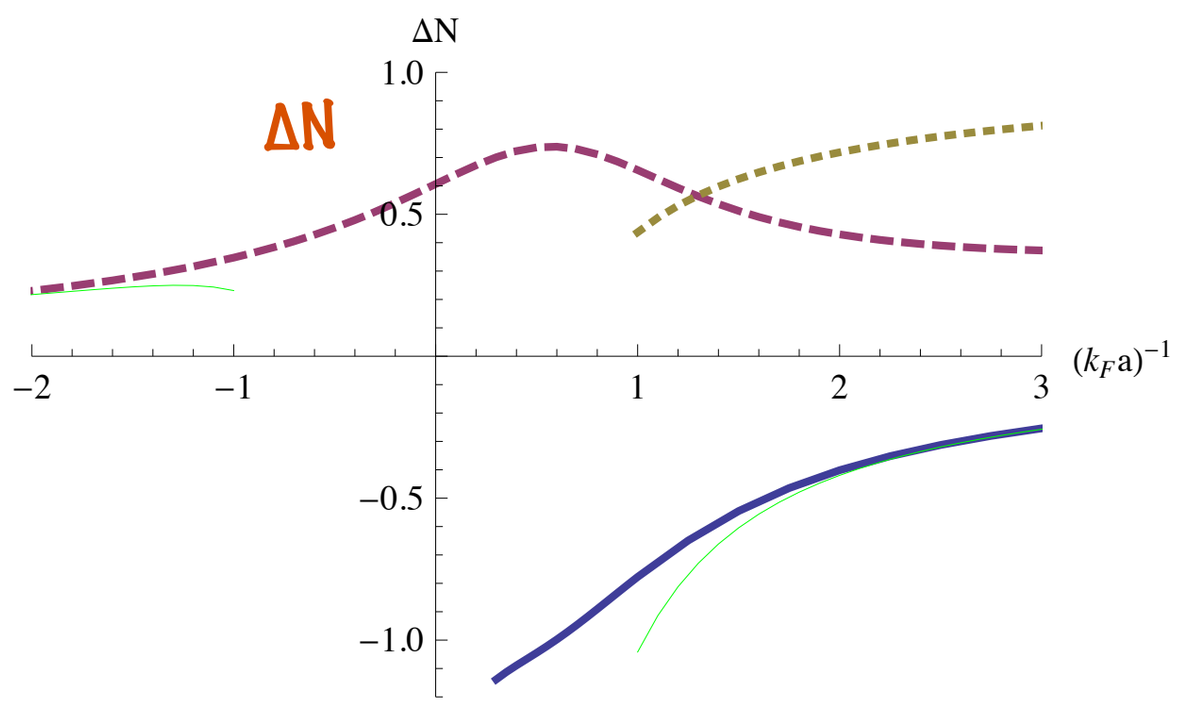


weak coupling: $\Delta N = -\frac{2}{\pi}k_F a - \frac{4}{\pi^2}(k_F a)^2 + \ldots$

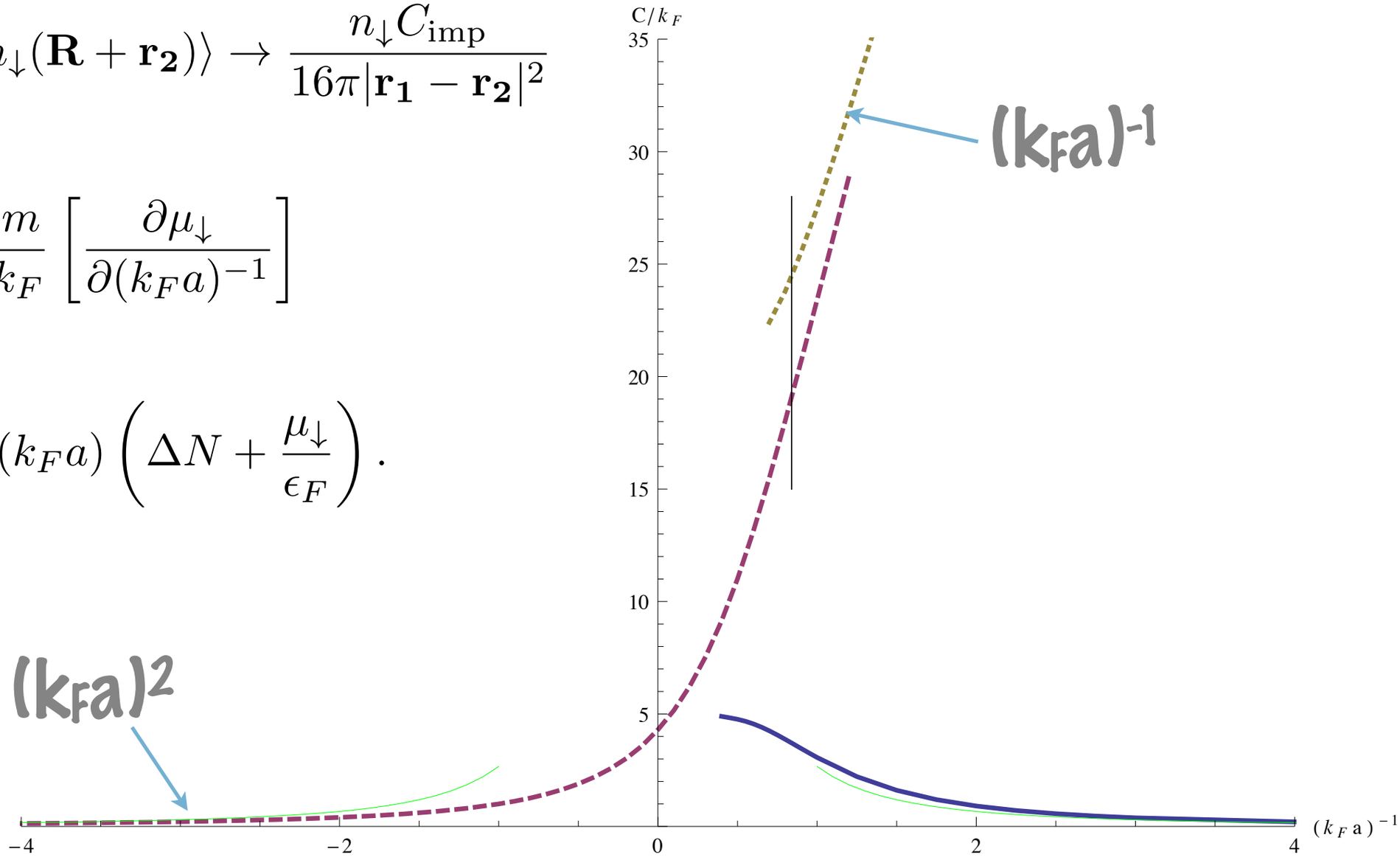
Tan's contact

C_{imp}: contact density per impurity

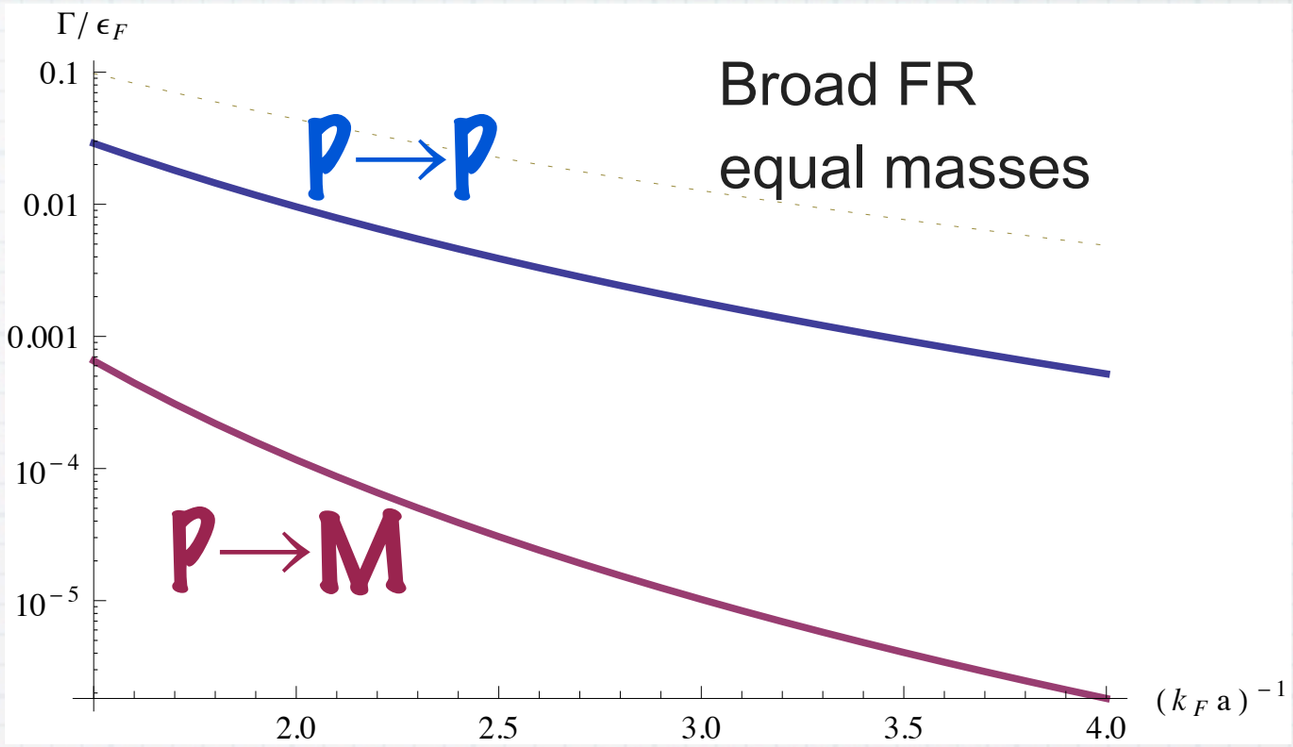
$$\langle n_\uparrow(\mathbf{R}+\mathbf{r_1})n_\downarrow(\mathbf{R}+\mathbf{r_2})\rangle \to \frac{n_\downarrow C_{\rm imp}}{16\pi|\mathbf{r_1}-\mathbf{r_2}|^2}$$

$$C_{\rm imp} = -\frac{4\pi m}{\hbar^2 k_F}\left[\frac{\partial \mu_\downarrow}{\partial (k_F a)^{-1}}\right]$$

$$\frac{C_{\rm imp}}{k_F} = -4\pi(k_F a)\left(\Delta N + \frac{\mu_\downarrow}{\epsilon_F}\right).$$



Decay: analytics BEC



2-body: $\Gamma_{(P_+ \to P_-)} \propto Z_- \dfrac{a^2}{a^*}$

3-body: $\Gamma_{(P_+ \to M)} \propto \dfrac{2\pi}{m_r^2 a^* \sqrt{1 + 4R^*/a}} T_{\mathrm{vac}}^2 (a^*)^5$

Universal limit $\propto (k_F a)^6$

(agrees with Petrov, PRA 2003)

Narrow limit $\propto (a)^{9/2} (R^*)^{3/2}$

Conclusions

- Complete characterization of the repulsive branch: energy, residue, decay rate, m^*, ΔN, C_{imp}

- RF spectra

- Many-body physics at narrow FR

I) G. Bruun and PM, PRL (2010)
II) K. Sadeghzadeh, G. Bruun, C. Lobo, PM, and A Recati, arXiv:Dec. 2010 (NJP in press)
III) PM and G. Bruun, arXiv:Feb 2011 (EPJD in press)
IV) more on narrow FR coming soon