# Impurities in a Fermi sea

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ICFO Institut de Ciències Fotòniques

UAB

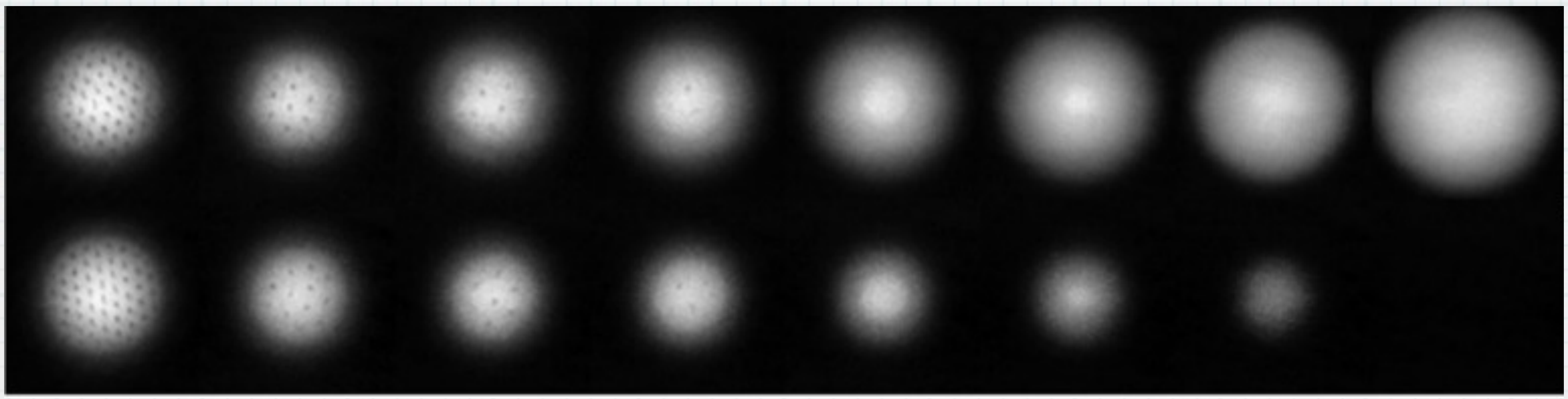
European Science Foundation
SETTING SCIENCE AGENDAS FOR EUROPE

FerMix - EuroQuam

# Fermi mixtures


BEC-BCS crossover

# Imbalanced Fermi mixtures



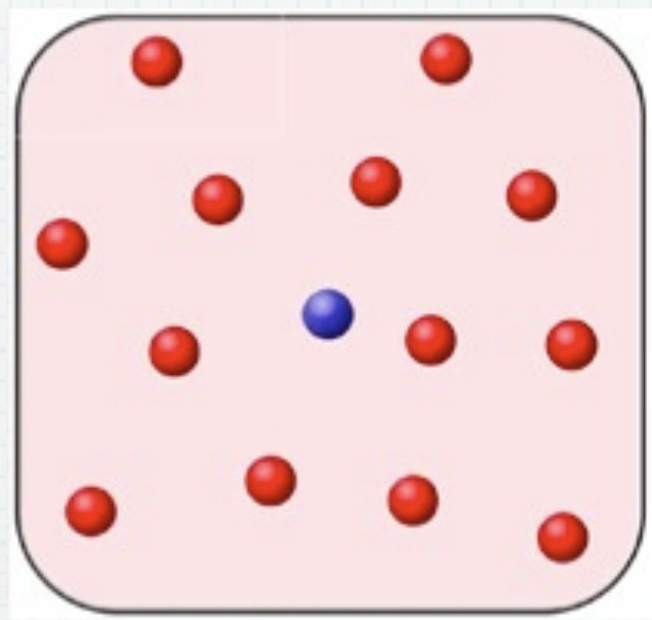
N=N

N>>N

SF-normal transition

# Very imbalanced Fermi mixtures

N>>1



normal Fermi liquid

# in collaboration with:

Georg Bruun (Aarhus)

Carlos Lobo (Southampton)

Alessio Recati (Trento)

Kayvan Sadegzadeh (Cambridge)

Matteo

# Outline

* Quasi-particles in many-body systems
* The MIT "impurity" experiment
* Polarons and molecules
* Repulsive polarons
* Itinerant FerroMagnetism
* RF spectra

# Many-body systems

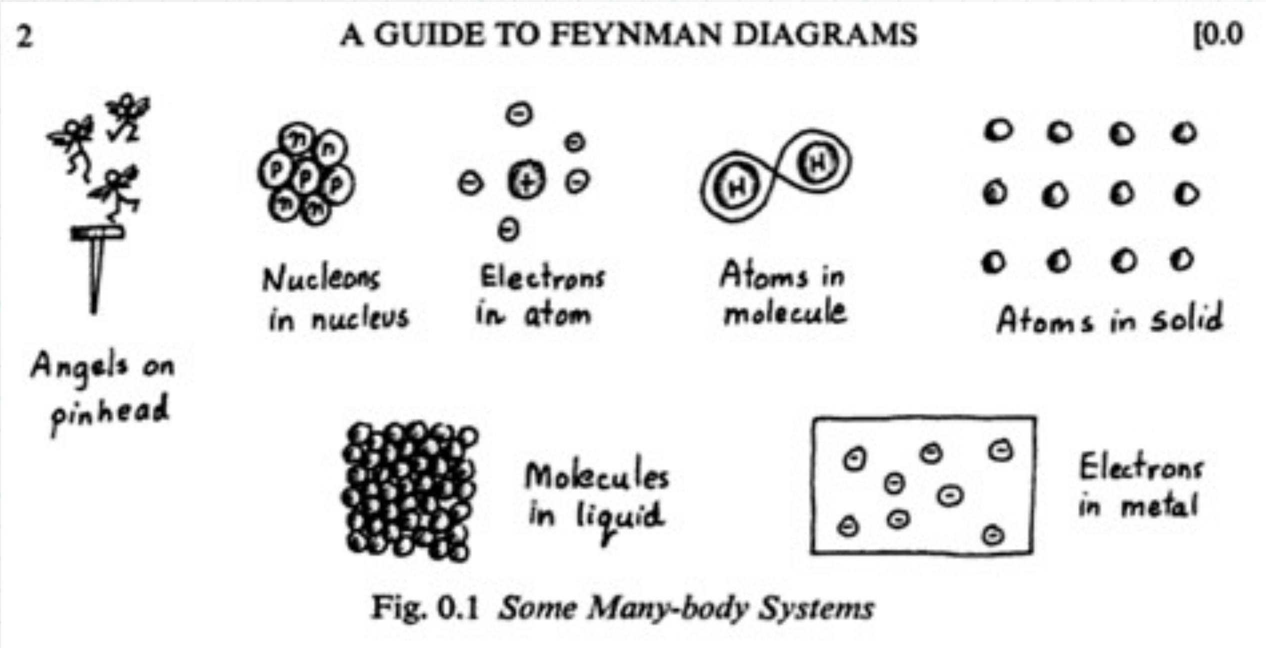


Fig. 0.1 *Some Many-body Systems*

(from Richard Mattuck's book)

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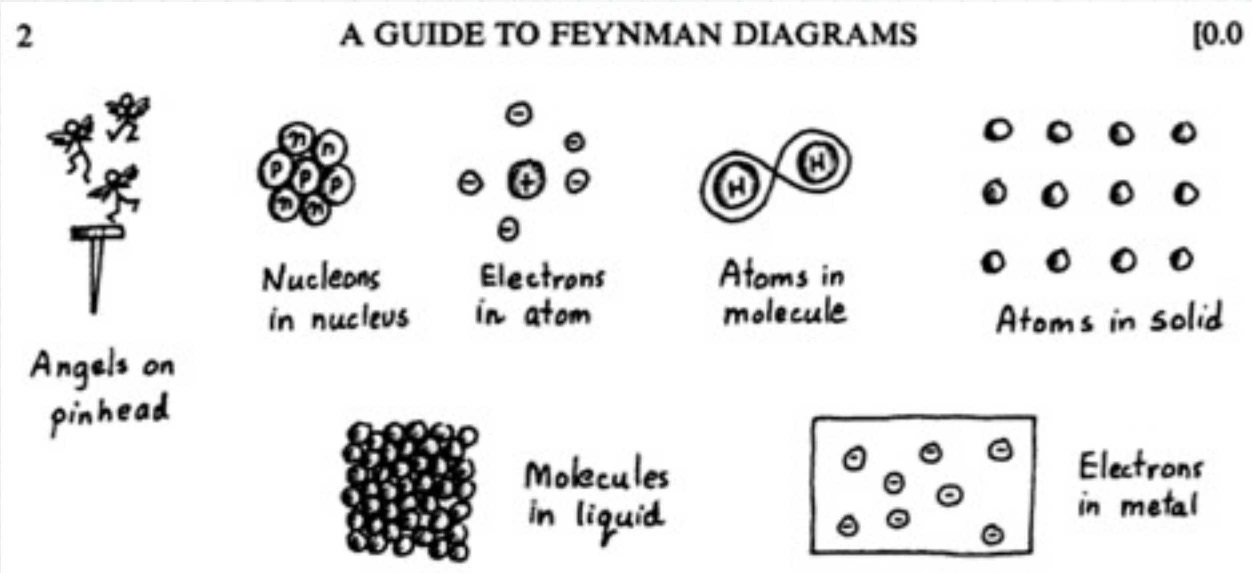


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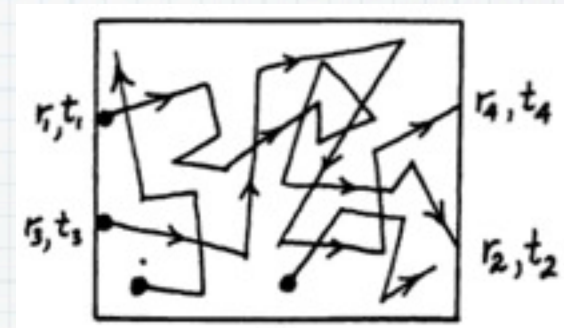

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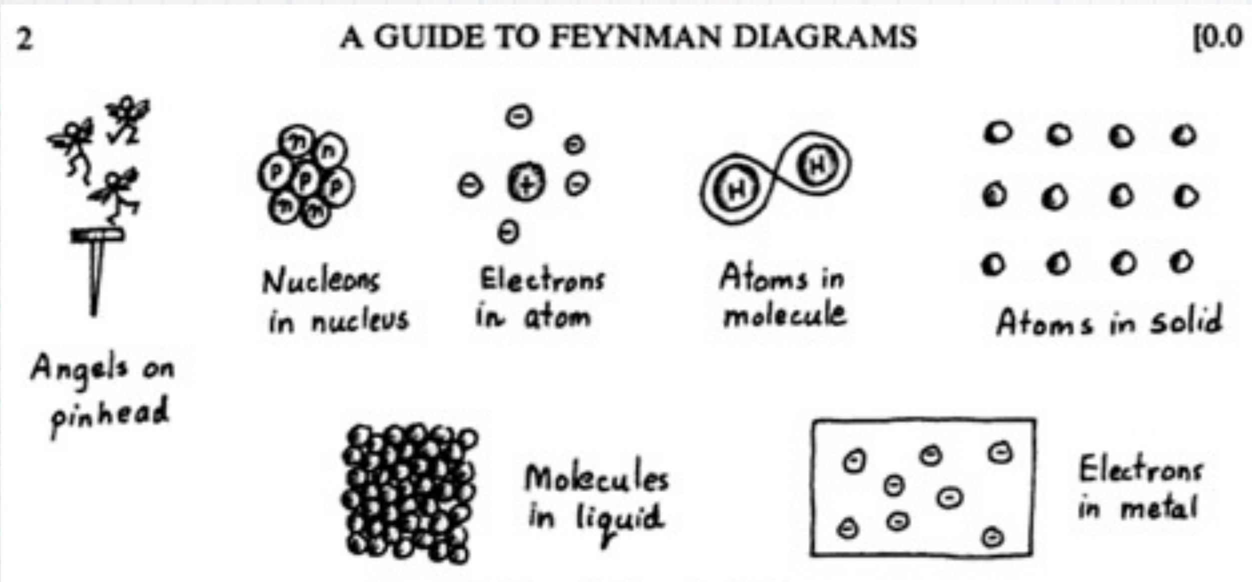


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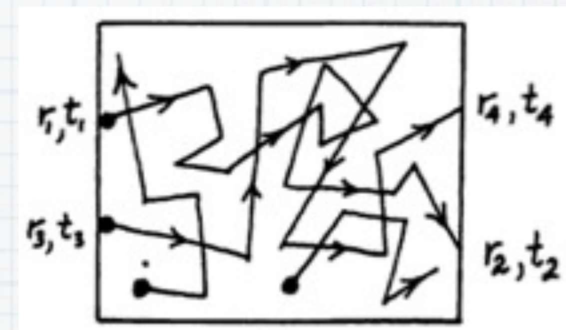
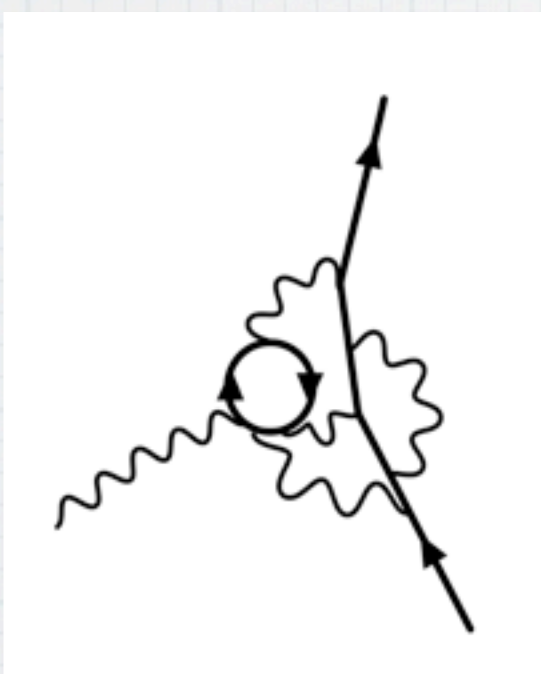



## Feynman diagrams:



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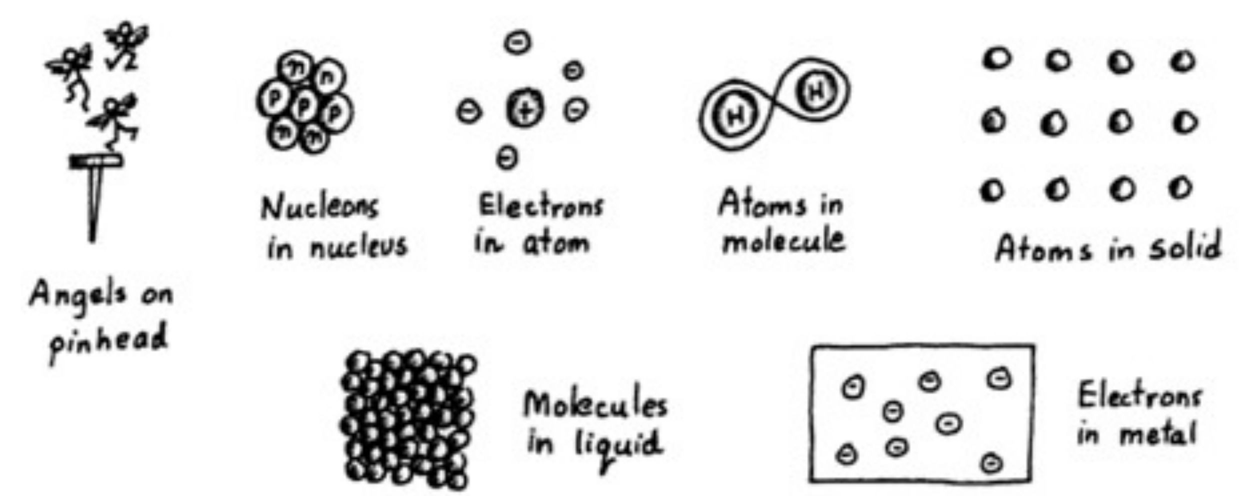
2 A GUIDE TO FEYNMAN DIAGRAMS [0.0

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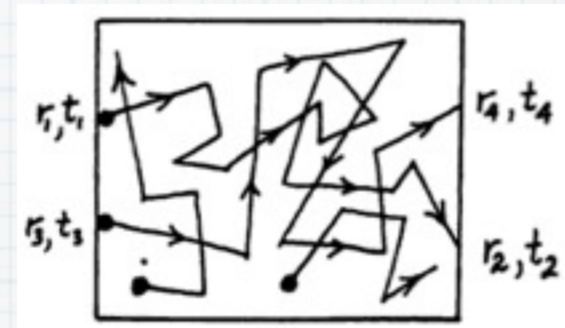


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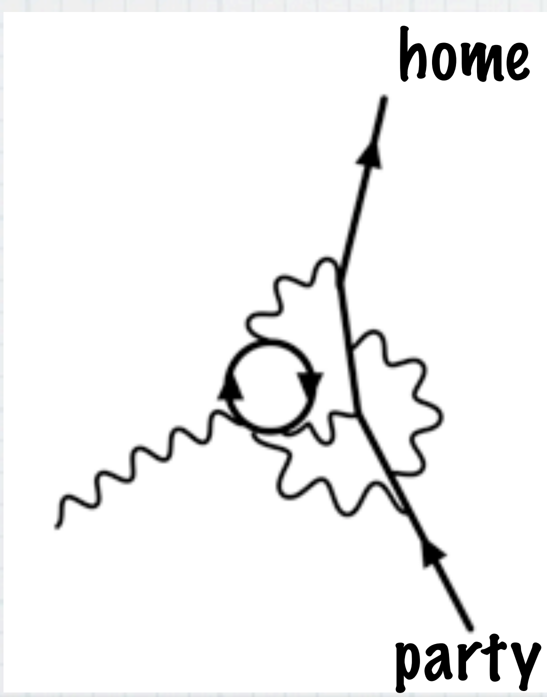


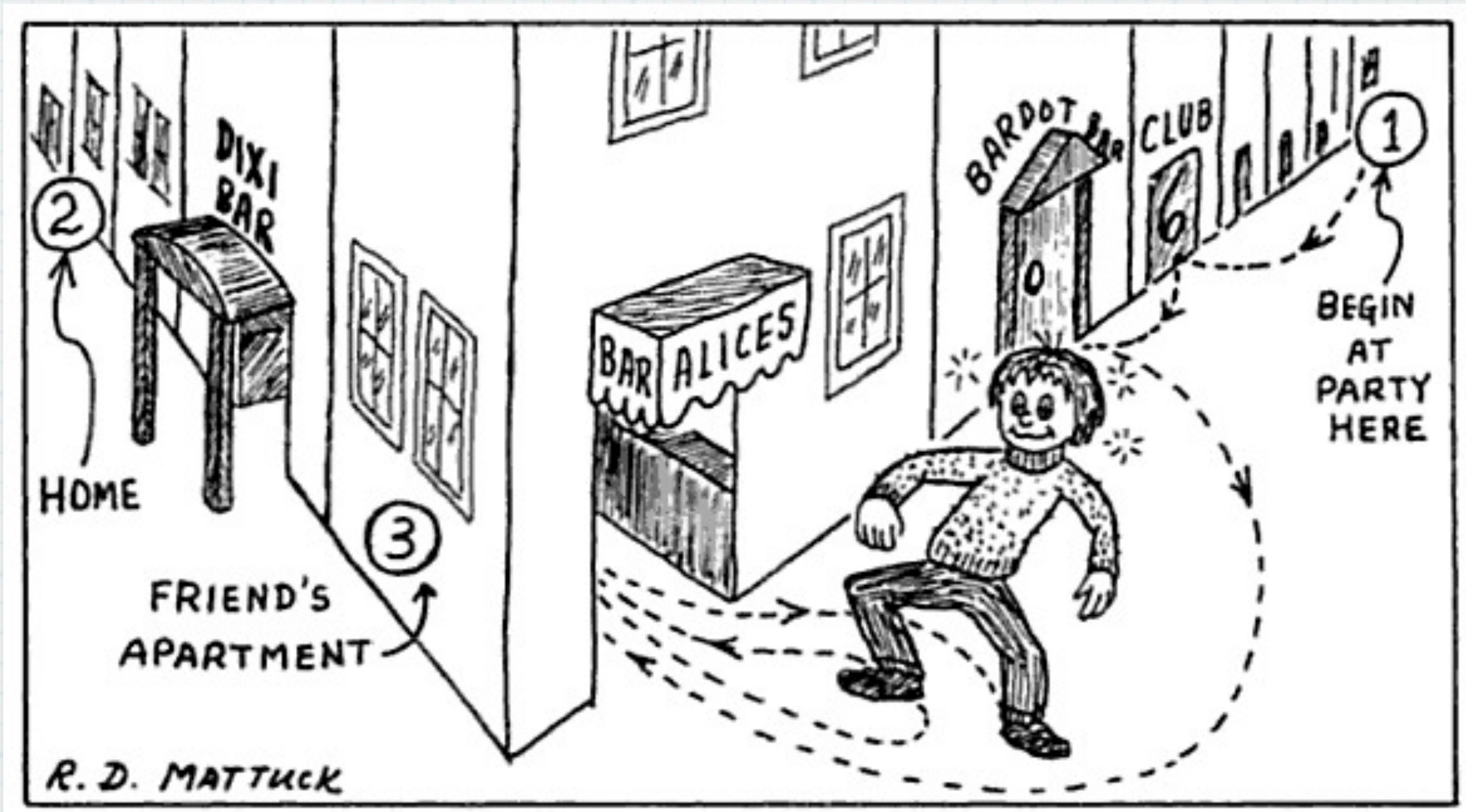
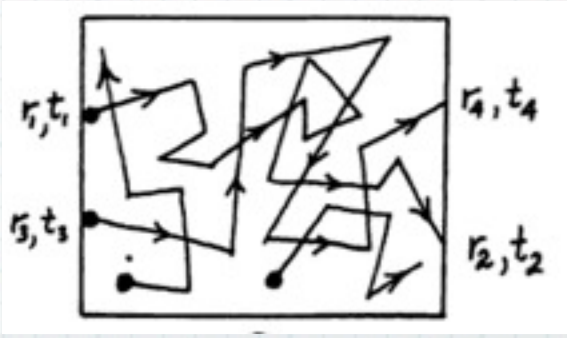




Fig. 1.1 *Propagation of Drunken Man*

# Quasi-Particles

Landau's idea:
who cares about real particles?



Of importance are the excitations,
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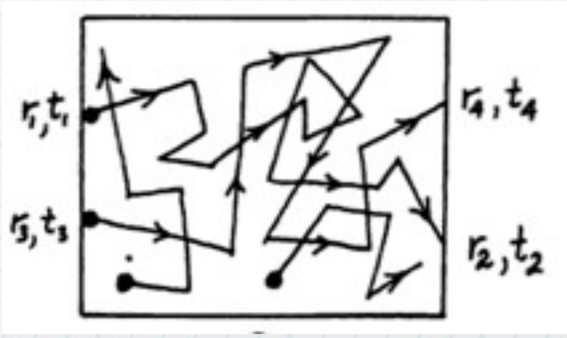


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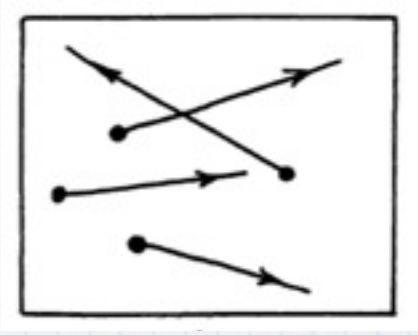

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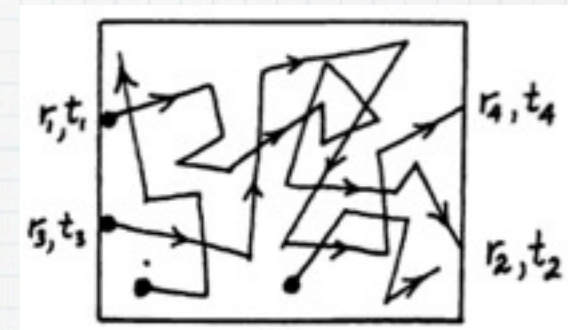


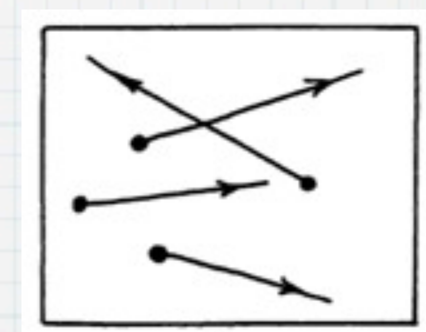

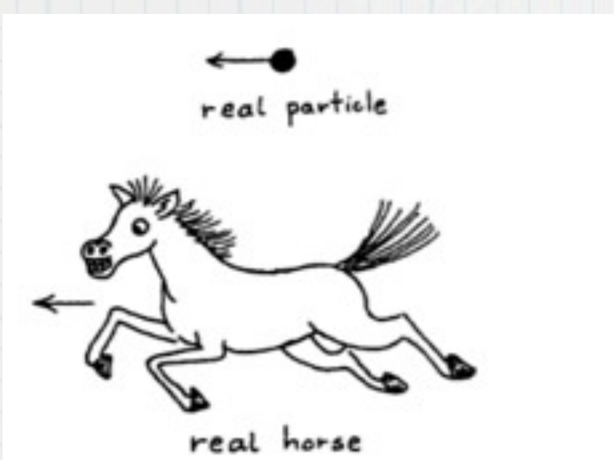

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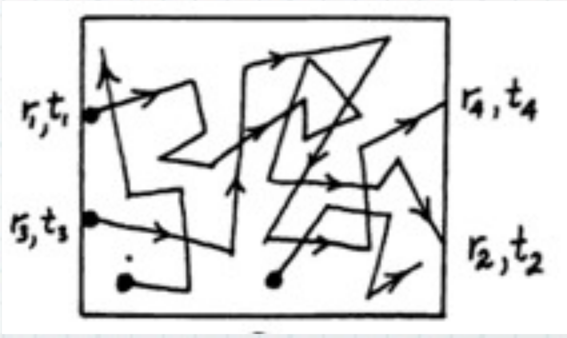


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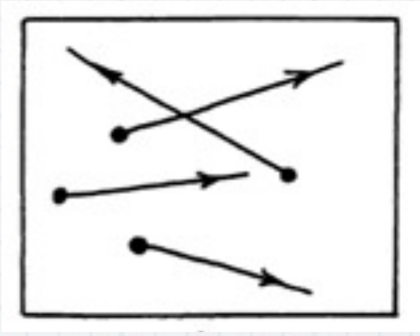

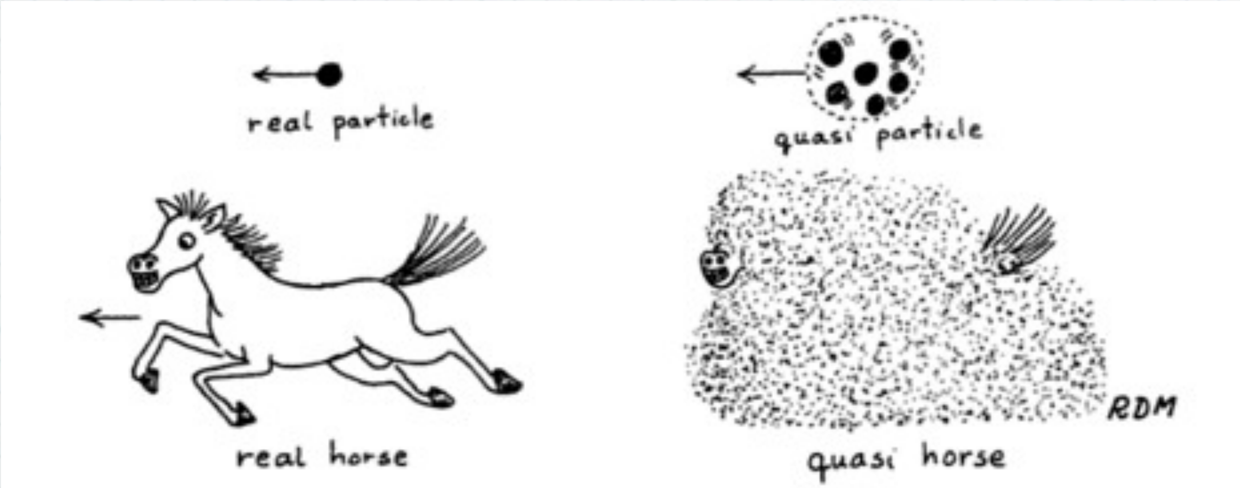


Fig. 0.4 *Quasi Particle Concept*

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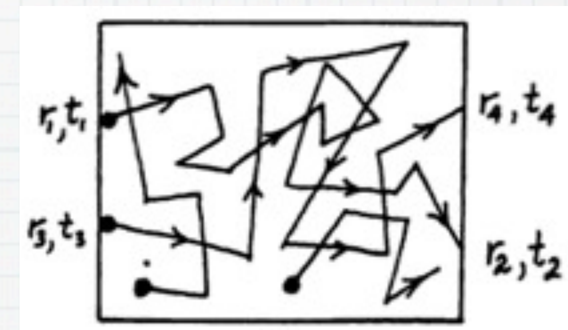


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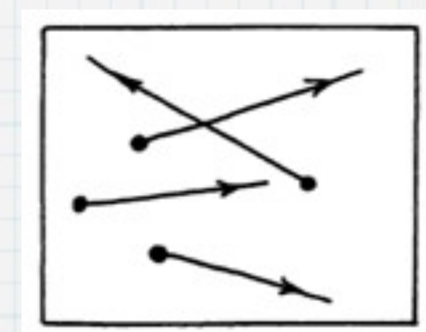


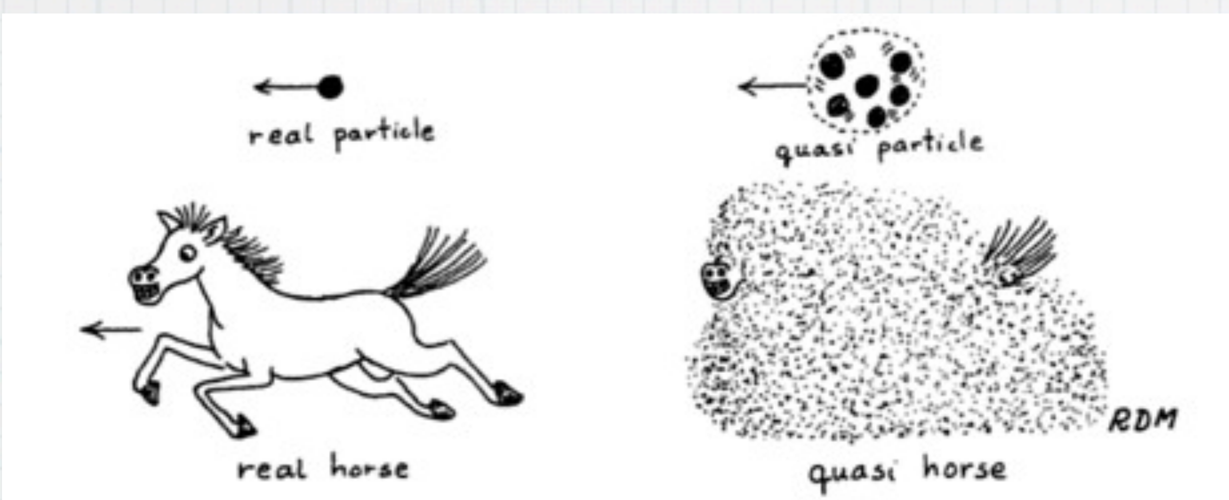


Fig. 0.4 *Quasi Particle Concept*

a QP is a "free particle" with:

@ renormalized mass
@ chemical potential
@ shielded interactions
@ q. numbers (charge, spin, ...)
@ lifetime

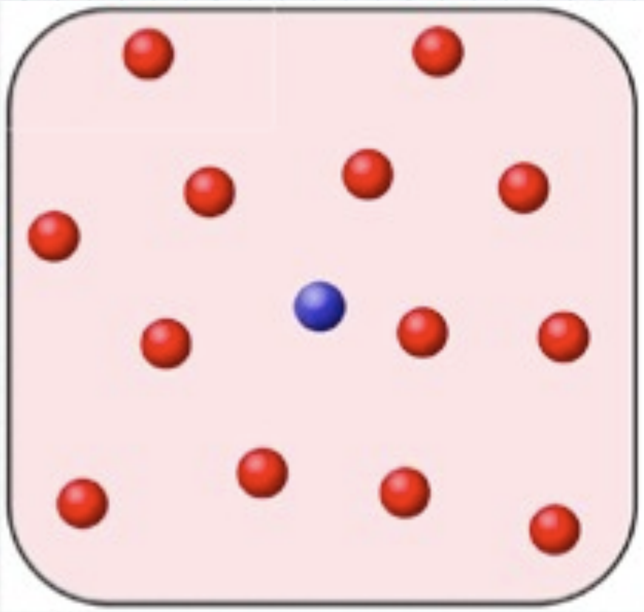
# The MIT experiment

Schirotzek, Wu, Sommer & Zwierlein, PRL 2009

- non-interacting ↑ Fermi sea (N>>1)
- a single ↓ impurity

BCS — Attraction strength → BEC

$(k_F a)^{-1} < 0$



free particle

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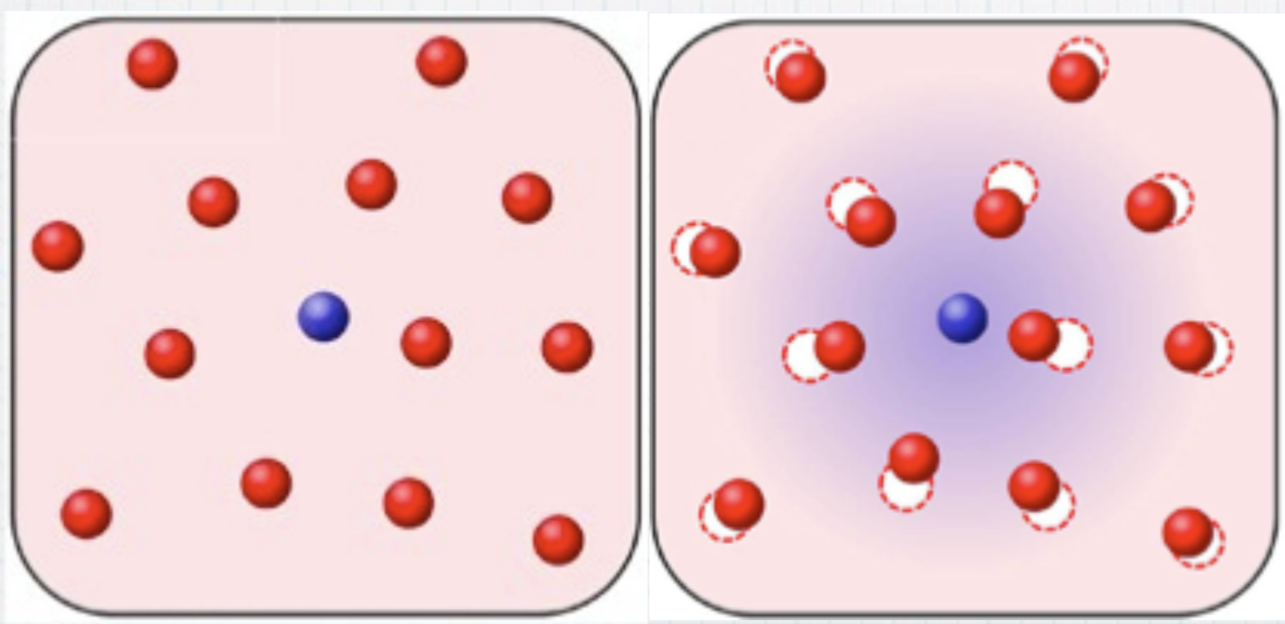

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**QP** (polaron)

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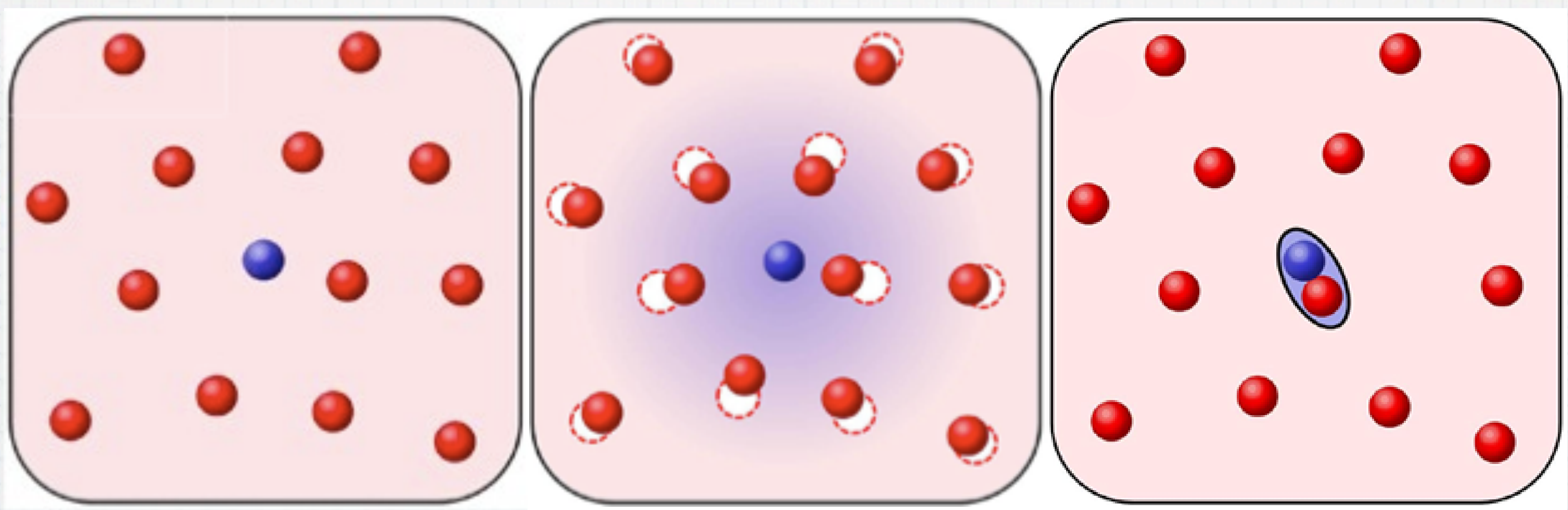

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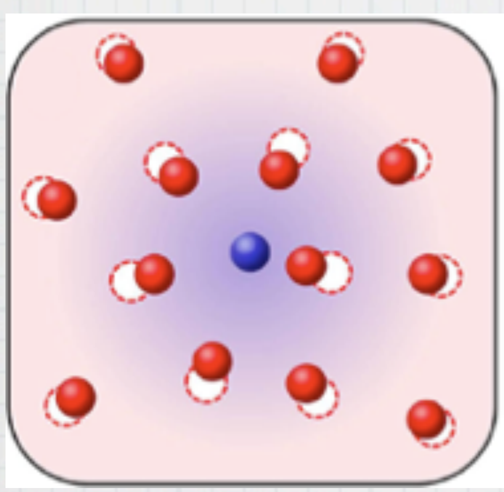
free particle | **QP** (polaron) | **QP** (molecule)

**P-M transition**: Prokof'ev&Svistunov, PRB 2008

# Polaron: variational Ansatz

(F. Chevy, PRA 2006)



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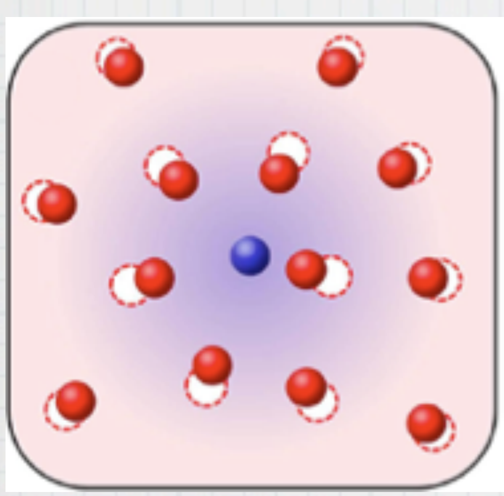
the ↓ impurity

$$|\psi_{\mathbf{p}}\rangle = \phi_0 c^{\dagger}_{\mathbf{p}\downarrow} |0\rangle_{\uparrow} + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{q}\mathbf{k}}\, c^{\dagger}_{\mathbf{p}+\mathbf{q}-\mathbf{k}\downarrow}\, \boxed{c^{\dagger}_{\mathbf{k}\uparrow}\, c_{\mathbf{q}\uparrow}\, |0\rangle_{\uparrow}}$$

non-interacting ↑ Fermi sea

Particle-Hole dressing



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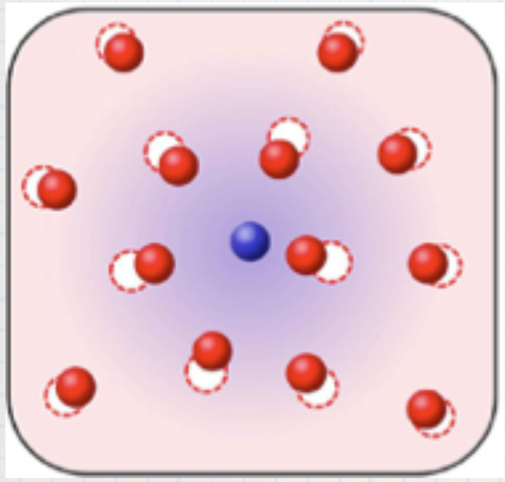

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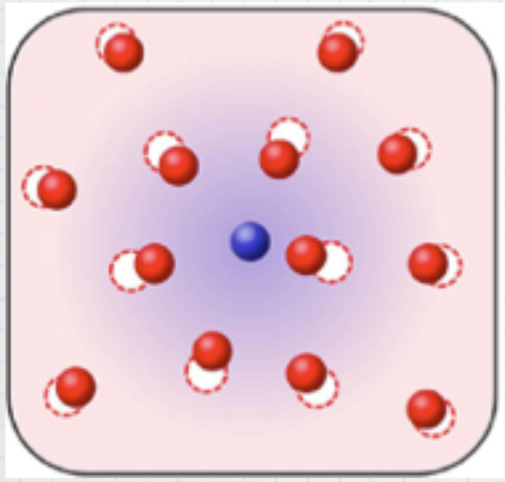


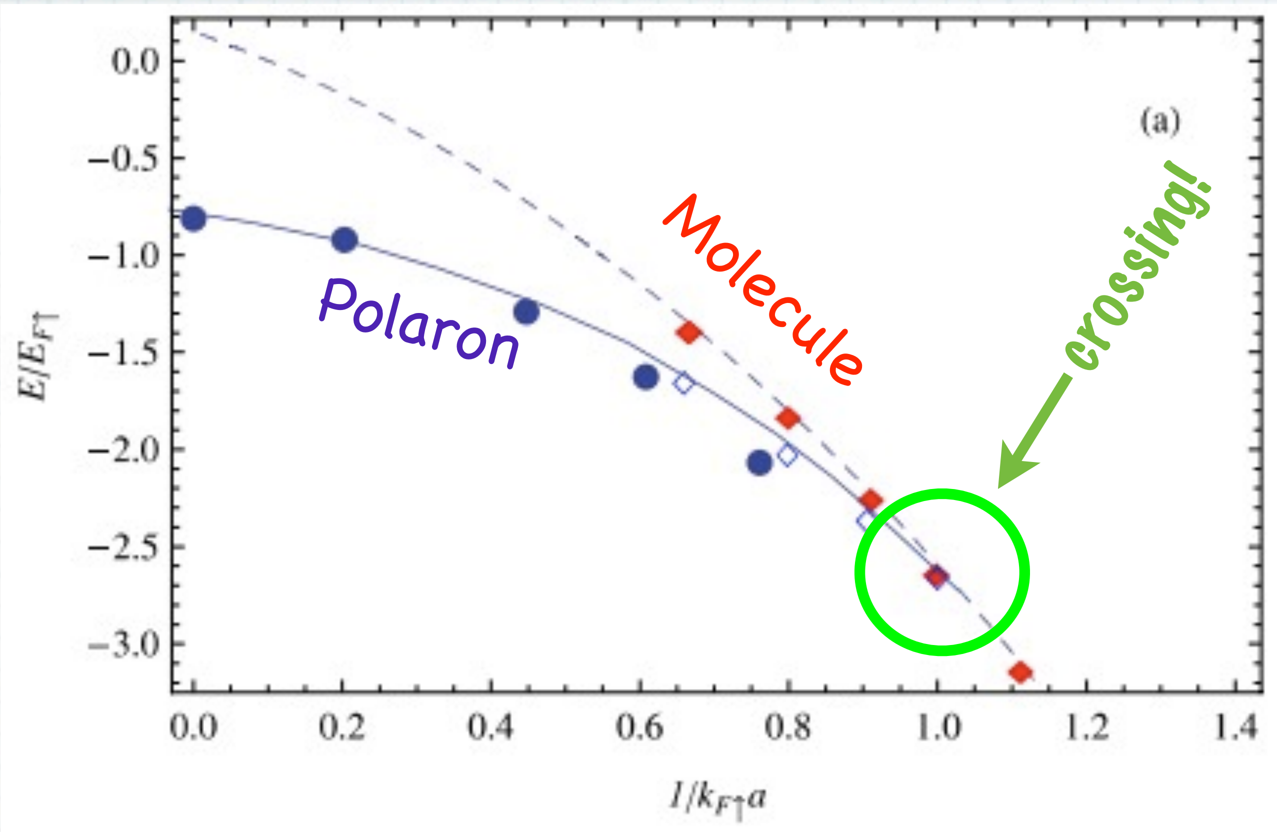
Very good agreement with QMC results for $\mu_\downarrow$ and $m^*$

This variational Ansatz has a diagrammatic equivalent:
the forward scattering, or ladder, approximation.

(Combescot et al., PRL 2007)

# QP parameters

## Chemical potential $\mu_\downarrow$




◇, ◆ : QMC

—, ⋯ :variat, diagr

● : MIT expmt

QMC: Prokof'ev&Svistunov

Variational and diagrammatic: Chevy, Recati, Lobo, Stringari, Combescot, Leyronas Massignan&Bruun, Zwerger, Punk, Stoof, Mora,...

Experiments: MIT, ENS

# QP parameters

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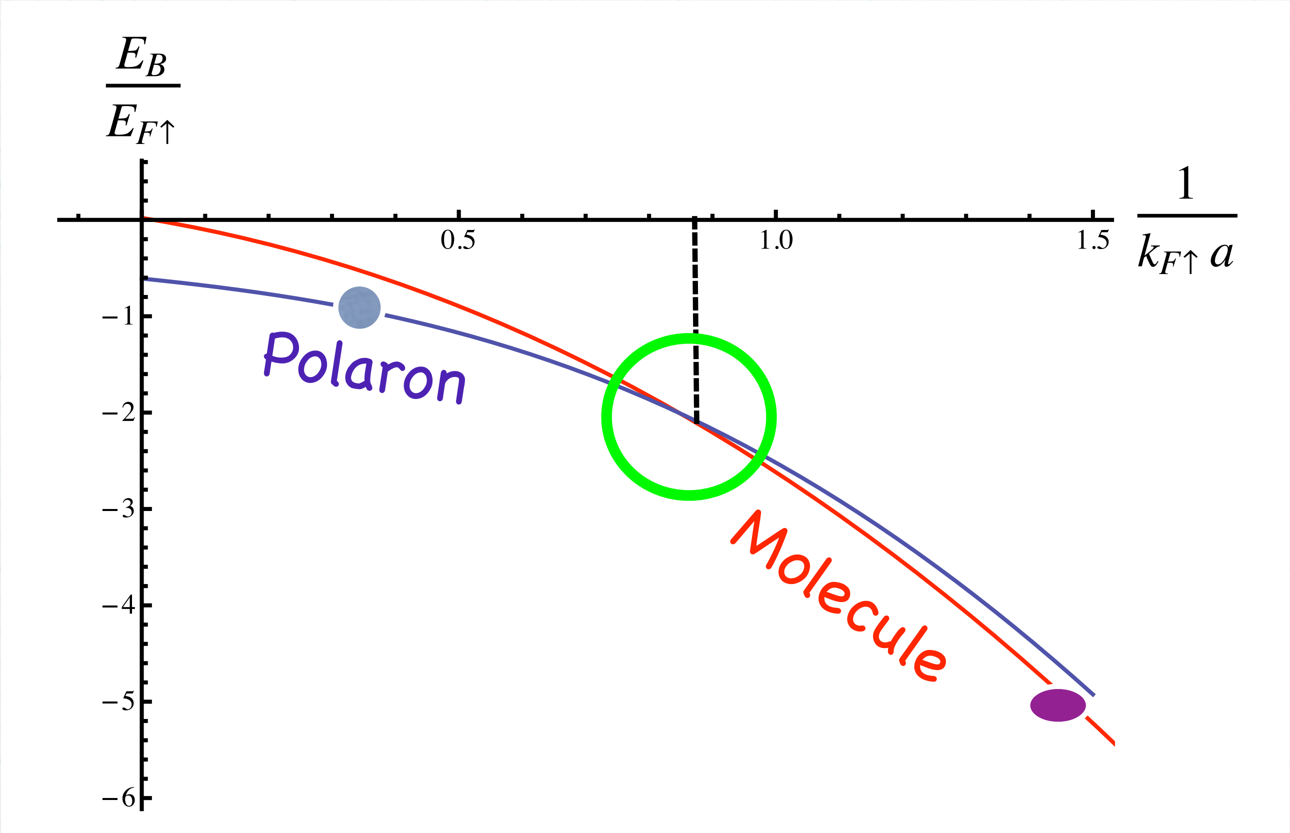

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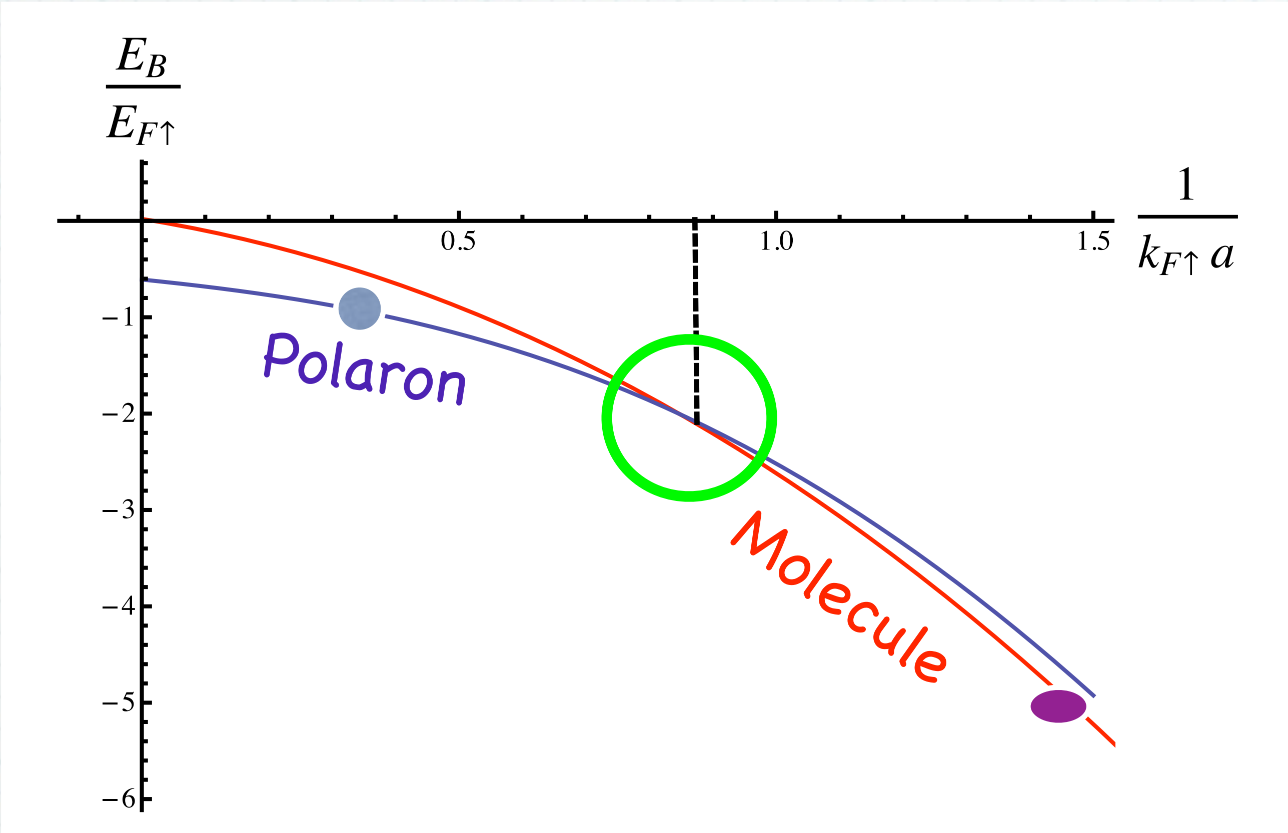
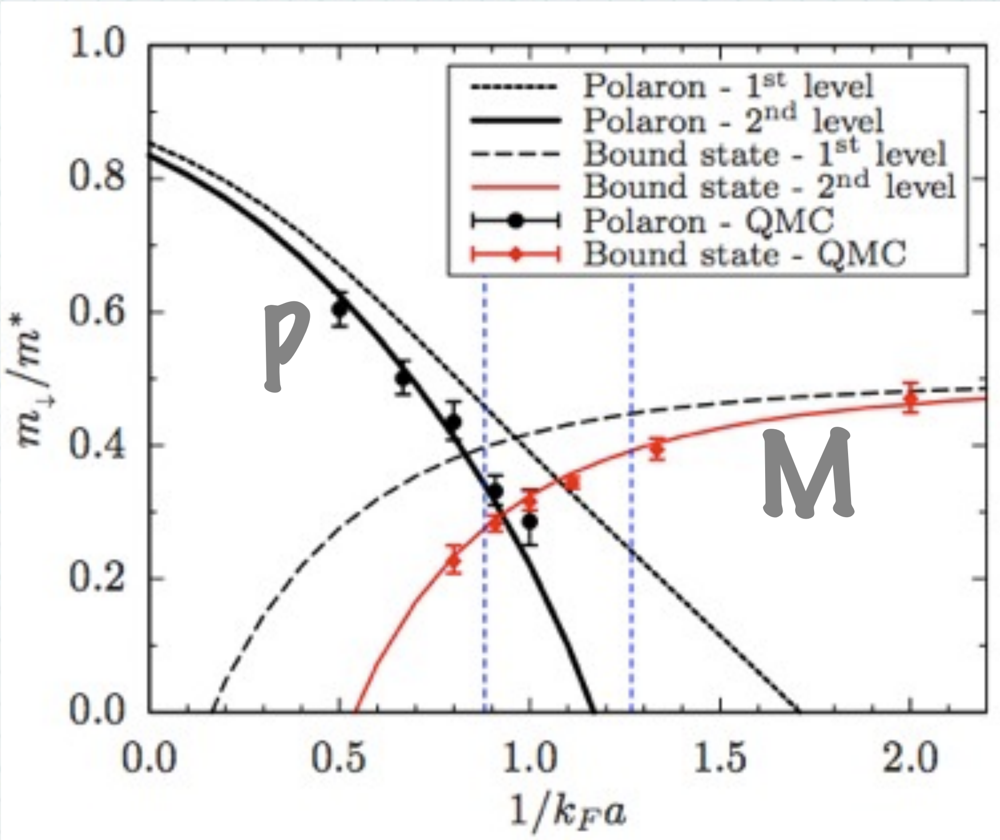



## Effective mass m*



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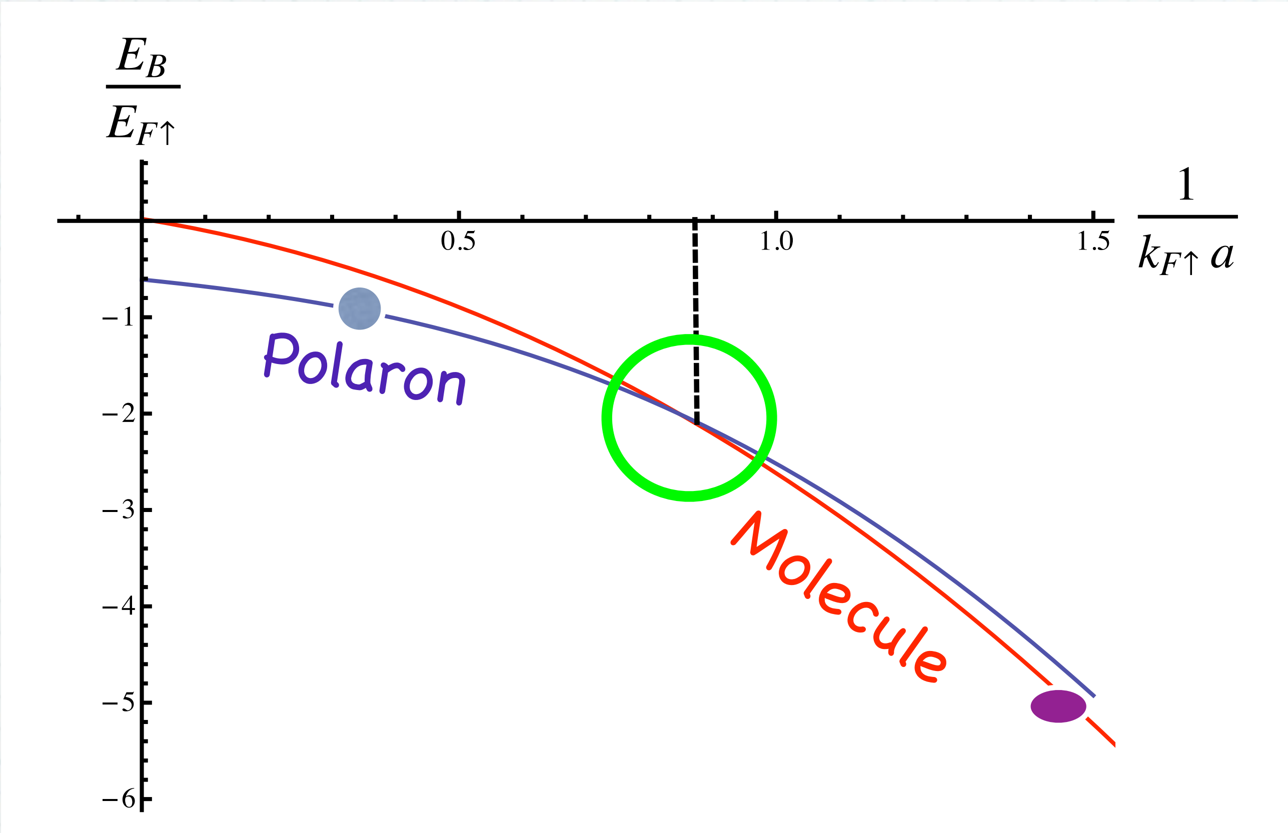


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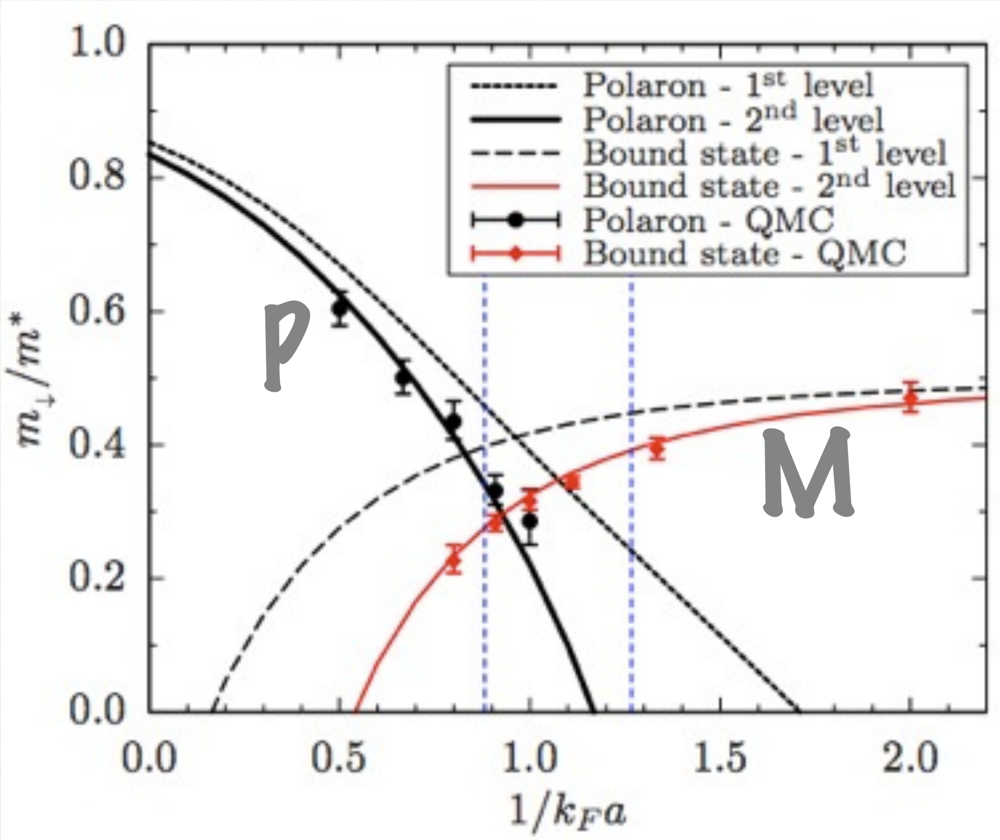



P-P Interactions: Mora&Chevy, PRL 2010

Zhenhua, Zöllner & Pethick, PRL 2010

# Equation of state of a unitary Fermi gas

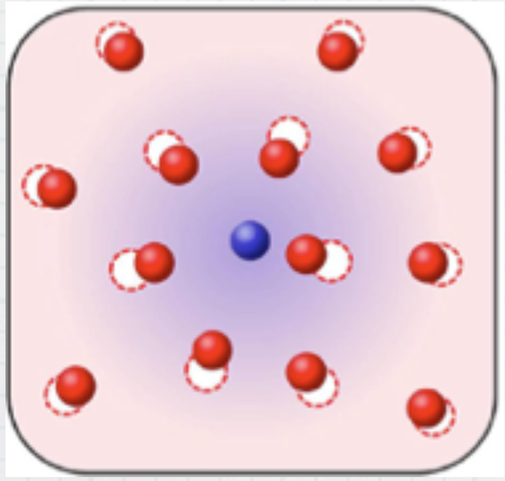
In the normal phase at T=0,

non-interacting $\uparrow$ non-interacting QP

$$P = \boxed{\frac{1}{15\pi^2}\left(\frac{2m_\uparrow}{\hbar^2}\right)^{3/2}\mu_\uparrow^{5/2}} + \boxed{\frac{1}{15\pi^2}\left(\frac{2m_\downarrow^*}{\hbar^2}\right)^{3/2}(\mu_\downarrow - A\mu_\uparrow)^{5/2}}$$

$A = -0.615$

$m_\downarrow^* = 1.2m$



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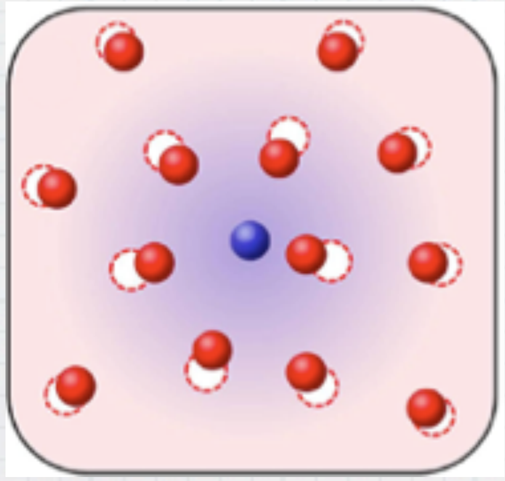


Same thermodynamics for:
- ultracold atoms
- dilute neutron matter

# What's left?

# What's left?

- ✓ chemical potential
- ✓ renormalized mass
- ✓ shielded interactions
- ✓ lifetimes (Georg & Pietro, PRL 2010)
- ✓ metastability (Kayvan, Georg, Carlos, Pietro & Alessio, arXiv:1012...)
- ✓ repulsive branch (Pietro & Georg, arXiv:1102...)

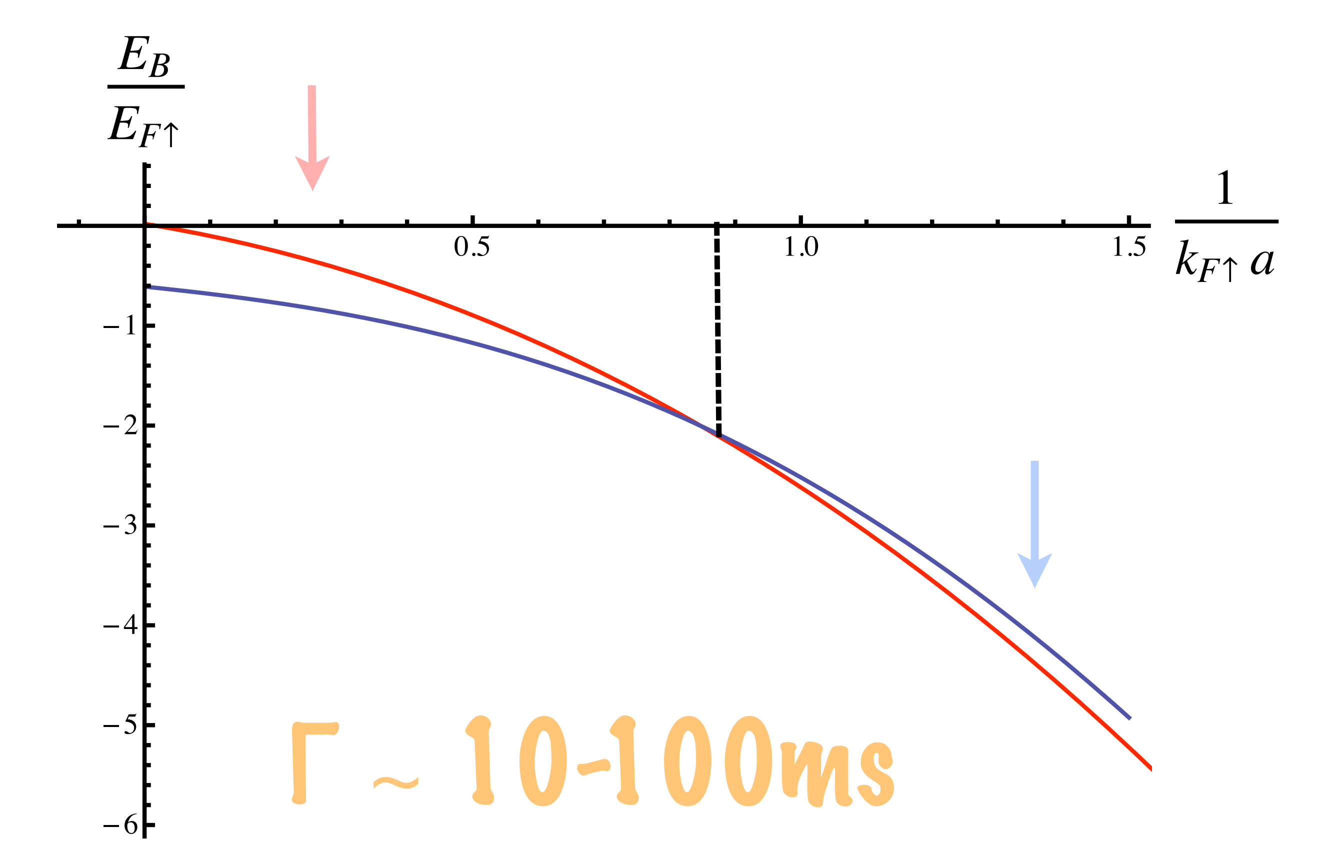
# Very long QP lifetimes!

G. Bruun & PM, PRL 2010

$$\Gamma_P \sim Z_M \left(\Delta\omega\right)^{9/2}$$

$$\Delta\omega = \omega_P - \omega_M$$

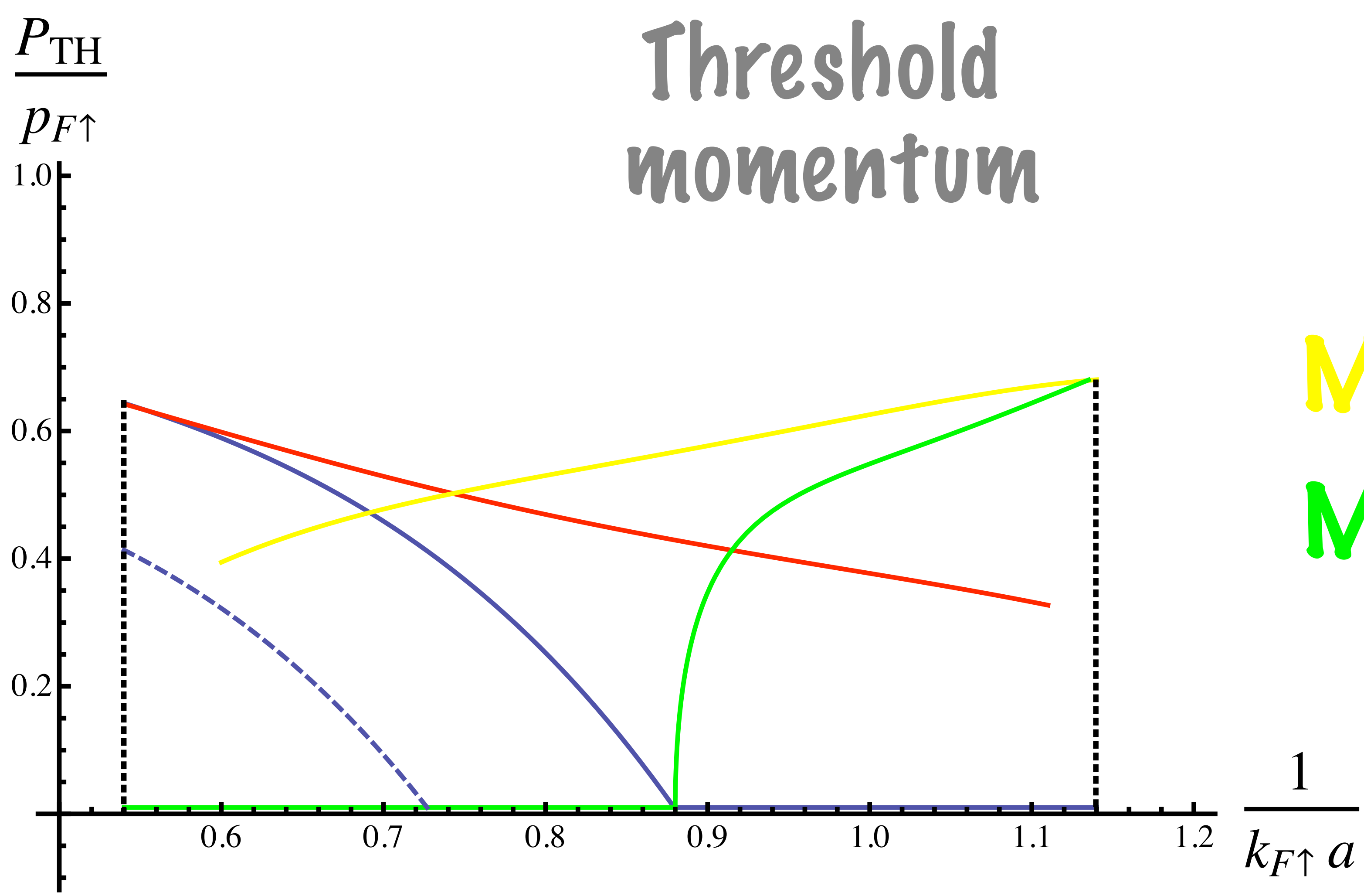
$$\Gamma_M \sim Z_P \left(-\Delta\omega\right)^{9/2}$$



# Decay of p≠0 QP

P→M+h

P→M+2h+p




M→P+p

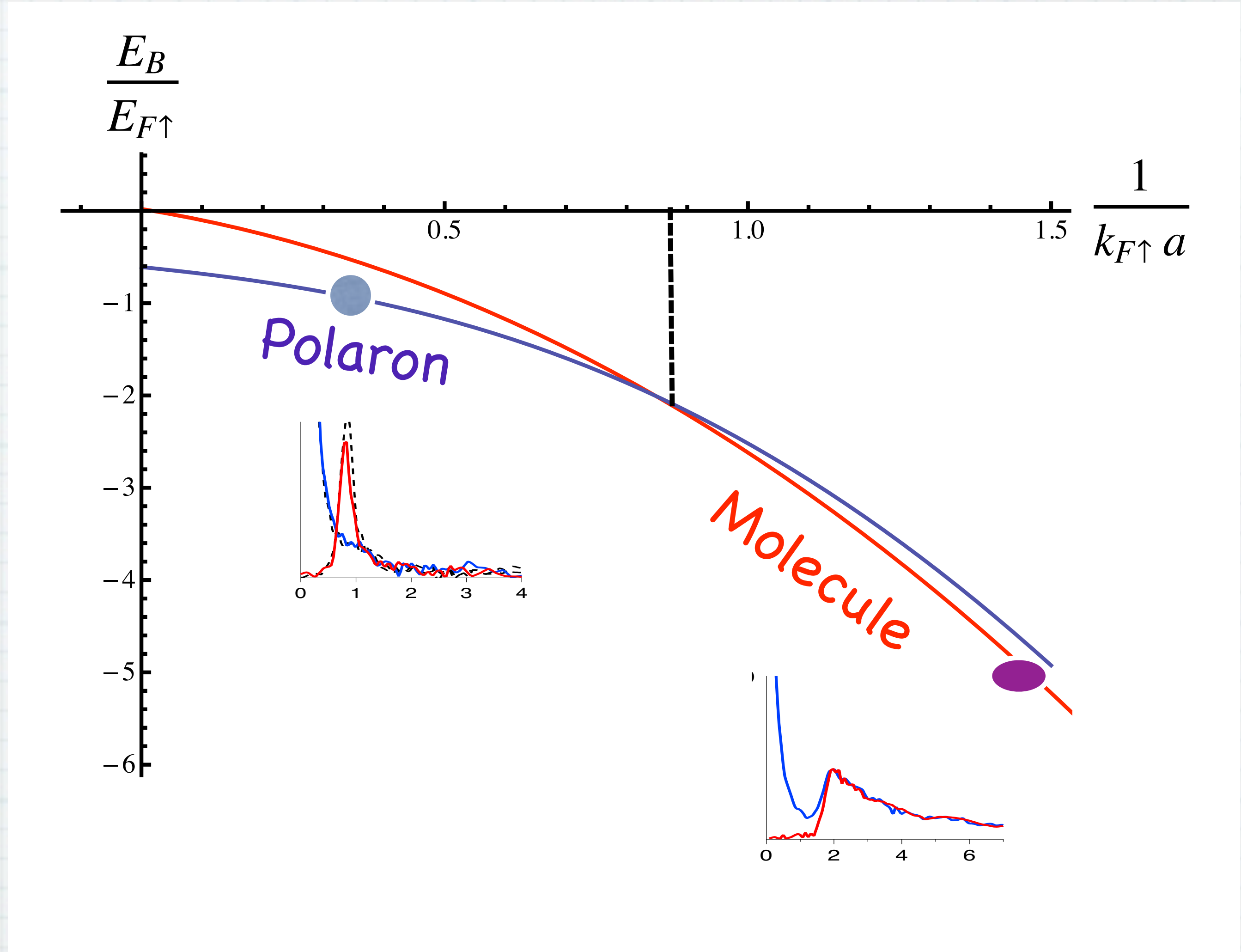
M→P+2p+h

(preliminary)

# Experimental observation

**Methods:**

- RF spectra
- Collective modes
  to measure m* vs. time
- Density profiles in the trap

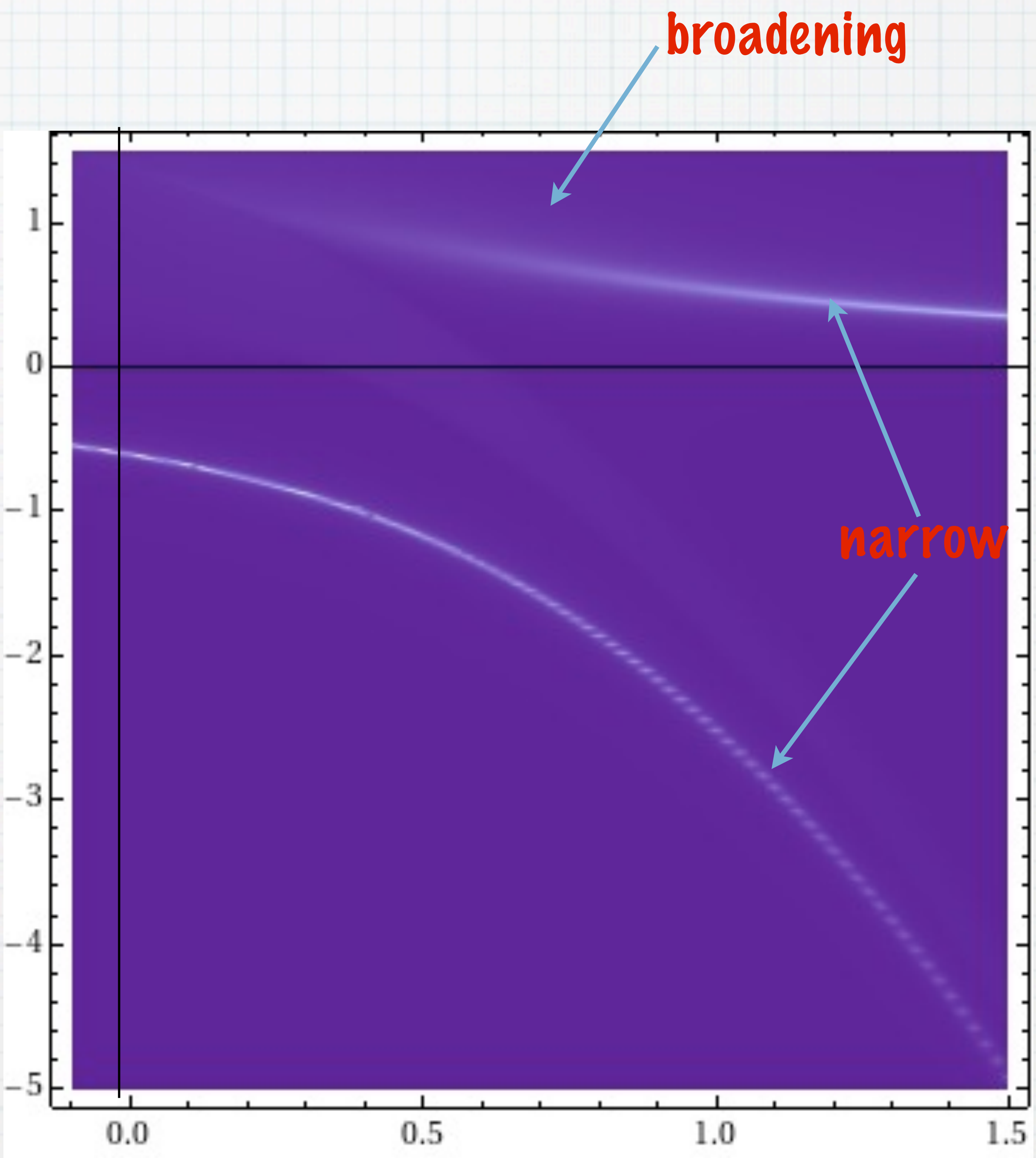



**Issues:**

- Phase separation?
  - stabilized by finite T
  - work with $m_\downarrow \neq m_\uparrow$
  - use bosonic impurities
- No decay to deeply bound molecular states

# ...is there more?

Yes..

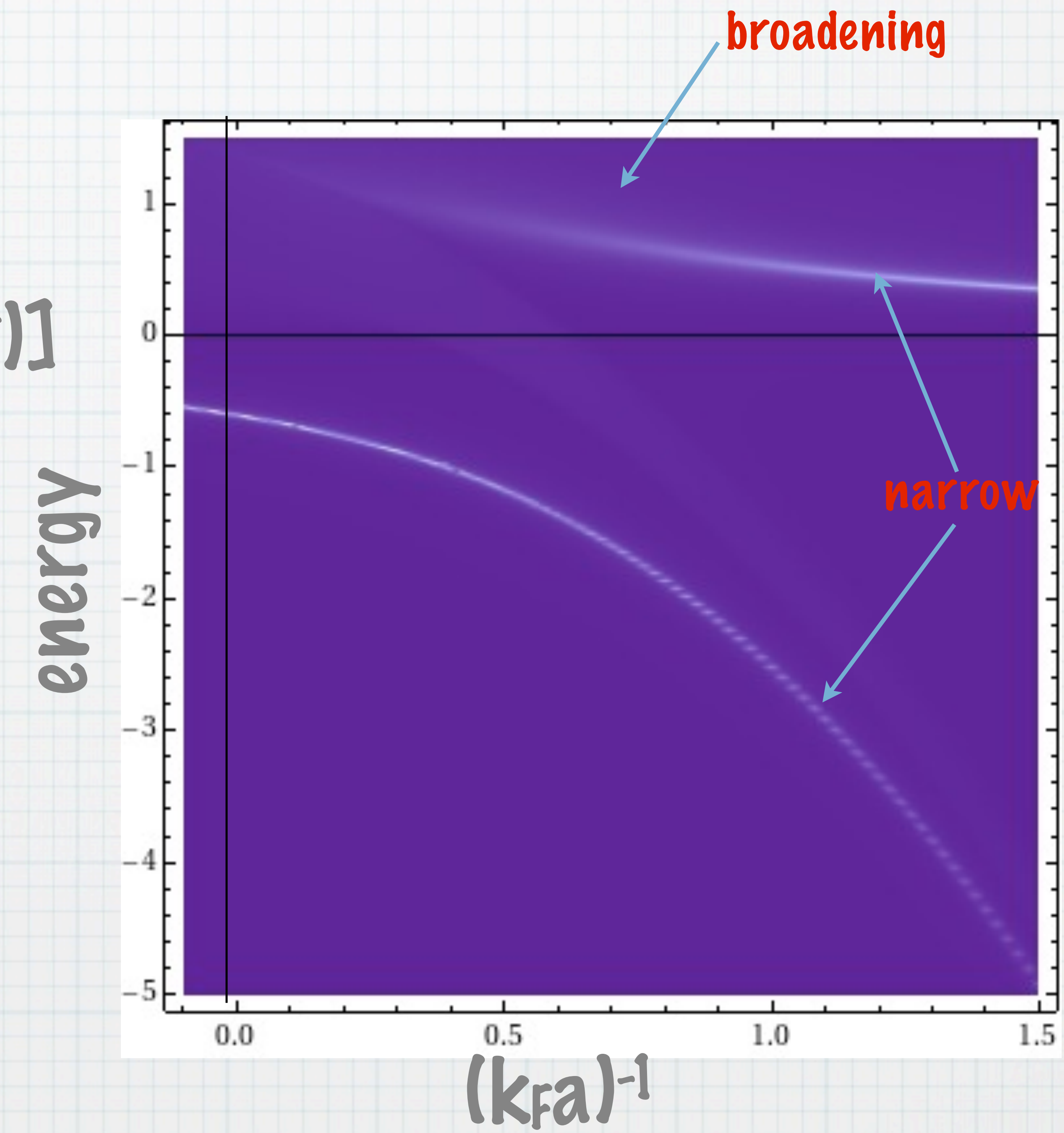


# ...is there more?

Yes..

spectral function

$A_{\downarrow}(\omega)=-\mathrm{Im}[G_{\downarrow}(k=0,\omega+i0^{+})]$



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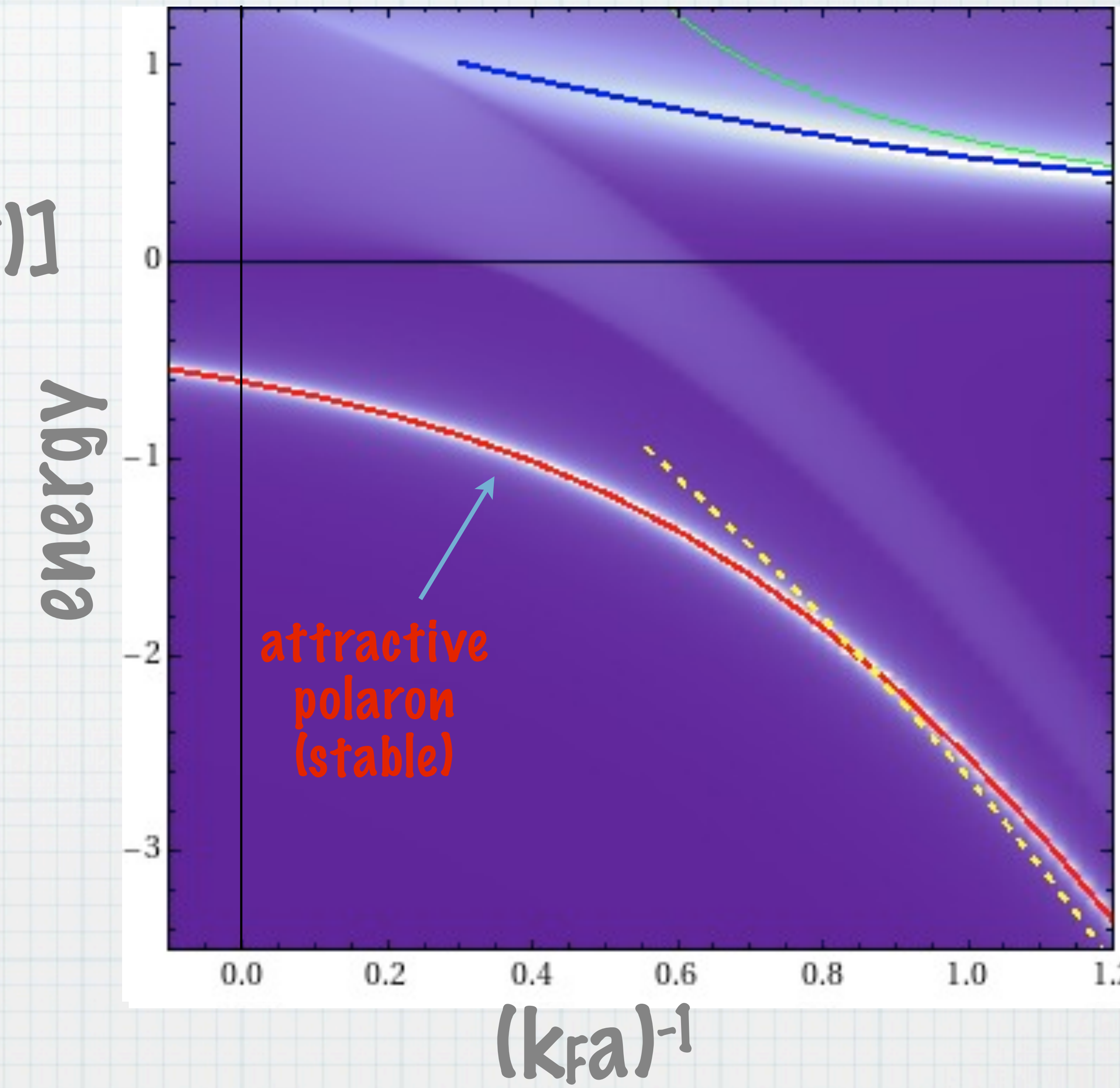

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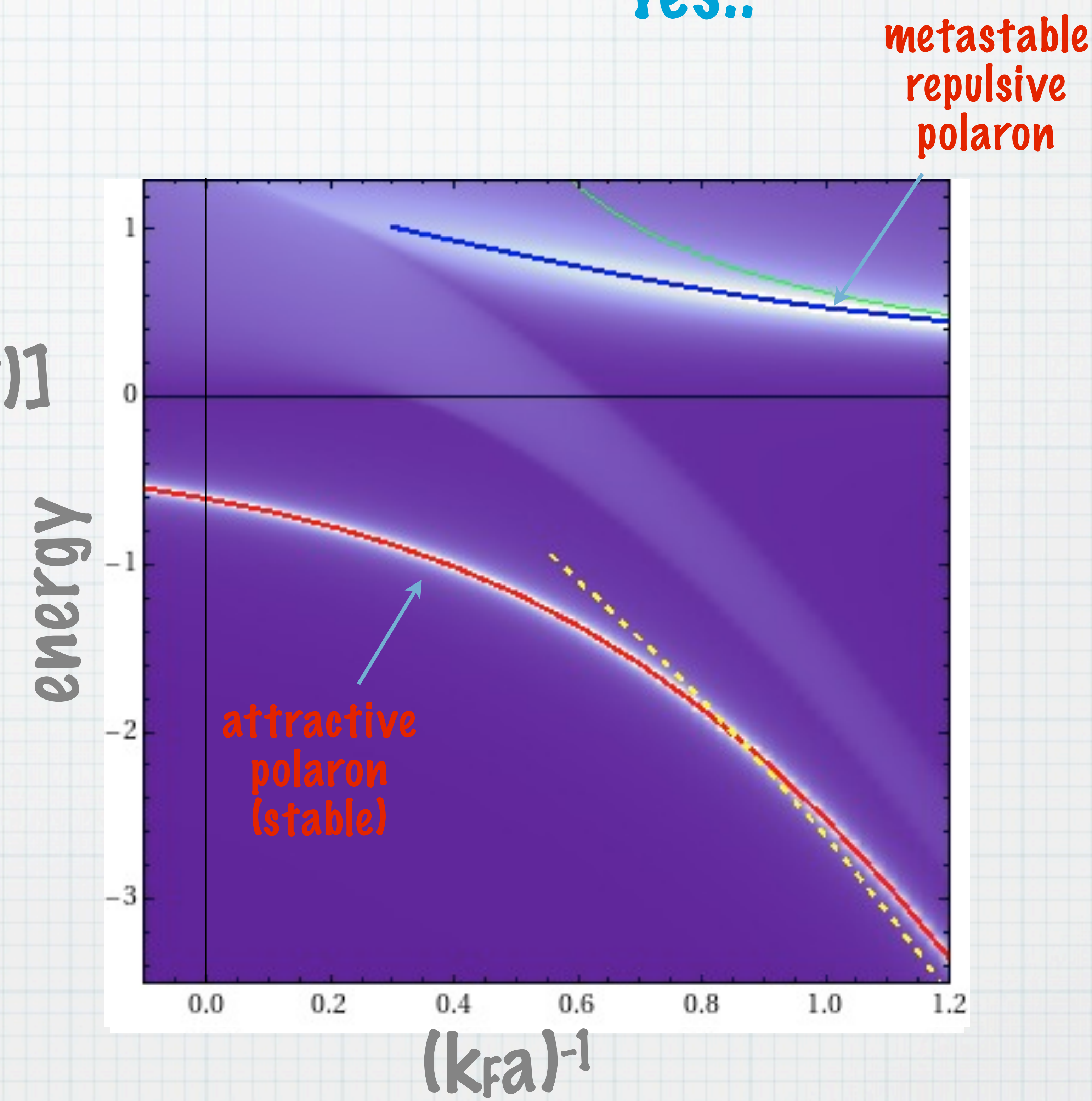

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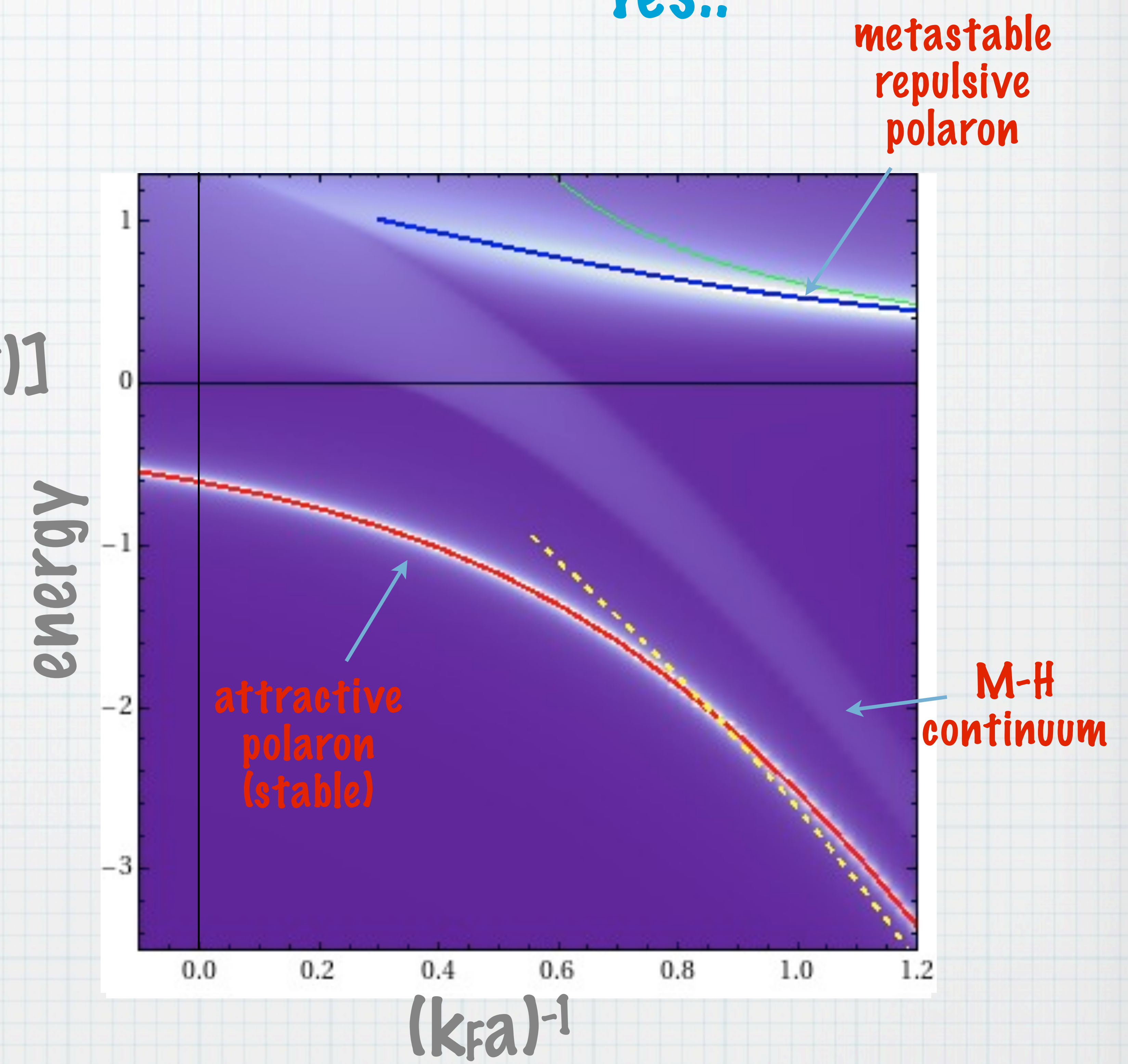

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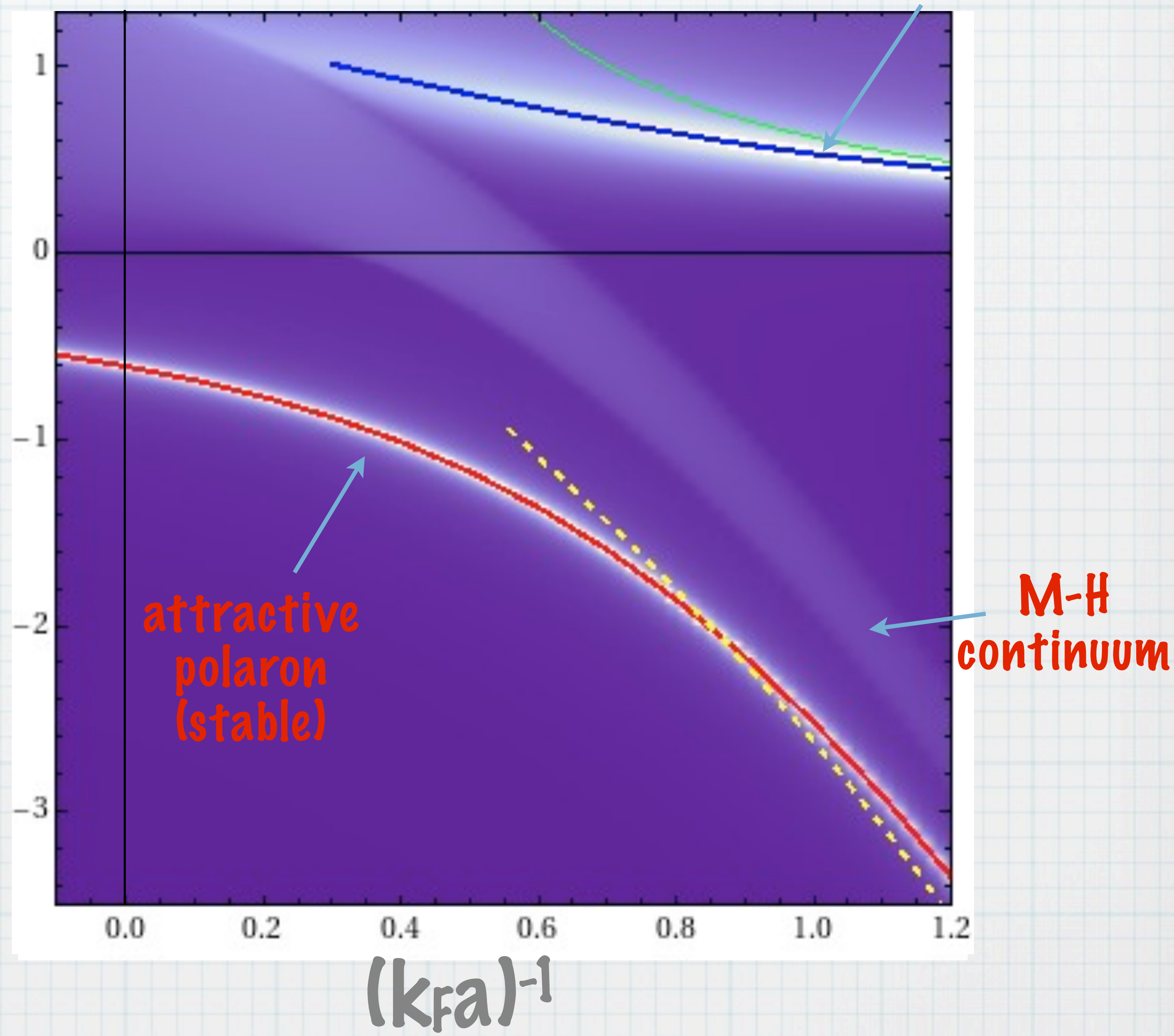
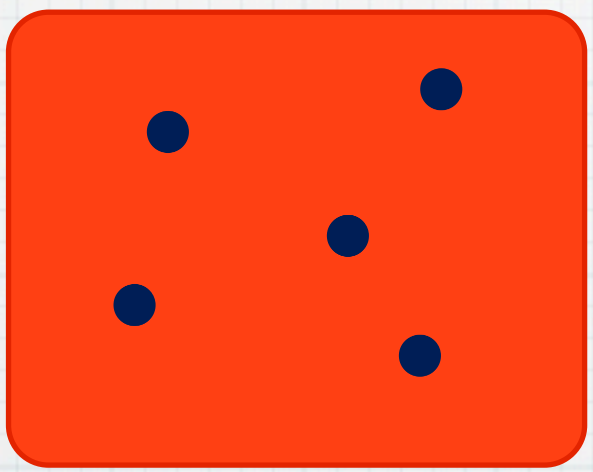



$$E_M = -\frac{\hbar^2}{2m_r a^2} - \epsilon_F + \frac{2\pi\hbar^2 a_3}{m_3} n_\uparrow$$

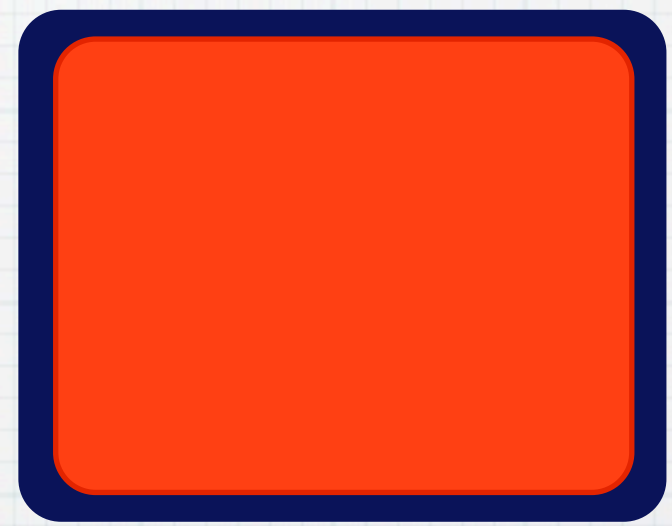
# IFM
# (Itinerant FerroMagnetism)

$\mu_{\downarrow} < E_F$



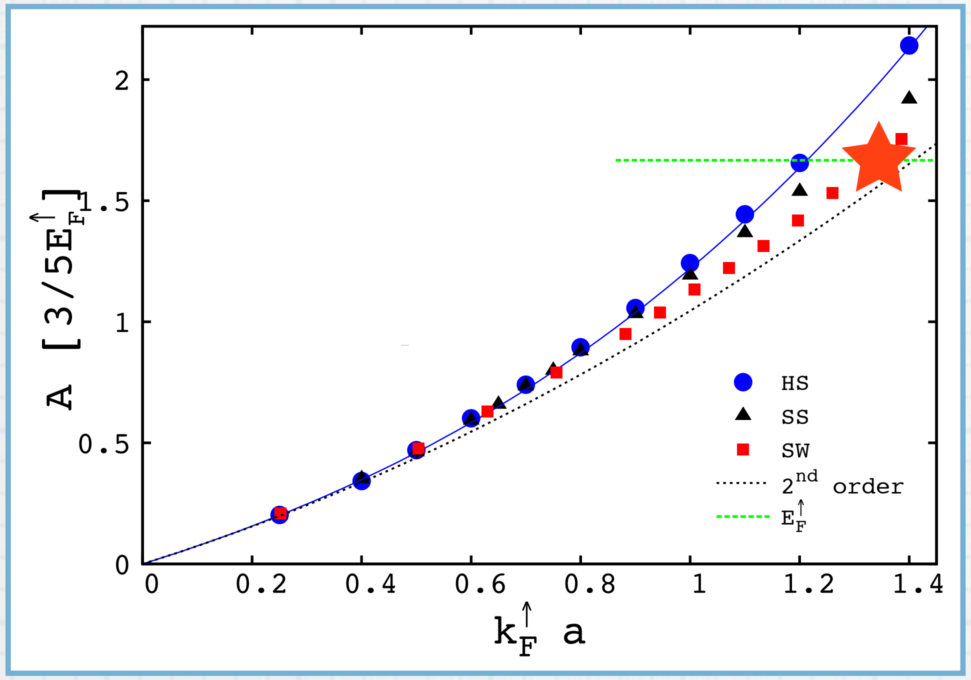
mixed state

$\mu_{\downarrow} > E_F$

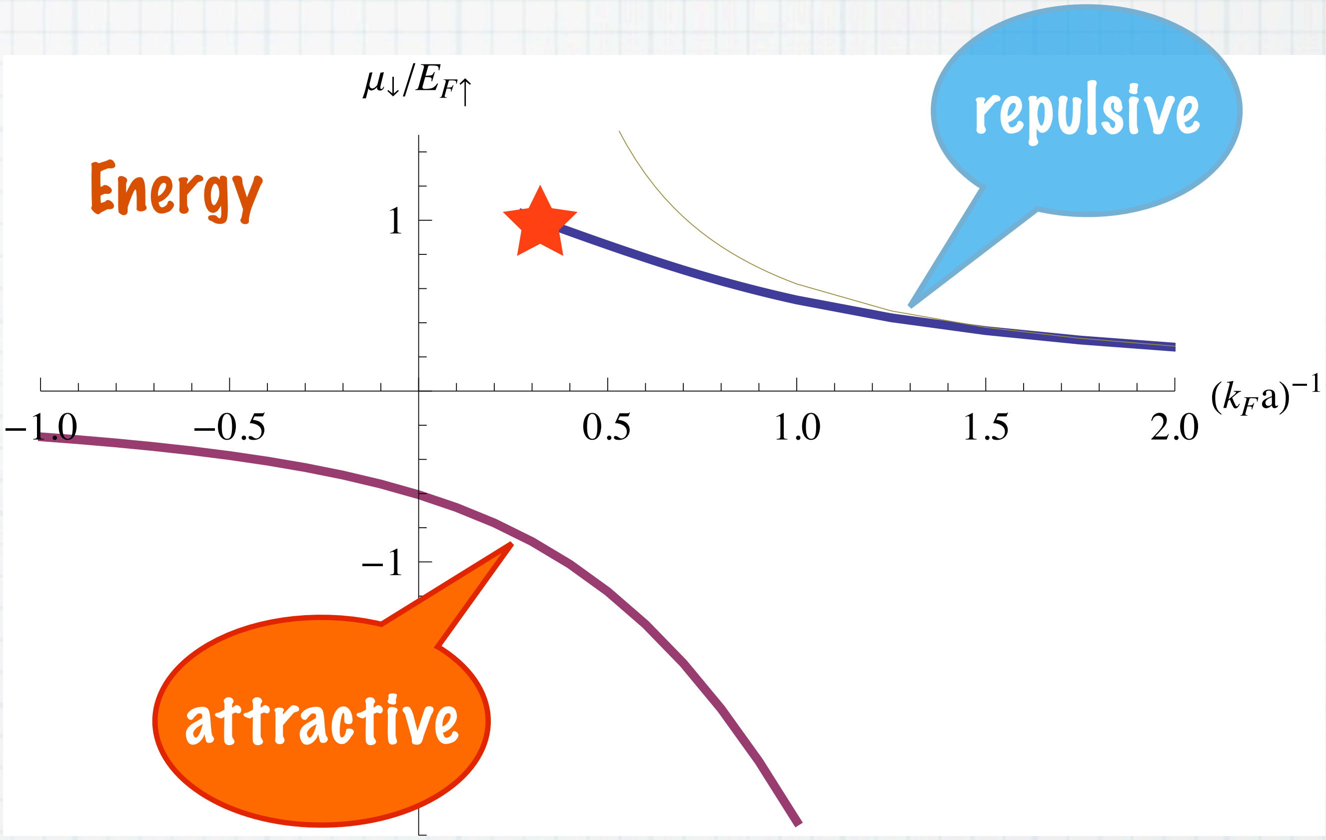


phase-separation

# Polaron energies



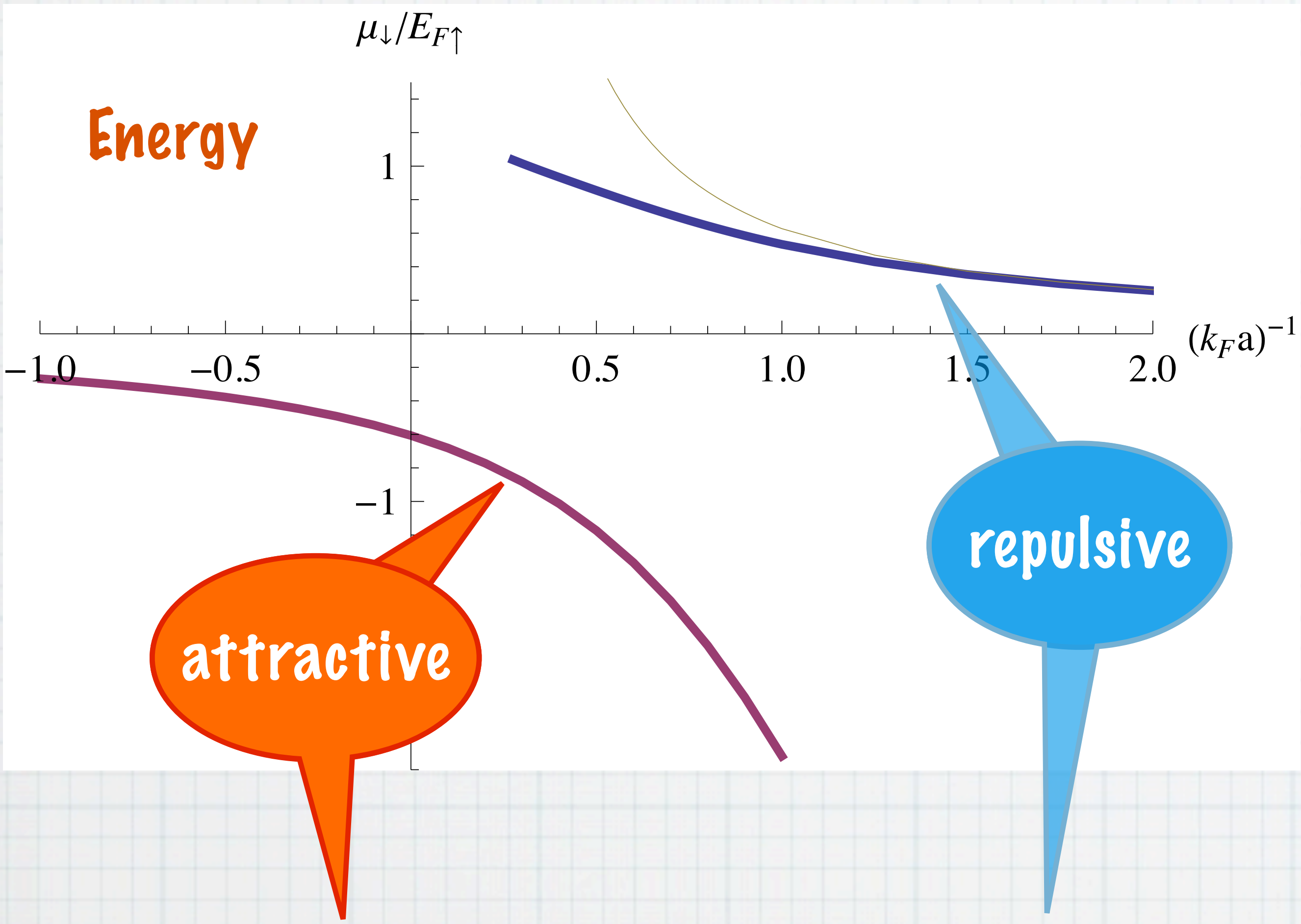

QMC by Pilati et al., PRL 2010



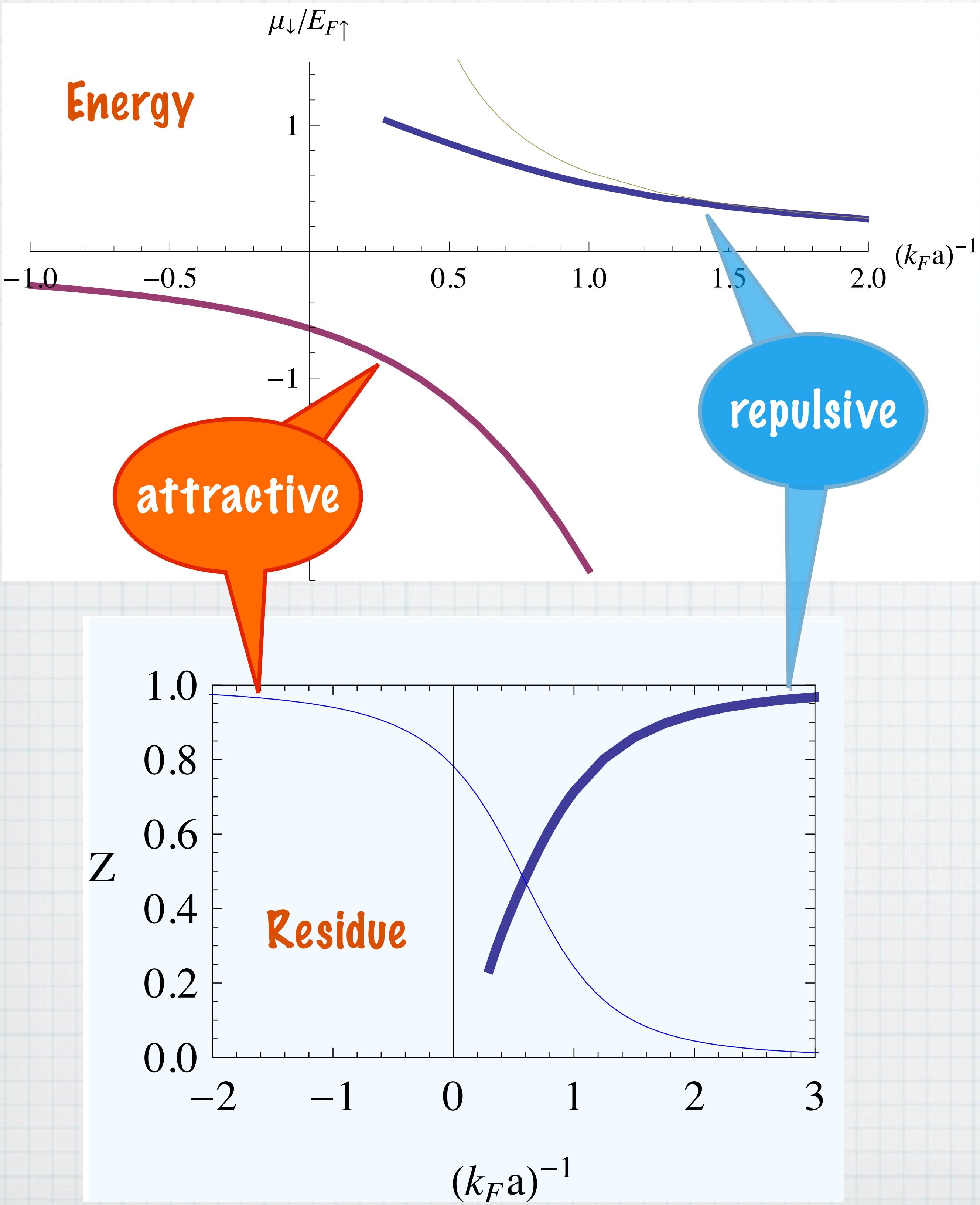

PM & G. Bruun
arXiv:1102.0121

weak coupling: $$\frac{\mu_\downarrow}{E_{F\uparrow}} = \frac{4}{3\pi}(k_F a) + \frac{2}{\pi^2}(k_F a)^2 + \ldots$$

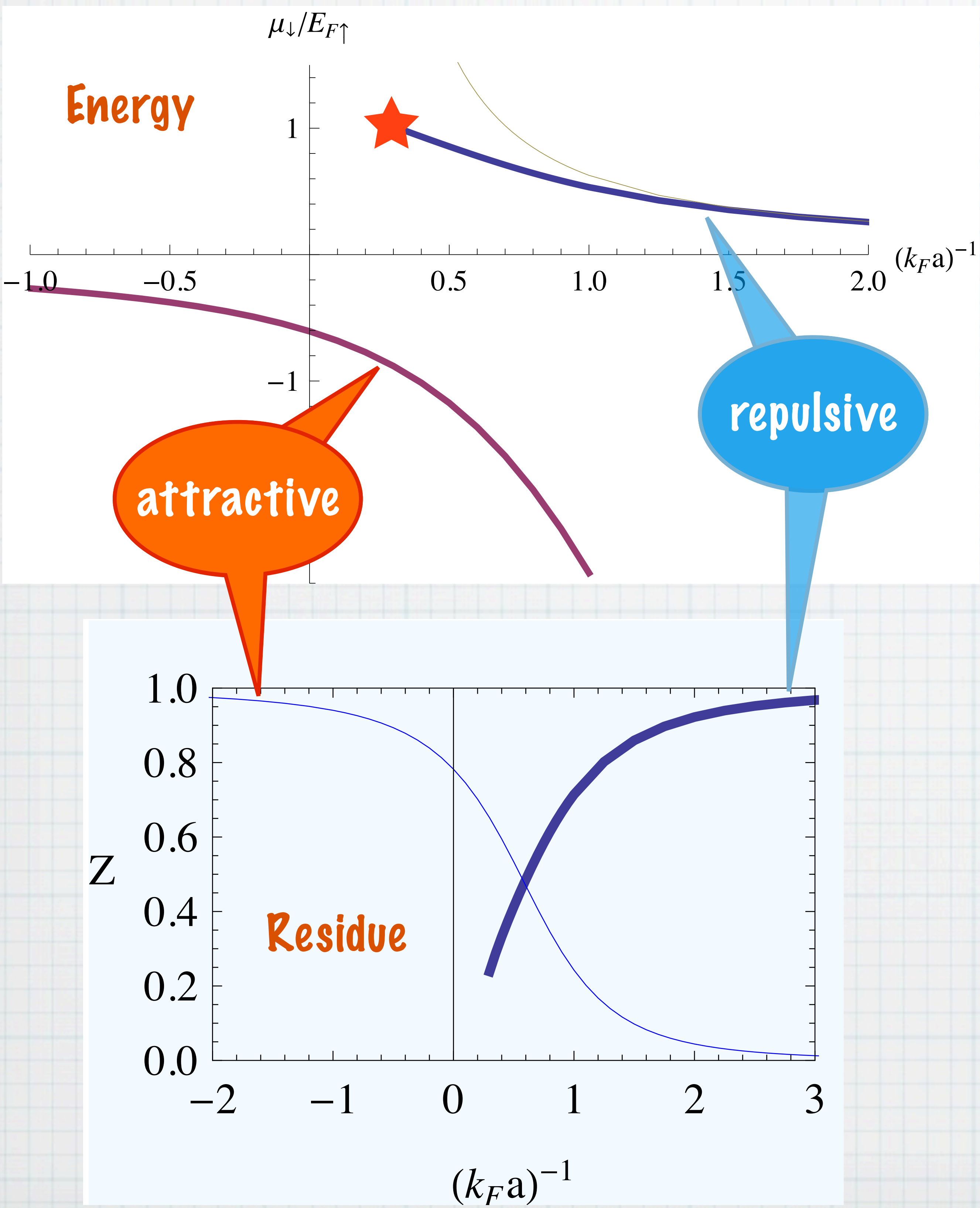
# Repulsive polaron ($m_\uparrow = m_\downarrow$)

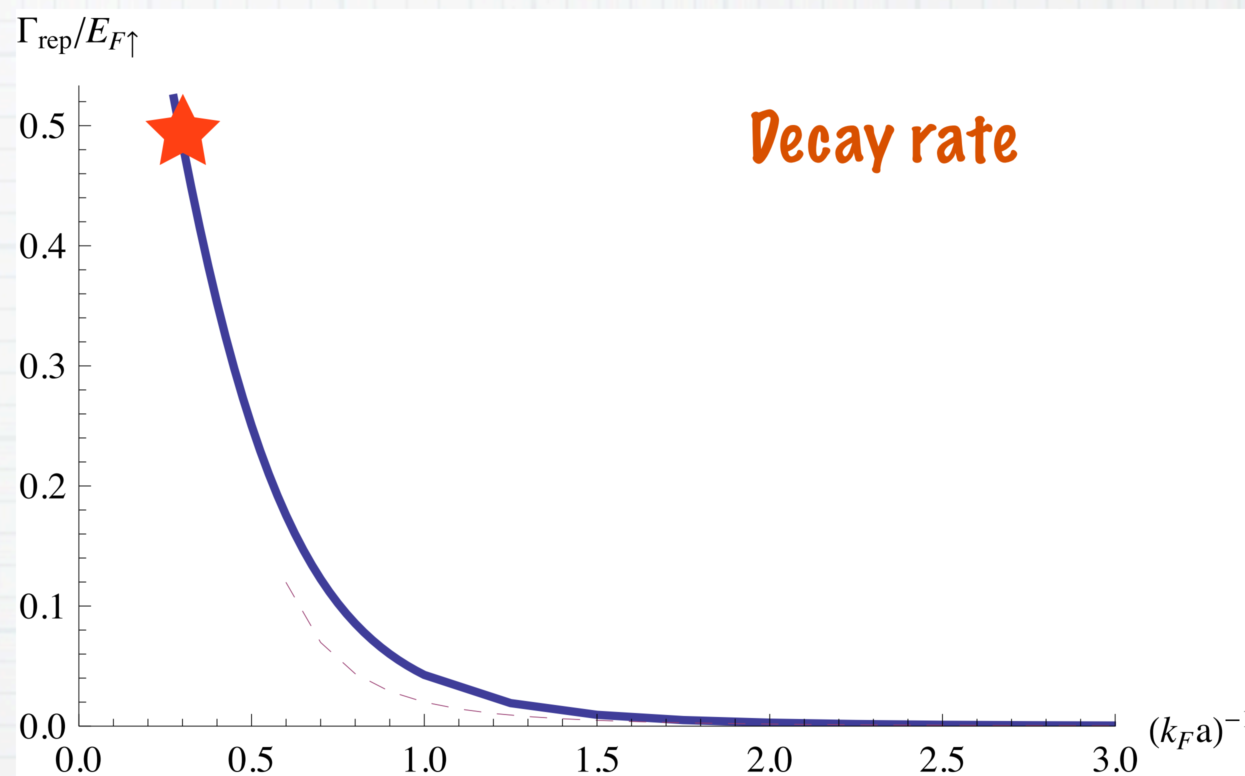


# Repulsive polaron ($m_\uparrow = m_\downarrow$)

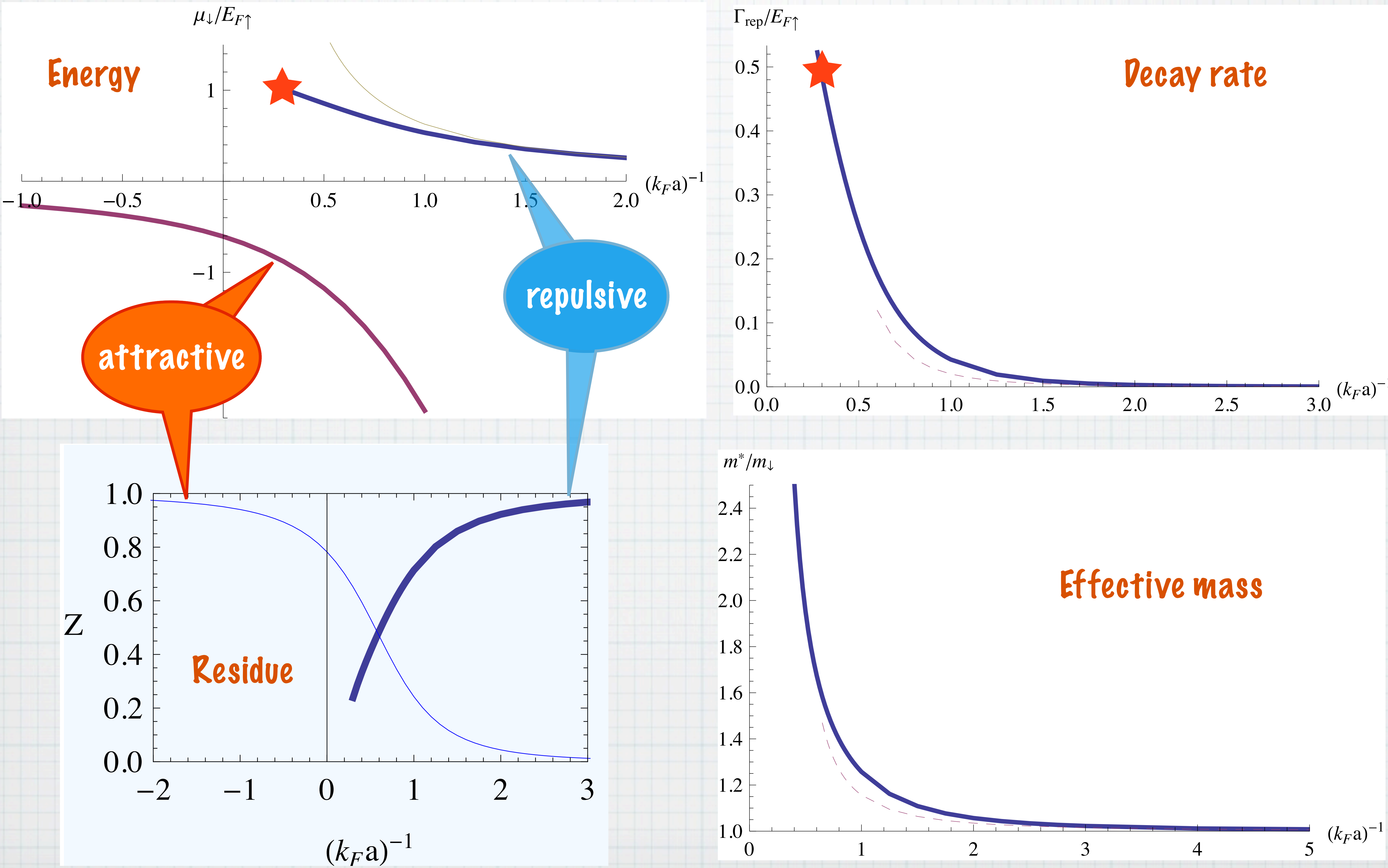


# Repulsive polaron ($m_\uparrow = m_\downarrow$)



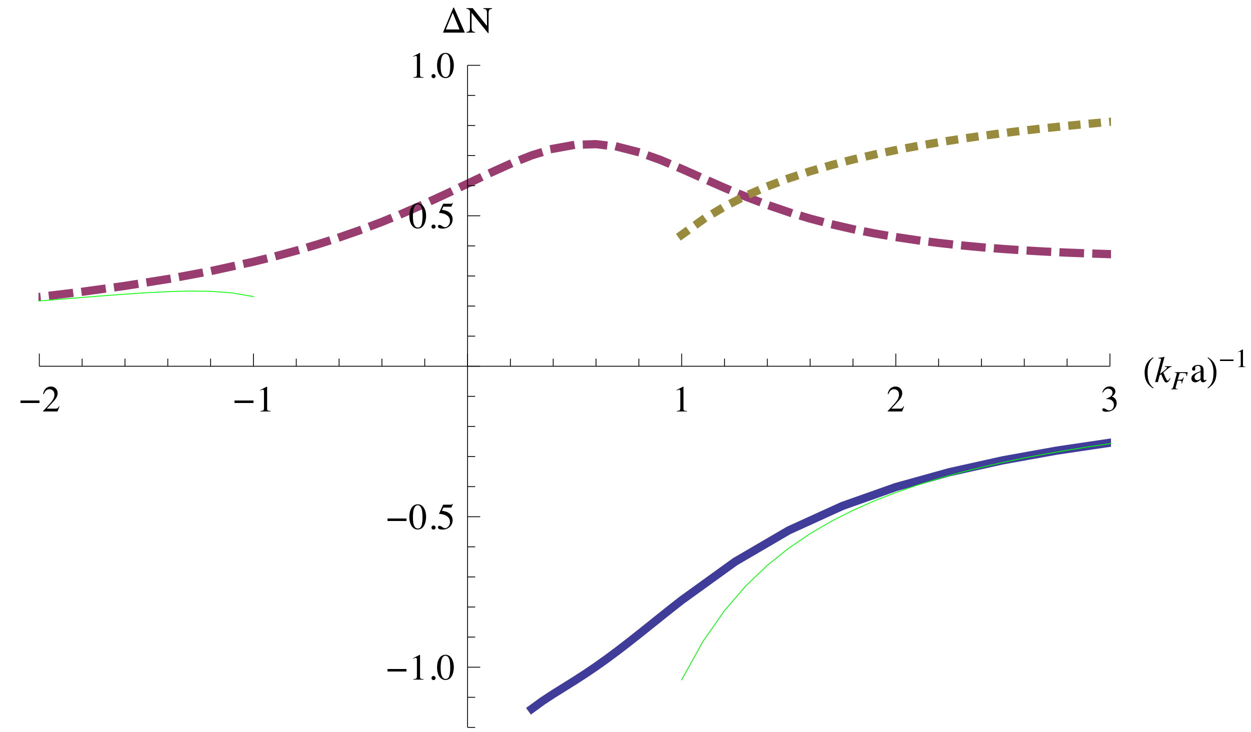
# Repulsive polaron ($m_\uparrow = m_\downarrow$)



# # of particles in the dressing cloud

$$\delta\mu_{\uparrow} = \frac{\partial^2\varepsilon}{\partial n_{\uparrow}\partial n_{\downarrow}} + \frac{\partial^2\varepsilon}{(\partial n_{\uparrow})^2}\Delta N = 0$$

$$\Delta N = -\left(\frac{\partial\mu_{\downarrow}}{\partial n_{\uparrow}}\right)_{n_{\downarrow}} / \left(\frac{\partial\mu_{\uparrow}}{\partial n_{\uparrow}}\right)_{n_{\downarrow}} \approx -\left(\frac{\partial\mu_{\downarrow}}{\partial\epsilon_F}\right)_{n_{\downarrow}}$$

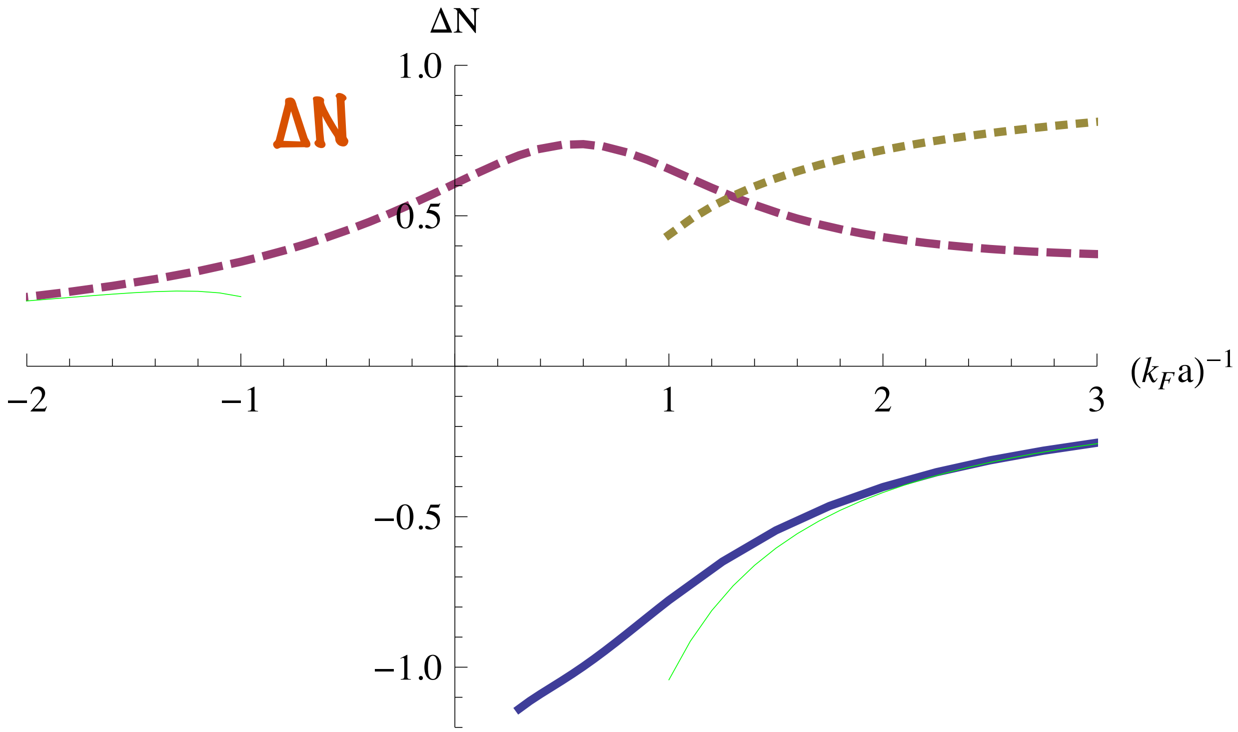



weak coupling: $\Delta N = -\frac{2}{\pi}k_F a - \frac{4}{\pi^2}(k_F a)^2 + \ldots$

# # of particles in the dressing cloud

$$\delta\mu_\uparrow = \frac{\partial^2\varepsilon}{\partial n_\uparrow \partial n_\downarrow} + \frac{\partial^2\varepsilon}{(\partial n_\uparrow)^2}\Delta N = 0$$

$$\Delta N = -\left(\frac{\partial\mu_\downarrow}{\partial n_\uparrow}\right)_{n_\downarrow} \Big/ \left(\frac{\partial\mu_\uparrow}{\partial n_\uparrow}\right)_{n_\downarrow} \approx -\left(\frac{\partial\mu_\downarrow}{\partial\epsilon_F}\right)_{n_\downarrow}$$




weak coupling: $\Delta N = -\frac{2}{\pi}k_F a - \frac{4}{\pi^2}(k_F a)^2 + \ldots$

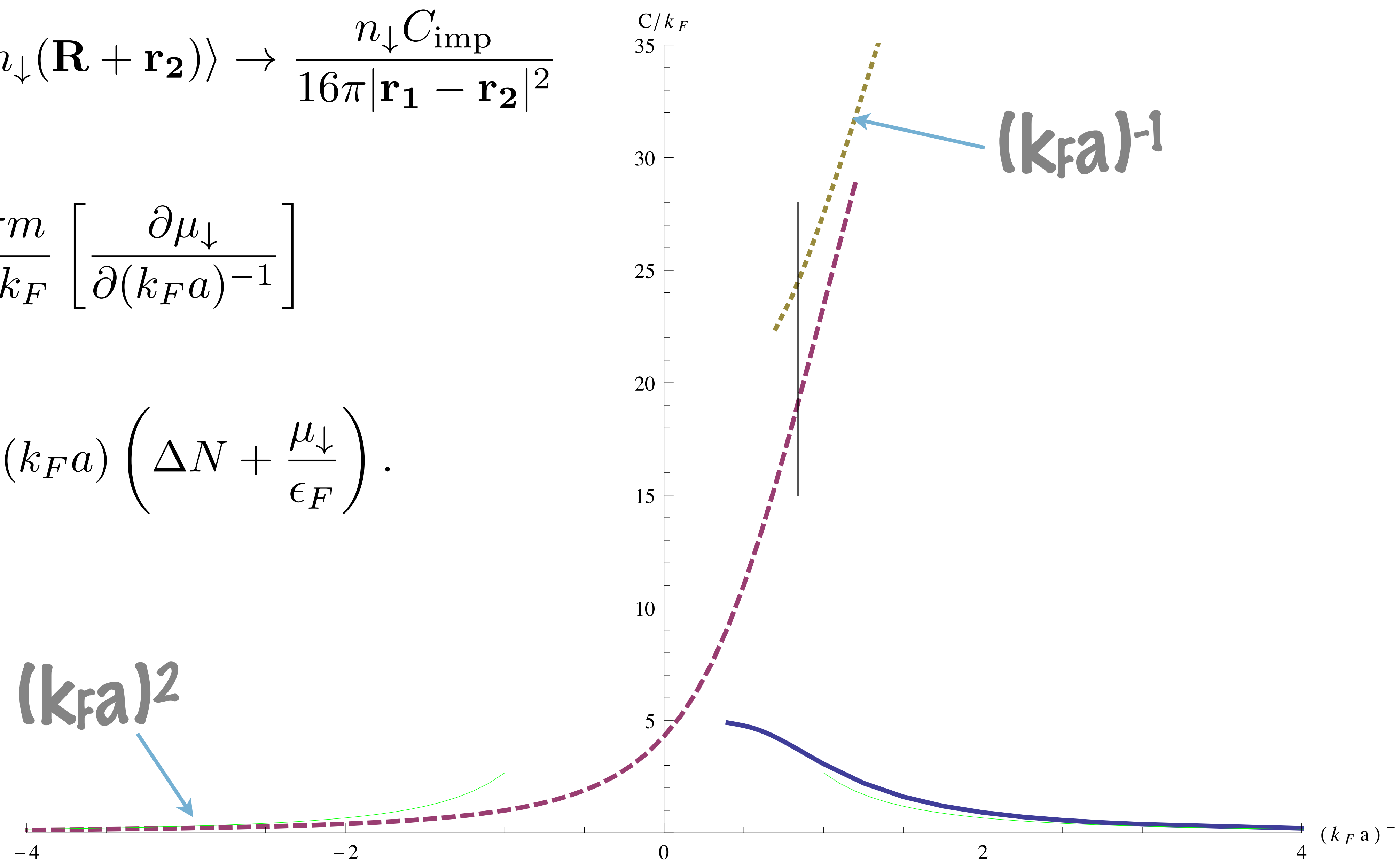
# Tan's contact

## $C_{imp}$: contact density per impurity

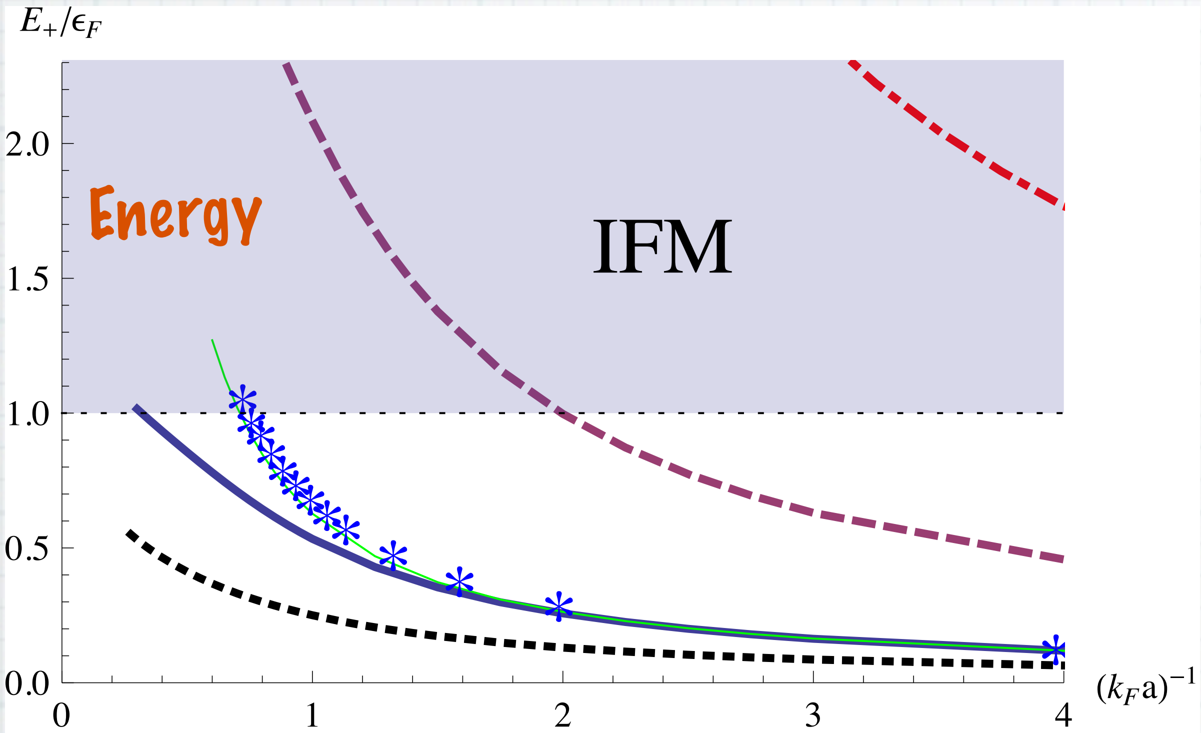
$$\langle n_\uparrow(\mathbf{R}+\mathbf{r_1})n_\downarrow(\mathbf{R}+\mathbf{r_2})\rangle \to \frac{n_\downarrow C_{\rm imp}}{16\pi|\mathbf{r_1}-\mathbf{r_2}|^2}$$

$$C_{\rm imp} = -\frac{4\pi m}{\hbar^2 k_F}\left[\frac{\partial \mu_\downarrow}{\partial (k_F a)^{-1}}\right]$$

$$\frac{C_{\rm imp}}{k_F} = -4\pi(k_F a)\left(\Delta N + \frac{\mu_\downarrow}{\epsilon_F}\right).$$

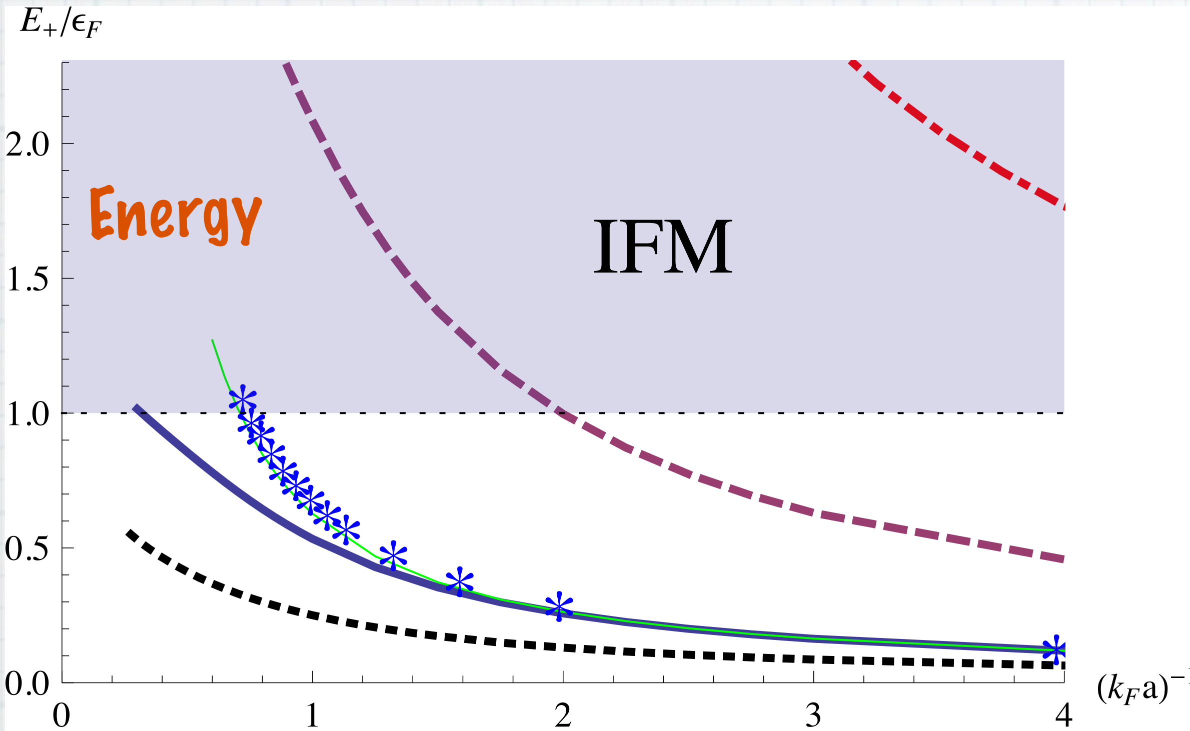


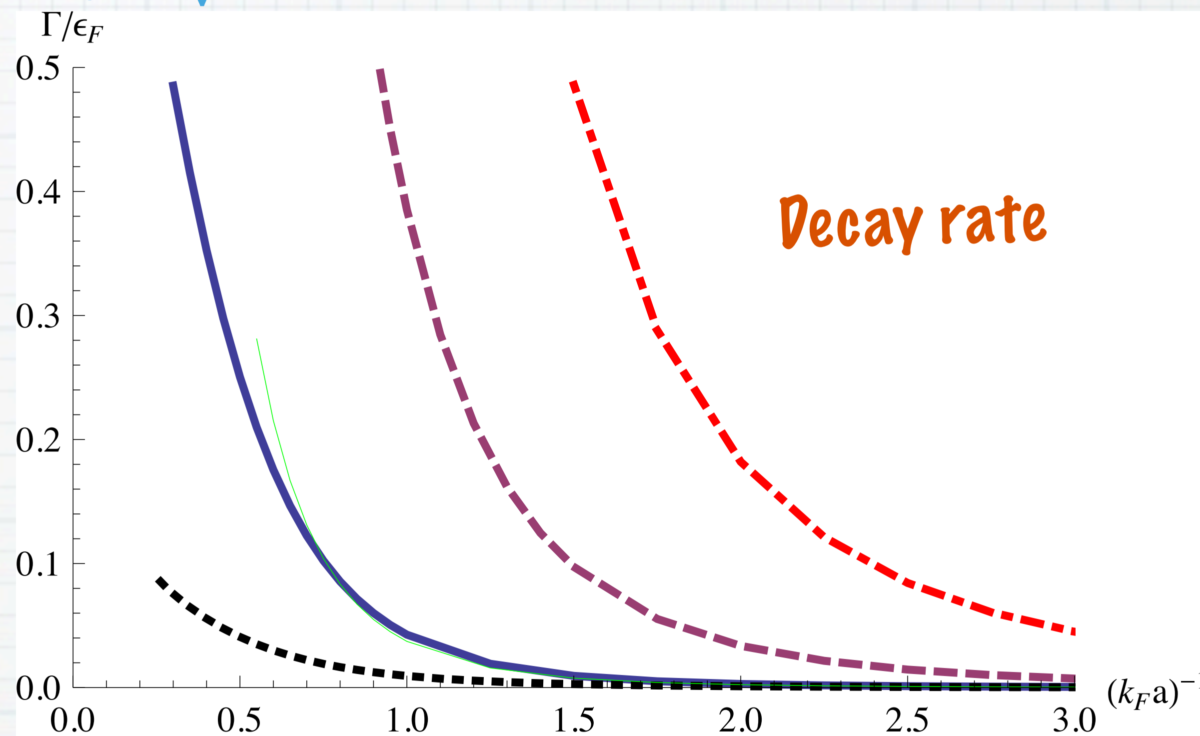
# $m_\uparrow \neq m_\downarrow$




| | $m_\downarrow/m_\uparrow$ |
|---|---|
| red dashed line | 6/173 |
| purple dashed line | 6/40 |
| solid dark blue line | 1 |
| black dashed line | 40/6 |

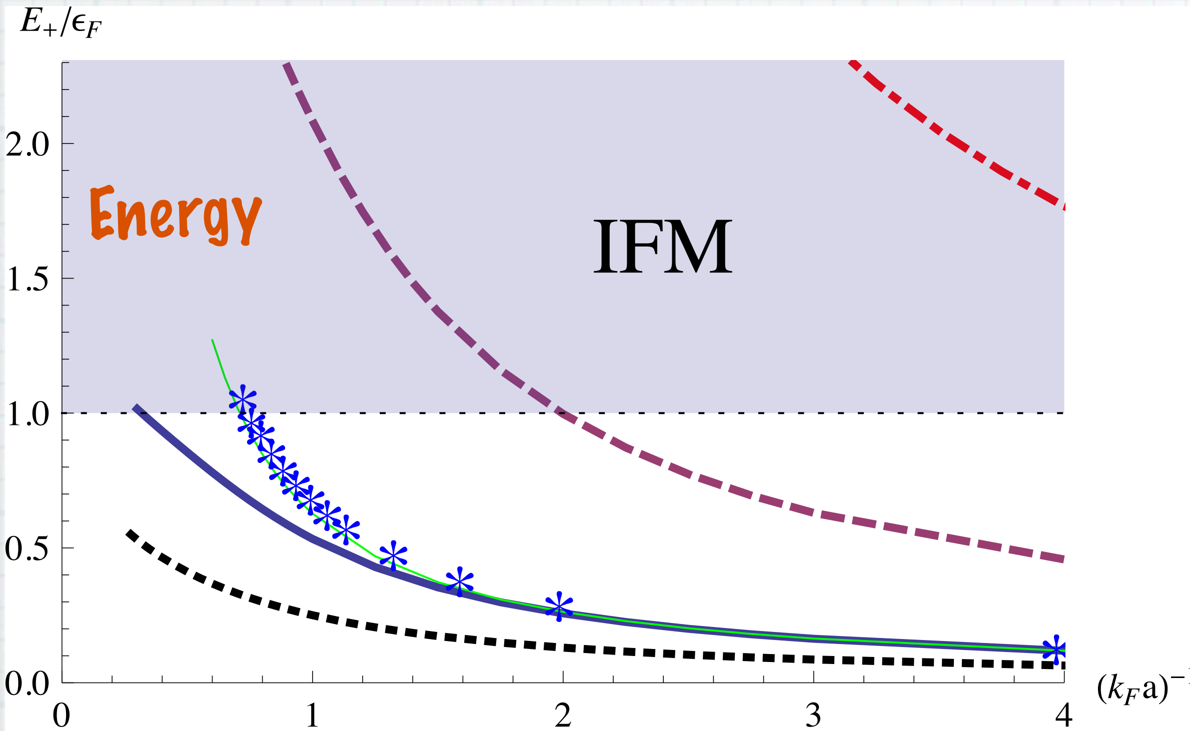
# $m_\uparrow \neq m_\downarrow$

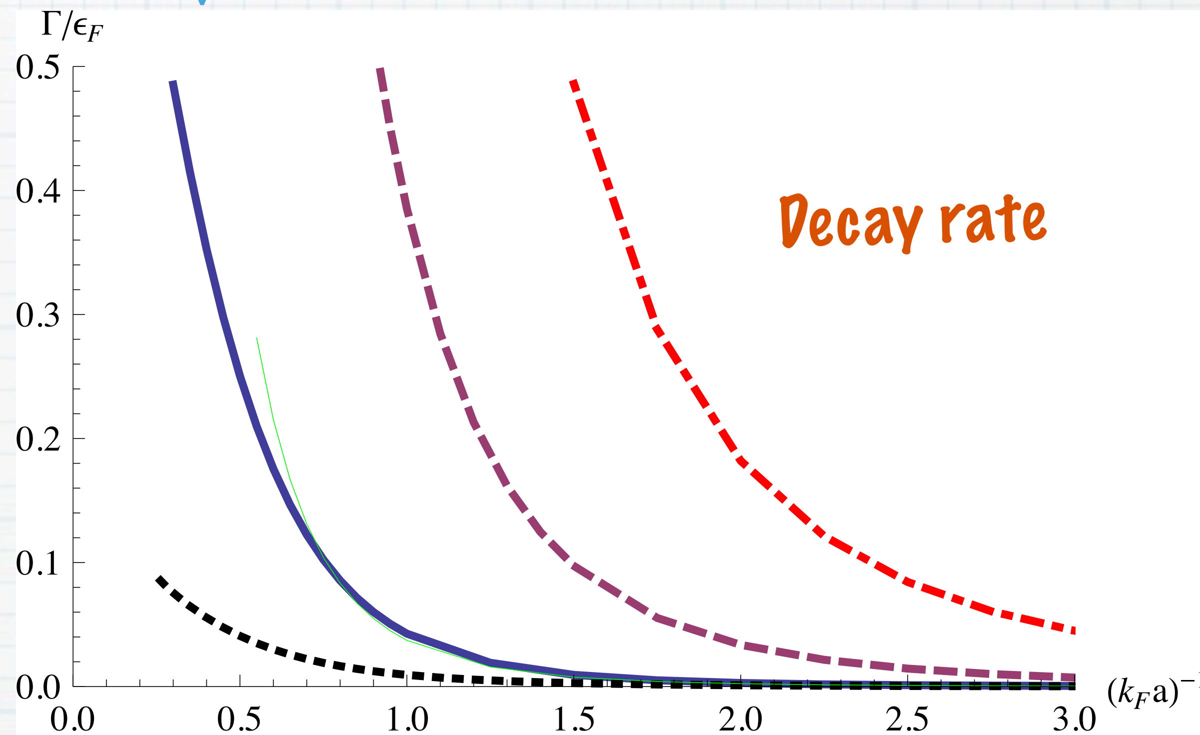


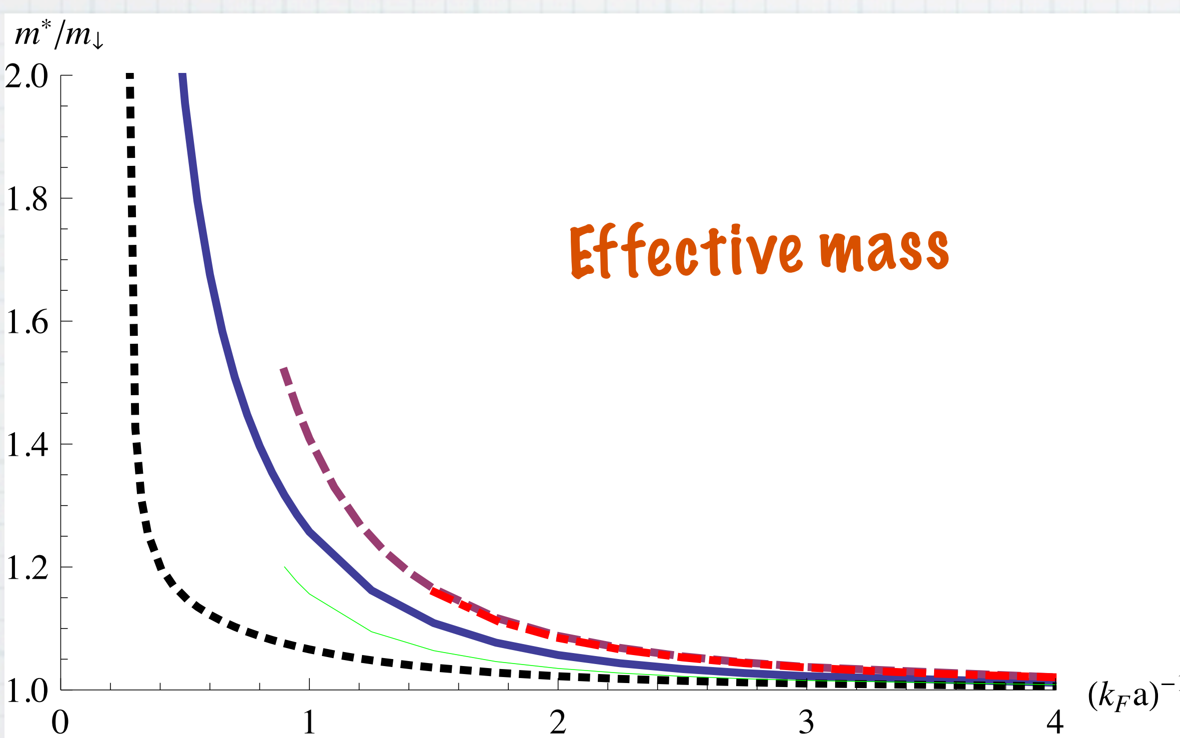




| | $m_\downarrow/m_\uparrow$ |
|---|---|
| | 6/173 |
| | 6/40 |
| | 1 |
| | 40/6 |

# $m_\uparrow \neq m_\downarrow$




| | $m_\downarrow/m_\uparrow$ |
|---|---|
| | 6/173 |
| | 6/40 |
| | 1 |
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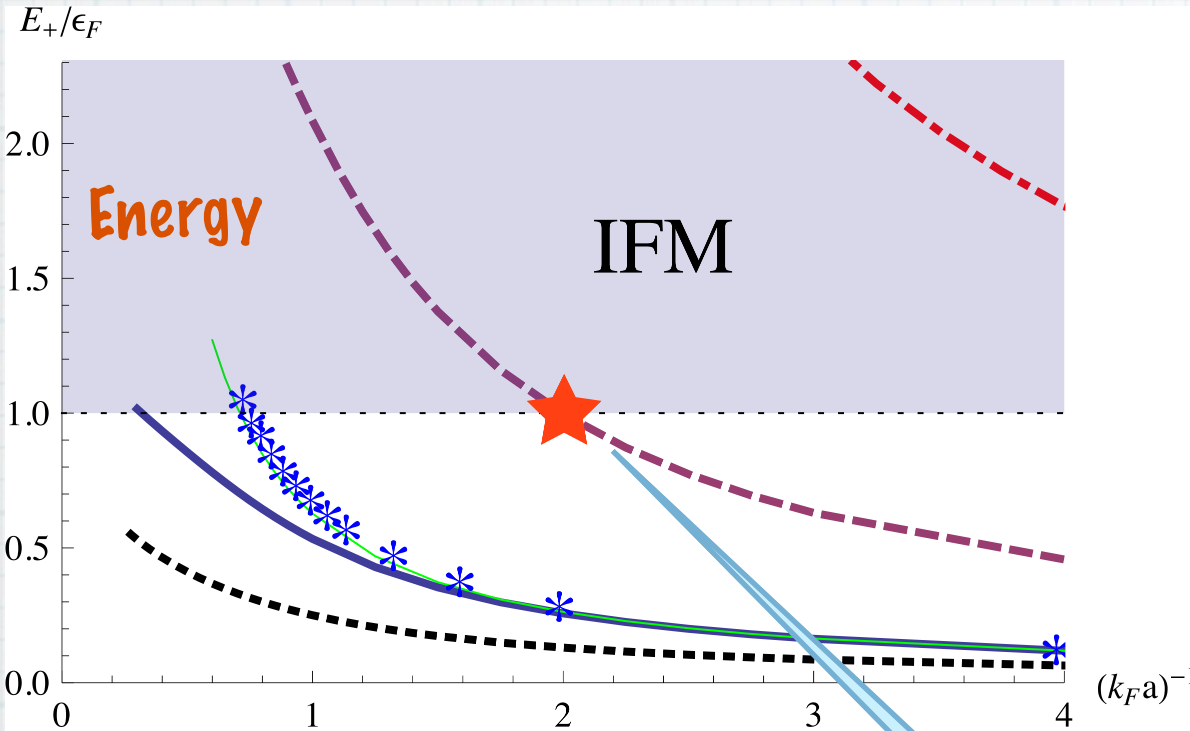
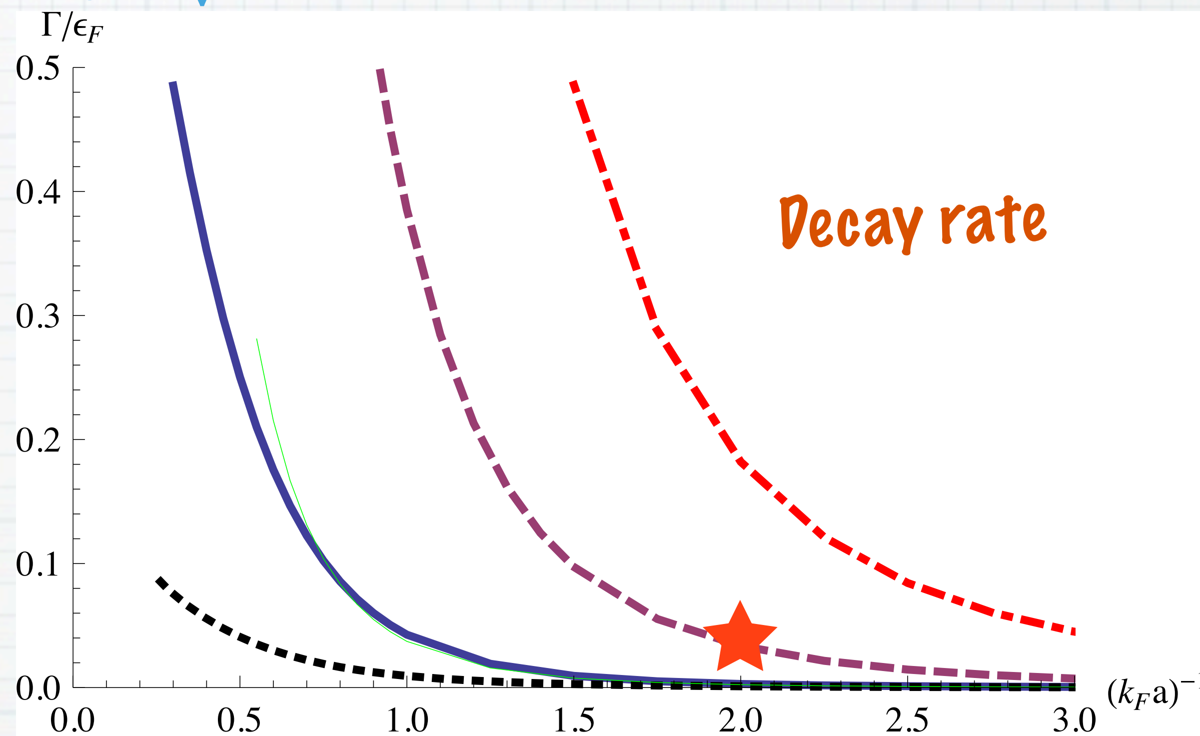
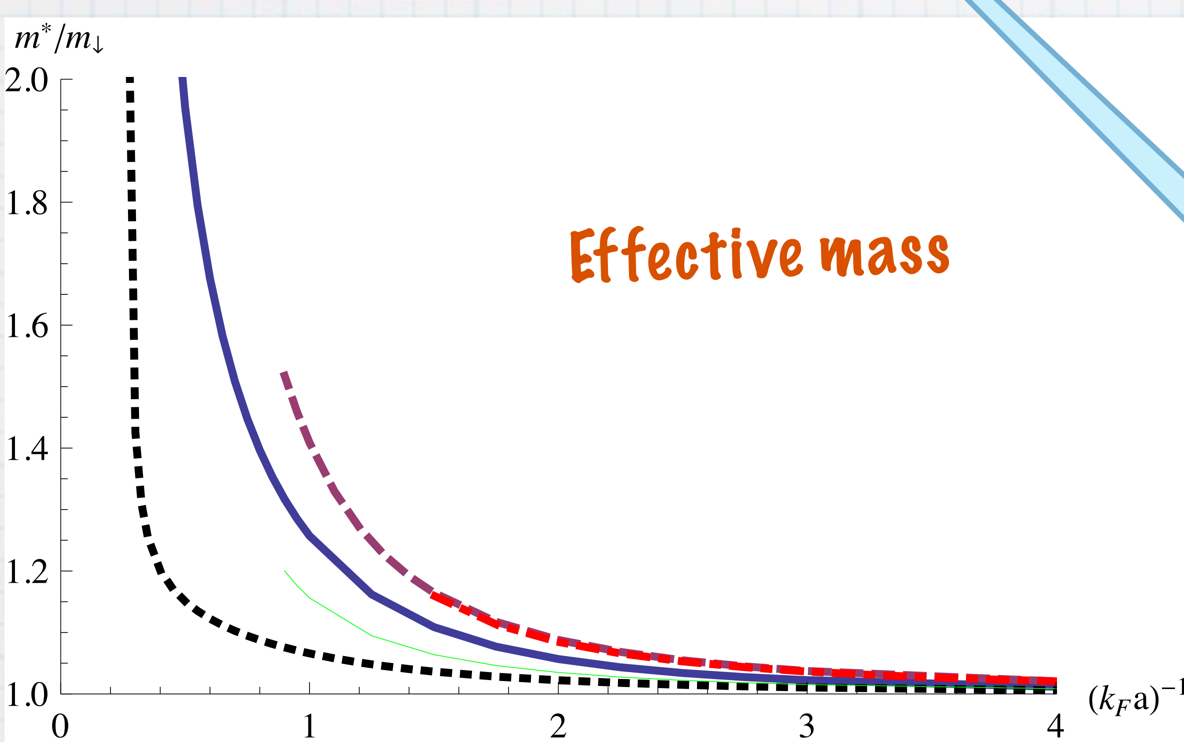
# $m_\uparrow \neq m_\downarrow$




| | $m_\downarrow/m_\uparrow$ |
|---|---|
| | 6/173 |
| | 6/40 |
| | 1 |
| | 40/6 |

Li impurities in a K gas:
IFM @ $k_F a$=0.5,
with $\Gamma/E_F$=0.04

# RF spectra

$\uparrow$ : 1

$\downarrow$ : 2,3

$\omega_0 = \varepsilon_3 - \varepsilon_2 > 0$

$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] = -\frac{1}{2}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\int \frac{\mathrm{d}\epsilon}{2\pi}\left[f(\epsilon)-f(\epsilon+\omega)\right]A_2(\mathbf{k},\epsilon)A_3(\mathbf{k},\epsilon+\omega),$$

# RF spectra

$\uparrow$ : 1

$\downarrow$ : 2,3

$\omega_0 = \varepsilon_3 - \varepsilon_2 > 0$

$n_3 \approx 0$

$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] = -\frac{1}{2}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\int \frac{\mathrm{d}\epsilon}{2\pi}\left[f(\epsilon) - f(\cancel{\epsilon +} \omega)\right] A_2(\mathbf{k},\epsilon) A_3(\mathbf{k},\epsilon+\omega),$$

# MIT: Int→nonInt

$$A_3(\mathbf{k},\omega) = 2\pi\delta(\omega - \xi_{k,3})$$

$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] = -\frac{1}{2}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3} f(\tilde{\omega} - \xi_{k,2}) A_\downarrow(\mathbf{k}, \tilde{\omega} - \xi_{k,2})$$

$$\tilde{\omega} = \omega - \mu_2 + \mu_3$$

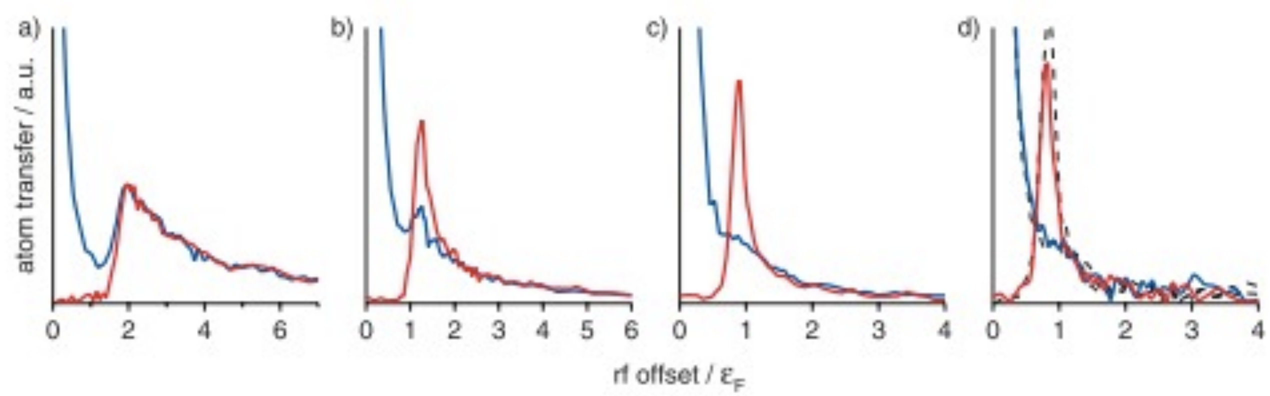


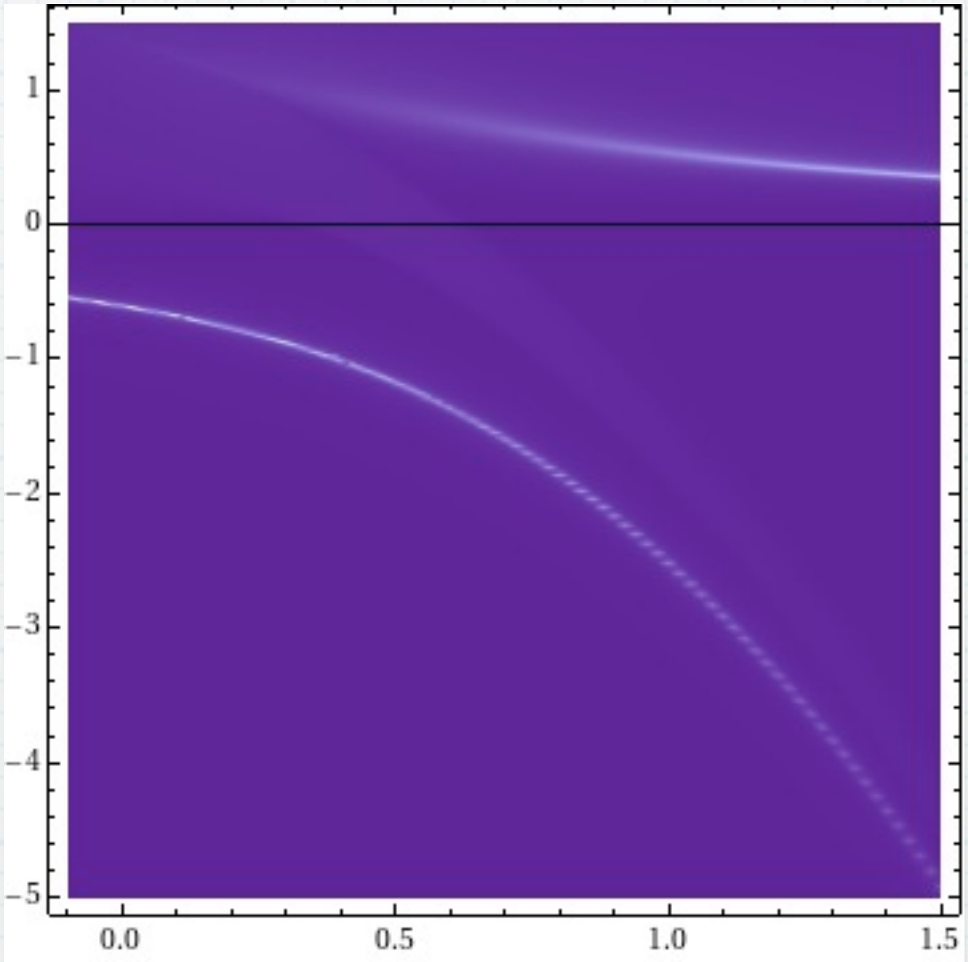

FIG. 2 (color online). rf spectroscopy on polarons. Shown are spatially resolved, 3D reconstructed rf spectra of the environment (blue, state $|1\rangle$) and impurity (red, state $|3\rangle$) component in a highly imbalanced spin-mixture. (a) Molecular limit; (b),(c) Emergence of the polaron, a distinct peak exclusively in the minority component. (d) At unitarity, the polaron peak is the dominant feature in the impurity spectrum, which becomes even more pronounced for $1/k_F a < 0$ (not shown). For the spectra shown as dashed lines in (d) the roles of states $|1\rangle$ and $|3\rangle$ are exchanged. The local impurity concentration was $x = 5(2)\%$ for all spectra, the interaction strengths $1/k_F a$ were (a) 0.76(2), (b) 0.43(1), (c) 0.20(1), and (d) 0 (unitarity).

# Innsbruck: nonInt→Int

$$A_2(\mathbf{k},\omega) = 2\pi\delta(\omega - \xi_{k,2})$$

$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] = -\frac{1}{2}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3} f(\xi_{k,2}) A_\downarrow(\mathbf{k},\tilde{\omega}+\xi_{k,2}).$$

$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] \propto A_\downarrow(\mathbf{k},\tilde{\omega}+\xi_{k,2}).$$



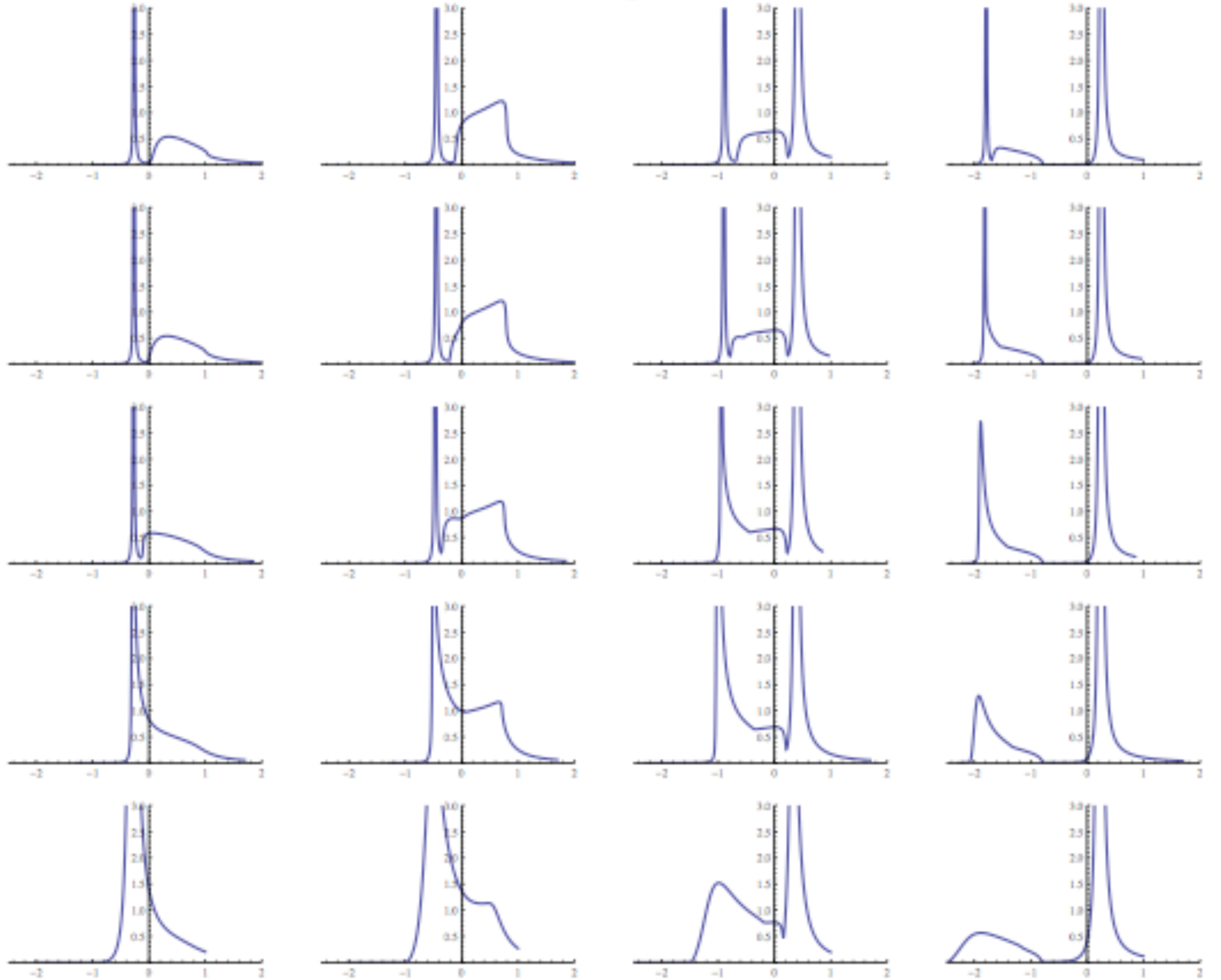
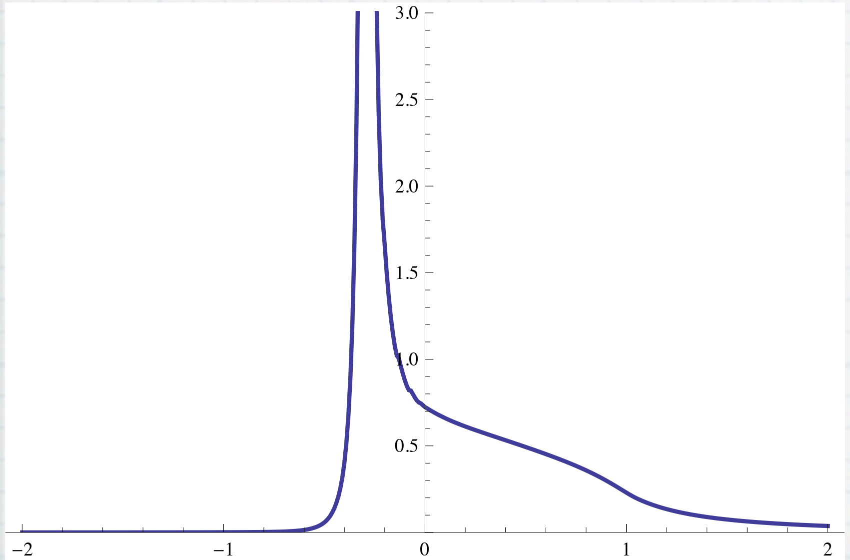
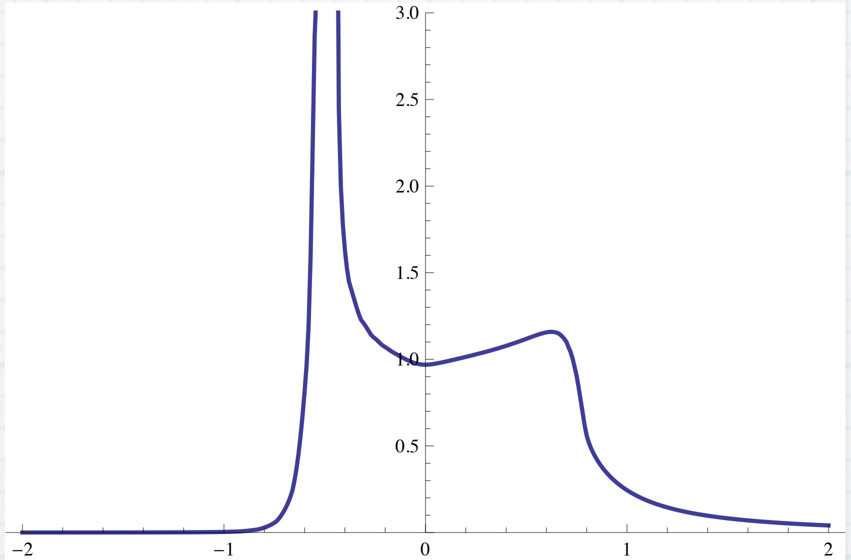


Figure 9: RF spectrum $A_{\downarrow}(\mathbf{k}, \tilde{\omega} + \xi_{k,2})$ of a single $^{40}$K impurity with finite momentum, in a Fermi sea of $^{6}$Li atoms. **From top to bottom**: $|\mathbf{p}|/k_{F\uparrow} = 0.1, 0.5, 1, \sqrt{2}, \sqrt{r}$.
**From left to right**: $(k_F a)^{-1} = -0.5, 0, 0.5, 1$.
The x-axis is the energy $\omega/E_{F\uparrow}$. For the fourth row, $p^2/(2m_{\downarrow}) = 3k_BT/2$, taking $k_BT = 0.2E_{F\uparrow}$.
For the last row, $p^2/(2m_{\downarrow}) = E_{F\uparrow}$.

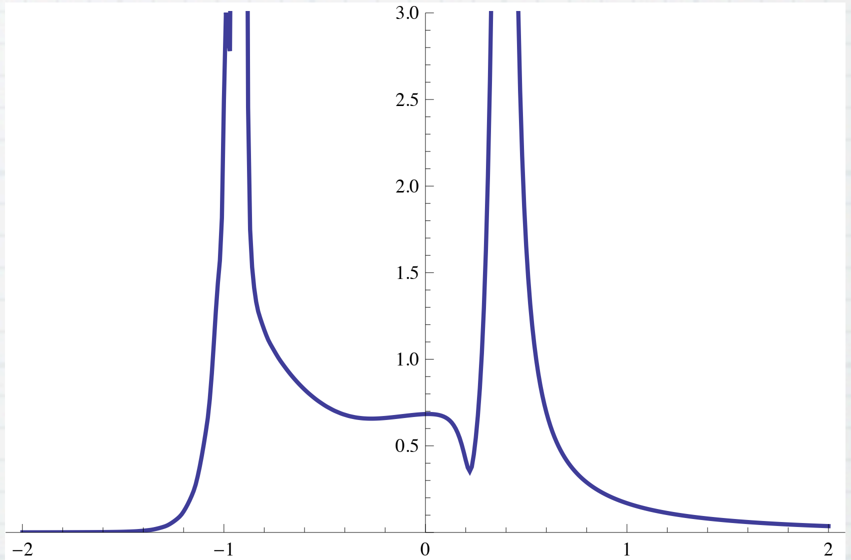
# Thermal average




$(k_F a)^{-1} = -0.5$



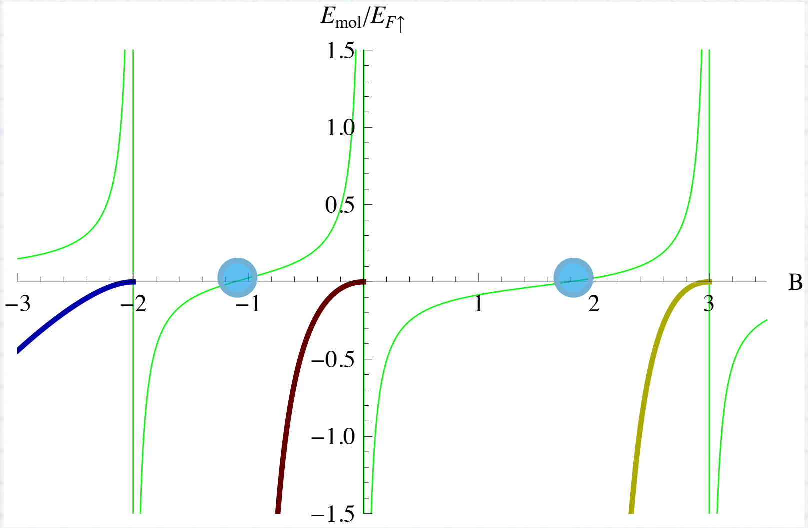

$(k_F a)^{-1} = 0$




$(k_F a)^{-1} = 0.5$

# a toy model with 3 FR

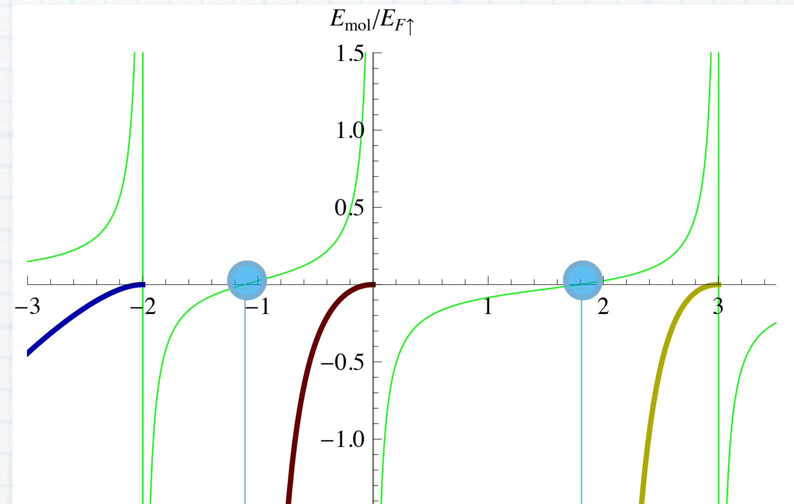
2-body bound states:




a=0

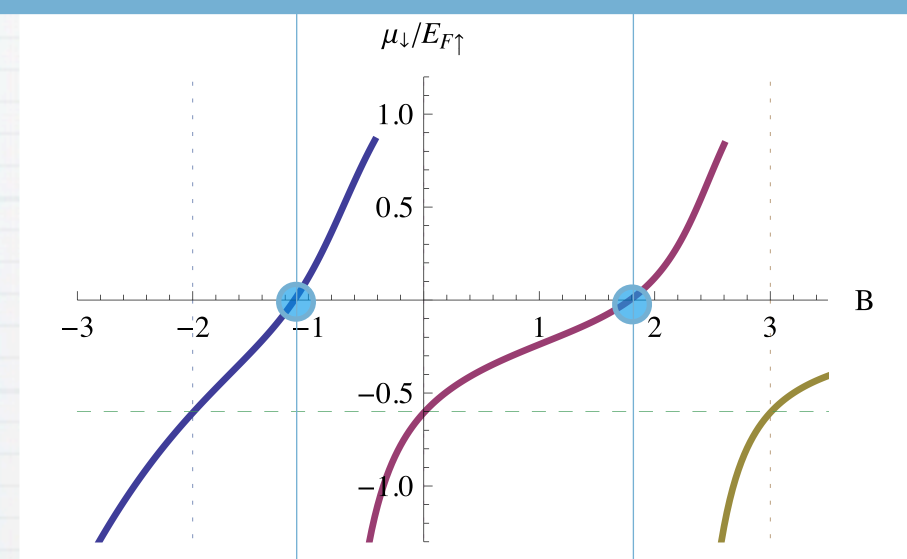
# a toy model with 3 FR

2-body bound states:




- a=0

Polaronic states:




weak coupling:

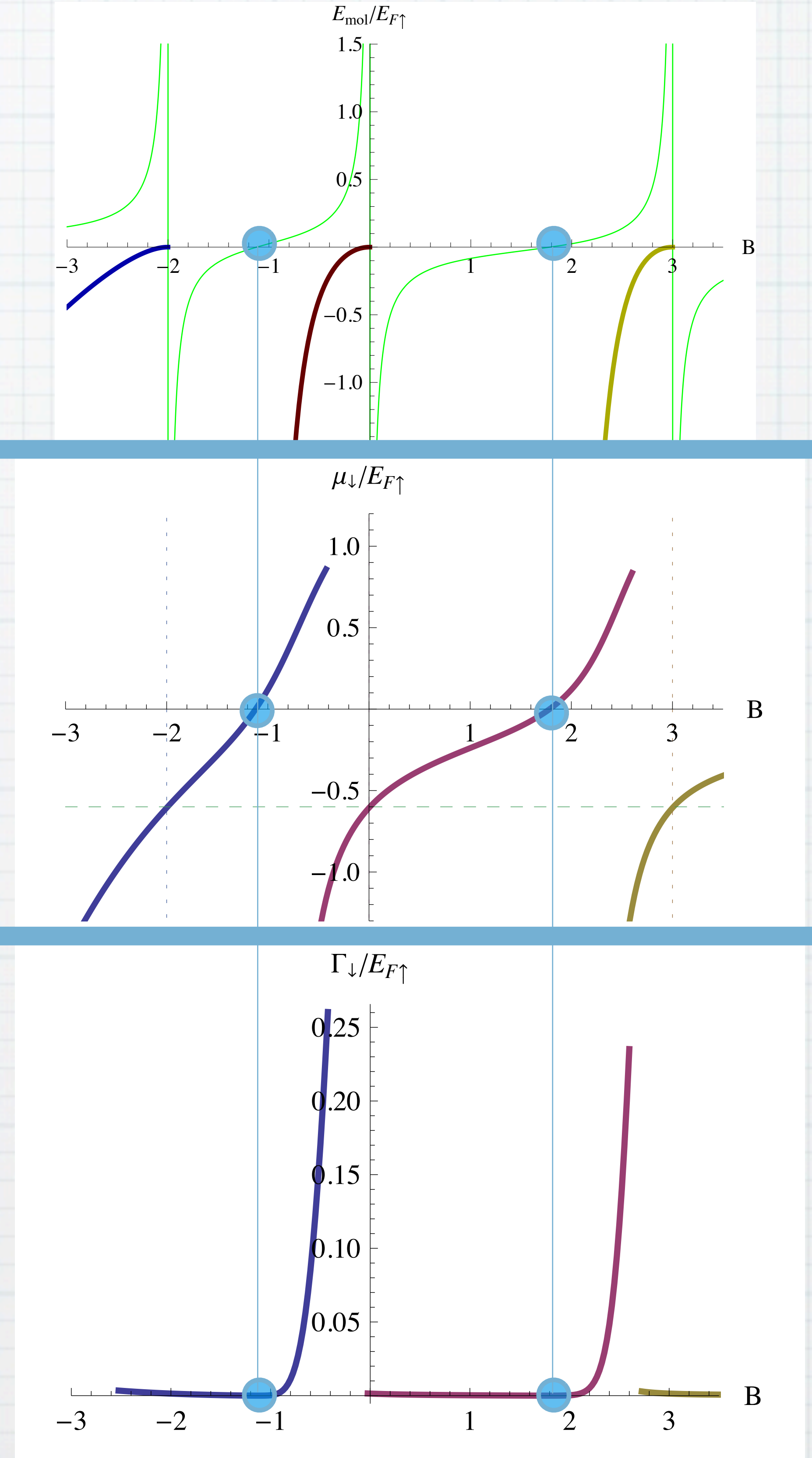
$E \propto a$

# a toy model with 3 FR

2-body bound states:

Polaronic states:

Decay rates:




a=0

weak coupling:

$E \propto a$

$\Gamma \propto \Theta(a)$

# Conclusions

- Complete characterization of the repulsive branch: energy, residue, decay rate, $m^*$, $\Delta N$, $C_{imp}$
- Quasiparticle properties fix completely the equation of state of the normal Fermi gas
- IFM easier to reach with lighter impurities
- RF spectra

G. Bruun and PM, Phys. Rev. Lett. 105, 020403 (2010)
K. Sadeghzadeh, G. Bruun, C. Lobo, PM, and A Recati, arXiv:1012.0484
PM and G. Bruun, arXiv:1102.0121