TOPOLOGICAL SUPERFLUIDS IN OPTICAL LATTICES

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UAB

EUROPEAN SCIENCE FOUNDATION

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Topological states... where?

- cond.mat. topological insulators
  (quantum wells, bismuth antimony alloys, Bi_2Se_3 crystals, ...)
- $\nu=5/2$ FQH state (Pfaffian)
- 2D p-wave SF of identical ↑ fermions
  Read&Green, PRB 2000
- 2D s-wave SF of imbalanced ↑↓ fermions with spin-orbit coupling
  Sato, Takahashi & Fujimoto, PRL 2009
  Sau Jay, Lutchyn, Tewari and Das Sarma, PRL 2010
- ... ...

Outlook of the talk

↑ 2D p-wave SF

↑↓ 2D s-wave SF
with $n_\uparrow \neq n_\downarrow$
and spin-orbit coupling

Why 2D?

Because in 2D particles have anyonic statistics
(anyons: any phase under exchange of two particles)

In particular, the statistic can be non-Abelian,
i.e., the exchange of two particles
may be described by a matrix

Non-Abelian anyons are a necessary ingredient
for topological quantum computation

↑ 2D p-wave SF

A stable p-wave SF?

3-body losses at a p-wave Feshbach resonance

A stable p-wave SF?

3-body losses at a p-wave Feshbach resonance

Ultracold proposals:

- "dissipation-induced stability" in optical lattices (1,2)
  (i.e., how to get no losses from large losses)
- time-dependent lattices (3,4)
- RF dressing of 2D fermionic polar molecules leads to long-range interactions ($\propto r^{-3}$) and high T_C (5)
- super-exchange interactions in Bose-Fermi mixtures (6,7)

1:Han, Chan, Yi, Daley, Diehl, Zoller & Duan, PRL 2009
2:Roncaglia, Rizzi & Cirac, PRL 2009
2:Lim, Lazarides, Hemmerich & Morais-Smith, EPL 2009
3:Pekker, Sensarma & Demler, arXiv:0906.0931
4:Dutta & Lewenstein, arXiv:0906.2115 & PRA 2010
5:Cooper & Shlyapnikov, PRL 2009
6:Lewenstein, Santos, Baranov & Fehrmann, PRL 2004
7:Massignan, Sanpera & Lewenstein, PRA 2010

Bose-Fermi mixture

1) $U_{BB} > 0$

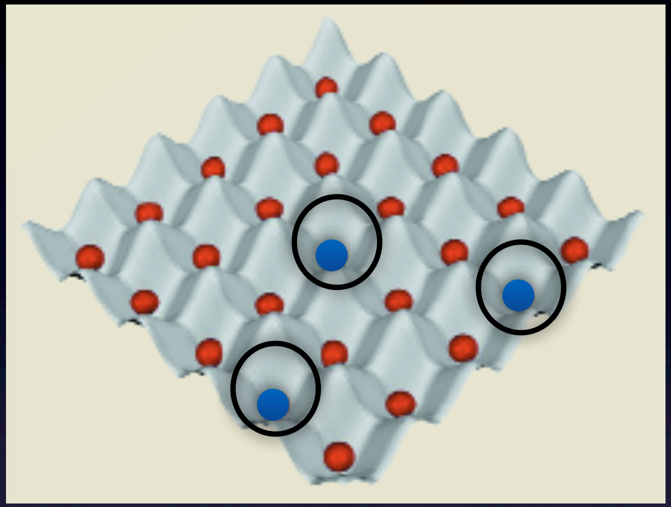
2) Strong coupling:

$$t_B \, , \, t_F \ll U_{BB} \, , \, |U_{BF}|$$

(bosons in n=1 Mott state)

Lewenstein, Santos, Baranov & Fehrmann, PRL 2004

Bose-Fermi mixture



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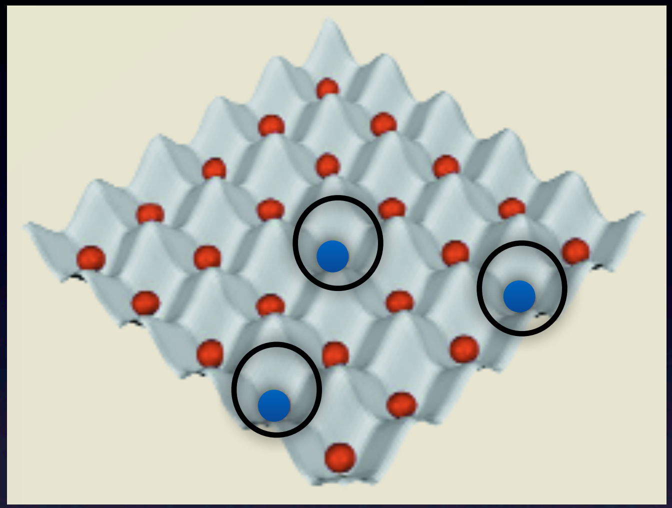

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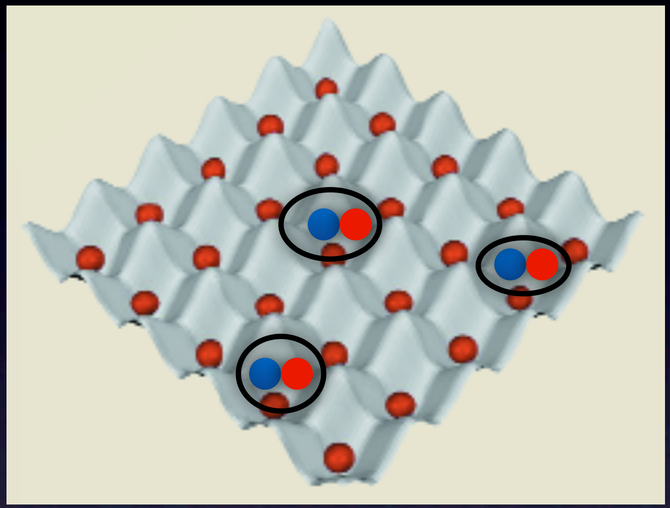

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composite fermions

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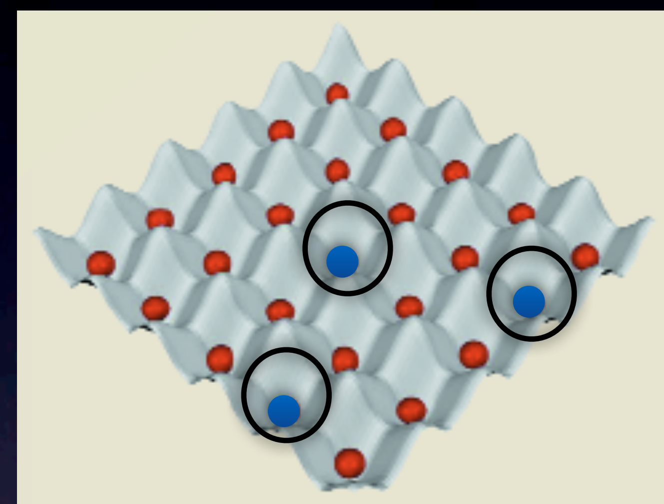

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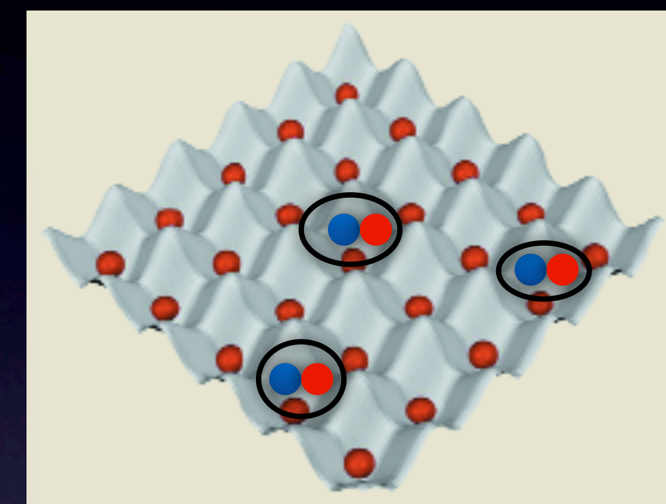

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Attractive interaction when $U_{BF}>U_{BB}$

Lewenstein, Santos, Baranov & Fehrmann, PRL 2004

Effective Fermi-Hubbard model

nearest-neighbor interaction (super-exchange)

$$H = -t \sum_{<i,j>} c_i^\dagger c_j - \frac{U}{2} \sum_{<i,j>} n_i n_j - \mu \sum_i n_i$$

$t \sim (t_B t_F)/U_{BF}$

$U>0$

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$U>0$

- BCS approach: introduce BdG operators

$$\gamma_n = \sum_i u_n(i) c_i + v_n(i) c_i^\dagger$$

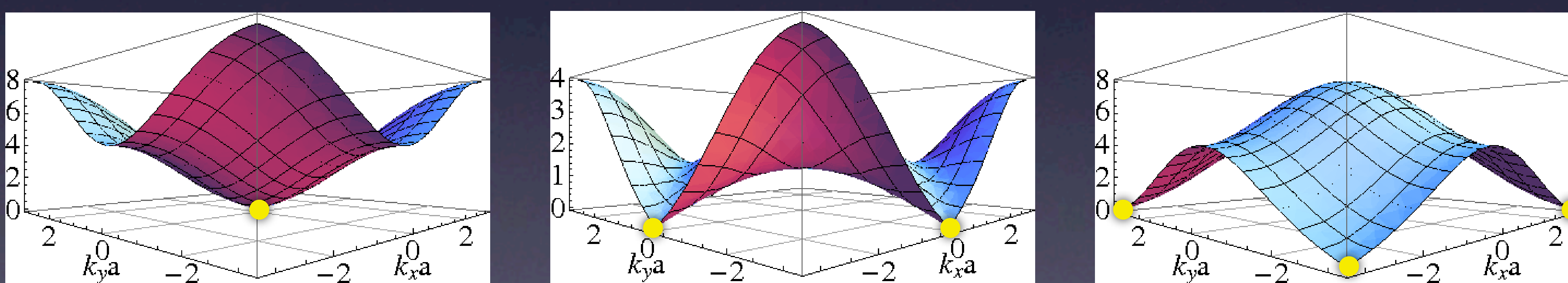
- Self-consistent "p-wave gap equation"

$$\Delta_{ij} = U \langle c_i c_j \rangle = U \sum_{E_n > 0} u_n^*(i) v_n(j) \tanh\left(\frac{E_n}{2 k_B T}\right)$$

Spectrum on a lattice
(homogeneous system)

2D chiral ($p_x \pm i p_y$) SF: $E(\mathbf{k}) = \sqrt{\xi(\mathbf{k})^2 + \Delta_{\mathbf{h}}(\mathbf{k})^2}$

Linear dispersion at the Dirac cones



μ=-4t μ=0 μ=4t

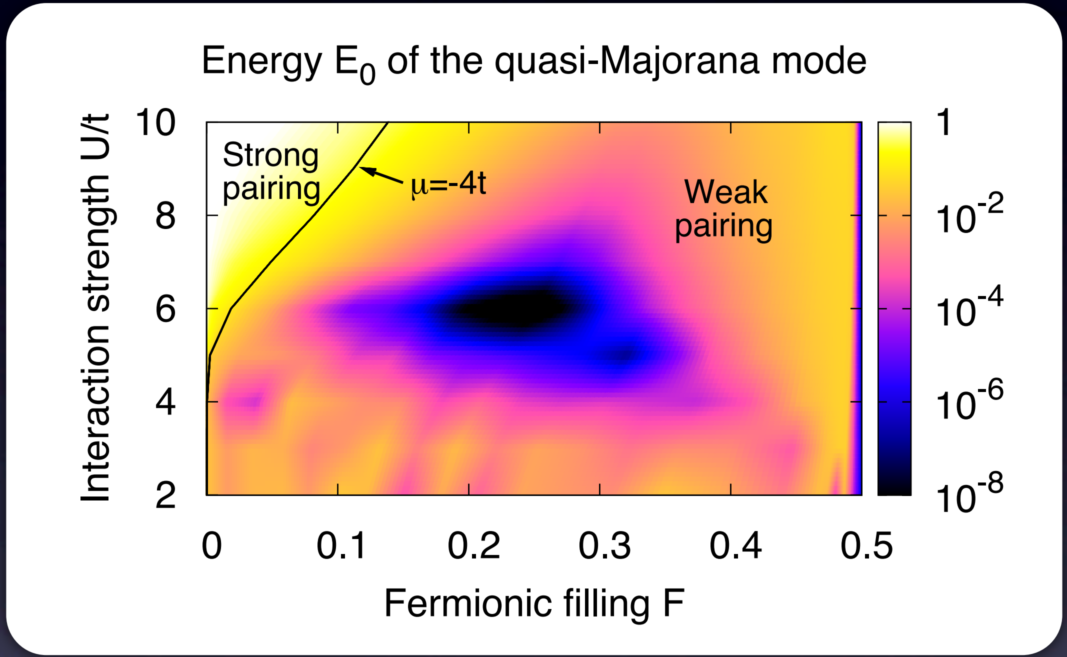
Two distinguishable topological phases for filling F<1/2 and F>1/2

Spectrum with vortex

Δ_0~t~10nK (super-exch.)

Low-lying spectrum: $E_n \approx n\omega_0$
$n=0,1,2,...$

The eigenstate with $E_0 \ll \Delta_0$ is a Majorana fermion.



Particle-hole symmetry of the BdG eqs.: $\{E_n, \psi_n\} \leftrightarrow \{-E_n, \sigma_1 \psi_n^*\}$. Then, if $E_0 = 0,\ u_0 = v_0^*$

P. Massignan, A. Sanpera & M. Lewenstein, PRA 2010

↑↓ 2D s-wave SF with $n_{\uparrow} \neq n_{\downarrow}$ and spin-orbit coupling

Ultracold atoms in synthetic gauge fields

Proposals: Jaksch&Zoller, NJP 2003
Osterloh et al., PRL 2005
Gerbier&Dalibard, NJP 2010
Bermudez et al., arXiv1004.5101

- adiabatic Raman passage
- adiabatic control of superpositions of degenerate dark states
- spatially varying Raman coupling
- Raman-induced transitions to auxiliary states in optical lattices

REVIEW:

Artificial gauge potentials for neutral atoms
J. Dalibard, F. Gerbier, G. Juzeliūnas, and P. Öhberg
submitted to RMP, arXiv:1008.5378

↑↓ fermions in synthetic gauge fields

$$\mathcal{H}_0 = -\mathrm{t}\sum_{\mathrm{i}}\left[\mathbf{c}^\dagger_{\mathrm{i}+\hat{x}} e^{i\sigma_y \alpha}\mathbf{c}_{\mathrm{i}} + \mathbf{c}^\dagger_{\mathrm{i}+\hat{y}} e^{i\sigma_x \beta}\mathbf{c}_{\mathrm{i}} + \mathrm{h.c.}\right]$$

$$\mathbf{c}^\dagger_{\mathrm{i}} = (c^\dagger_{\mathrm{i}\uparrow}, c^\dagger_{\mathrm{i}\downarrow})$$

External non-Abelian gauge fields yield a fictitious spin-orbit coupling

Add attractive interactions

↓

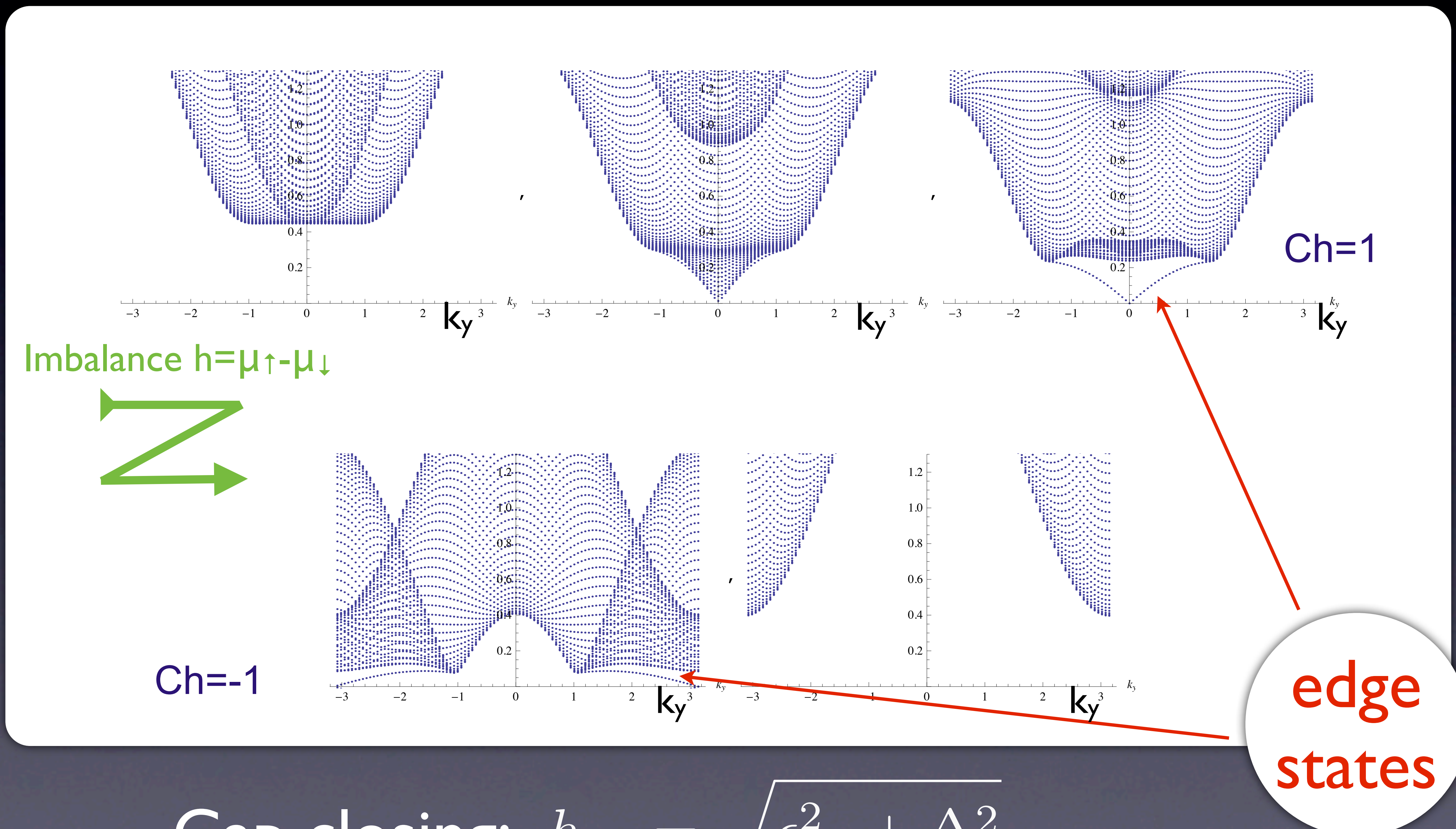
BCS superfluid

↓

strong imbalance ⇒ topological state

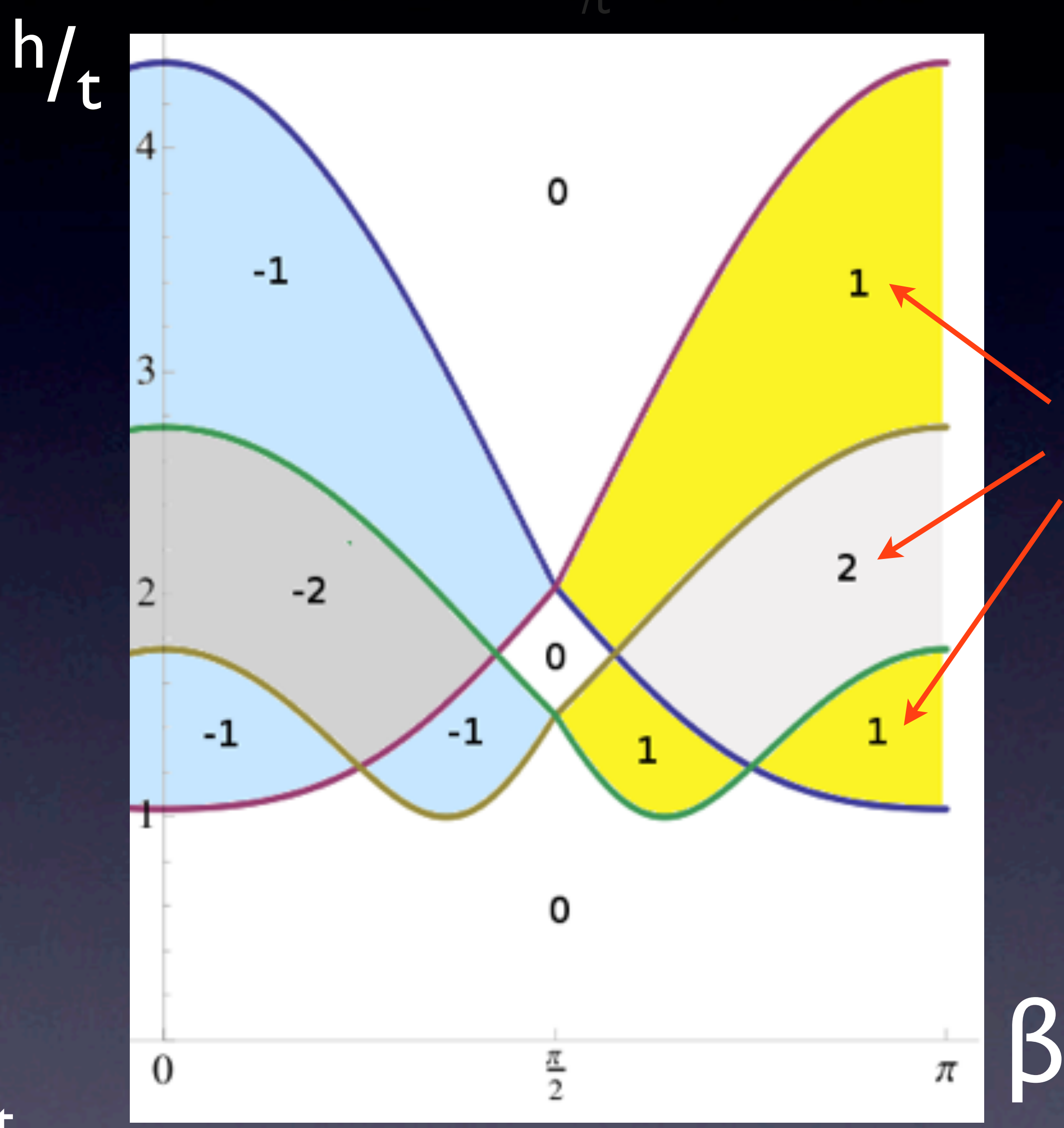
Spectrum on a cylinder

(open b.c. along x)



Gap closing: $h_{\mathbf{k}_0} = \sqrt{\epsilon_{\mathbf{k}_0}^2 + \Delta^2}$

Topological phases



h=μ↑-μ↓

Chern numbers

easy to calculate!
(see J. Bellissard, condmat/9504030)

Gap closing at $(\mathbf{k}_0, \tilde{h})$:

$$\mathcal{H}_{\text{eff}}(\mathbf{k}, h) = E(\mathbf{k}, h) + \vec{\sigma} \cdot \vec{f}(\mathbf{k}, h)$$
$$\Delta\text{CN}(\tilde{h}) = \text{sign}\{\det[J_{\vec{f}}(\mathbf{k}_0, \tilde{h})]\}.$$

Δ=t
α=π/4
μ=-0.5t[|cos(α)|+|cos(β)|]

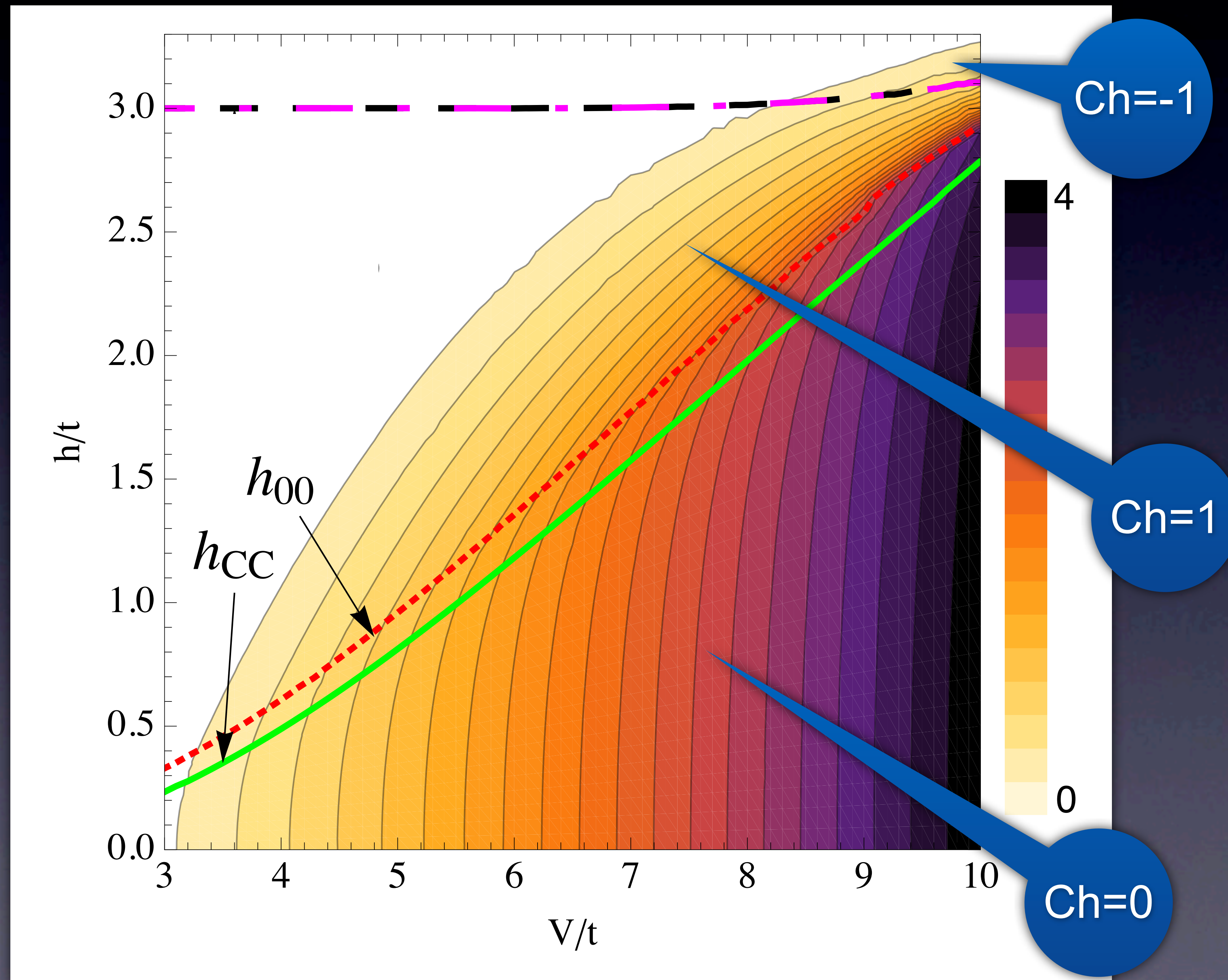
A. Kubasiak, P. Massignan & M. Lewenstein, arXiv:1007.4827 (EPL in press)

Spin imbalance vs. pair breaking

without SO coupling:
analytic CC limit
($h_{CC} = \Delta_0/\sqrt{2}$)

with SO coupling:
self-consistent calculation of Δ from the BCS gap equation

$\alpha=\beta=\pi/4$ $\mu=-3t$



A. Kubasiak, P. Massignan & M. Lewenstein, arXiv:1007.4827 (EPL in press)

Conclusions

- Ultracold SF fermions possess

  non-trivial topological phases

- Optical lattices stabilized p-wave SF ➢ FQH

- ↑↓ fermions in non-Abelian gauge fields ➢ QSH

P. Massignan, A. Sanpera & M. Lewenstein, PRA 2010
A. Kubasiak, P. Massignan & M. Lewenstein, EPL in press

Bose-Fermi mixture

1) $U_{BB}>0$

2) Strong coupling:

$$t_B \, , \, t_F \ll U_{BB} \, , \, |U_{BF}|$$

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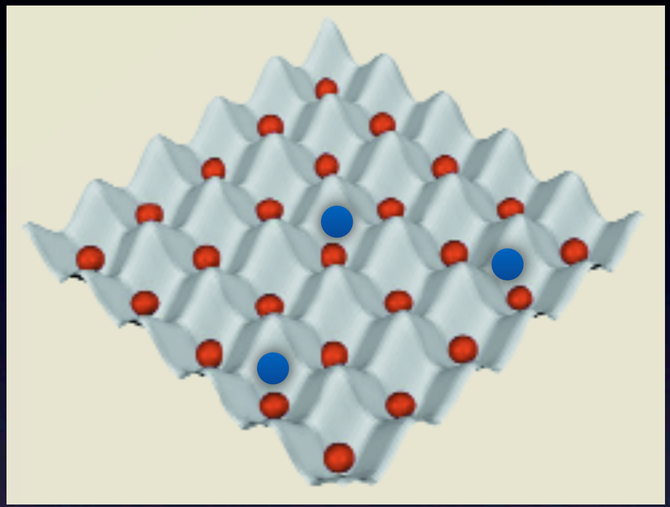

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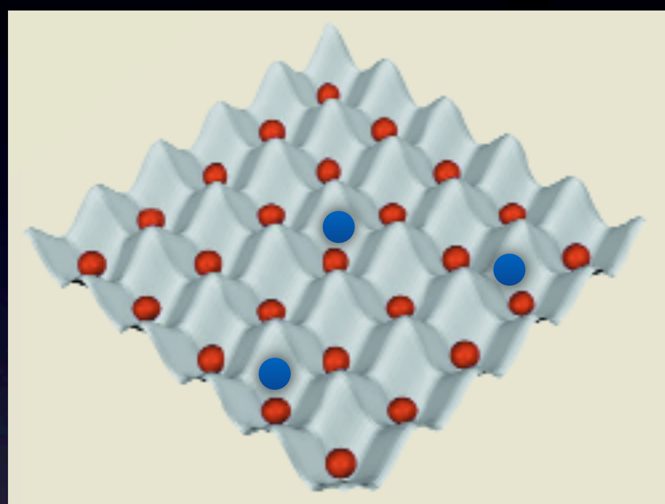

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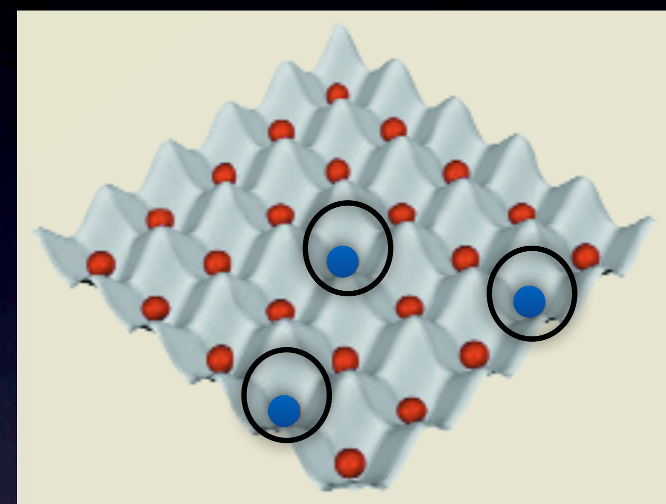

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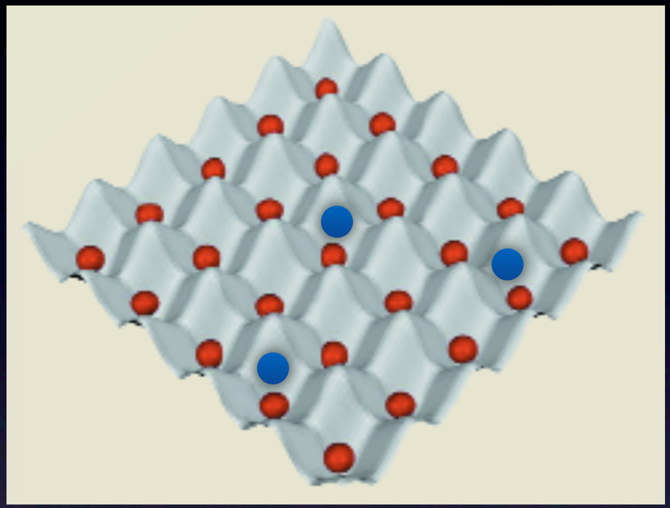

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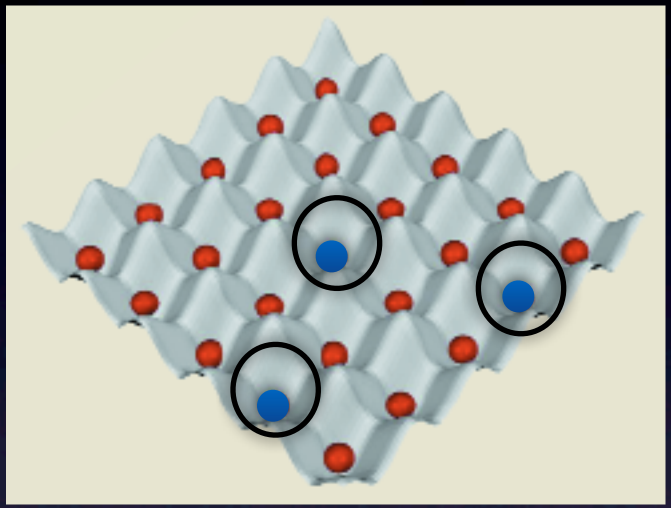

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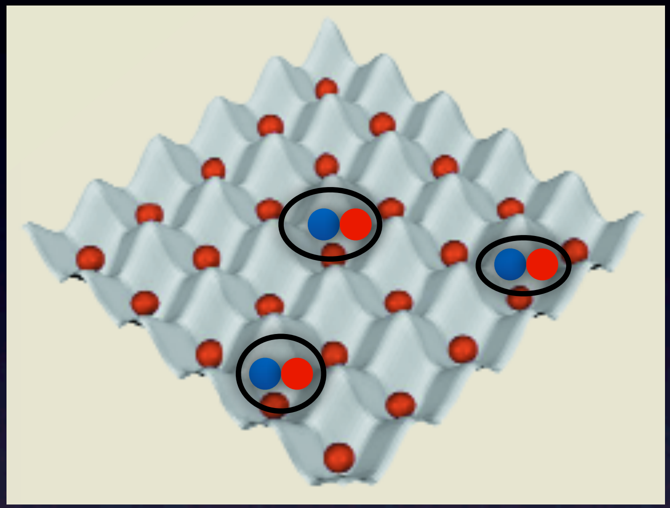

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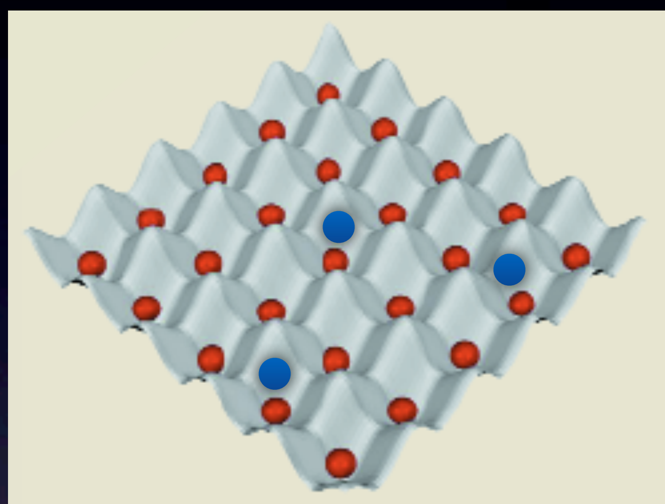

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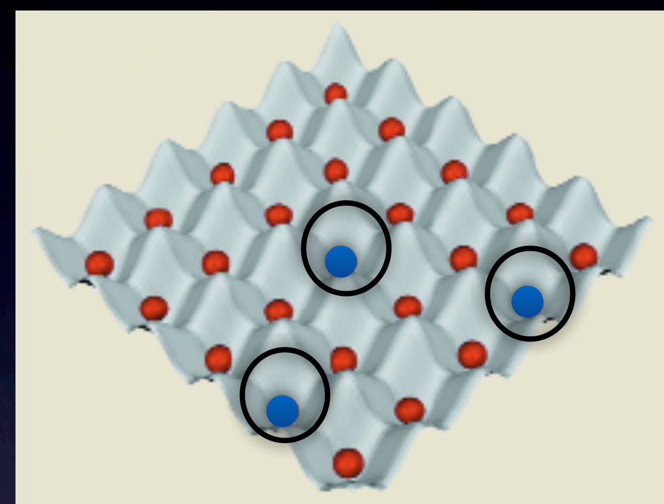

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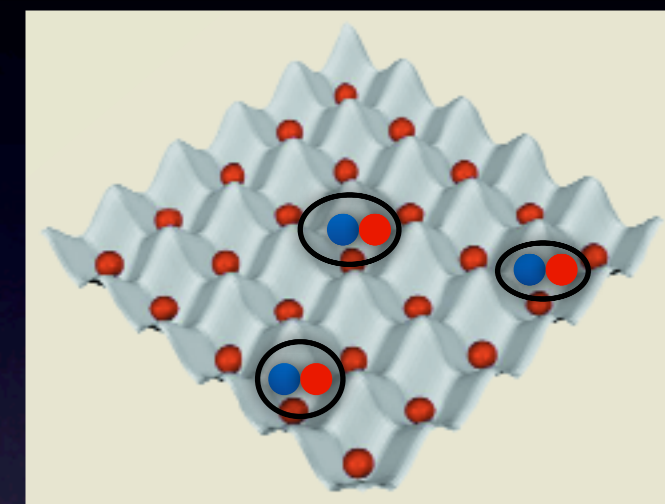

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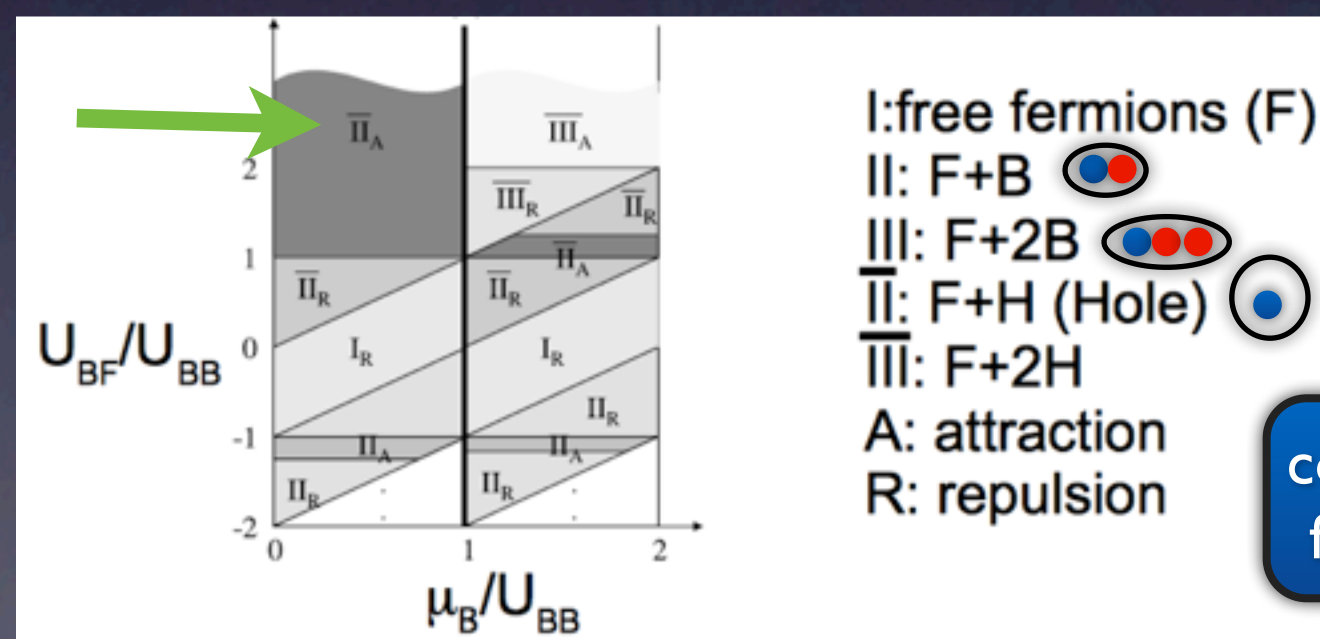

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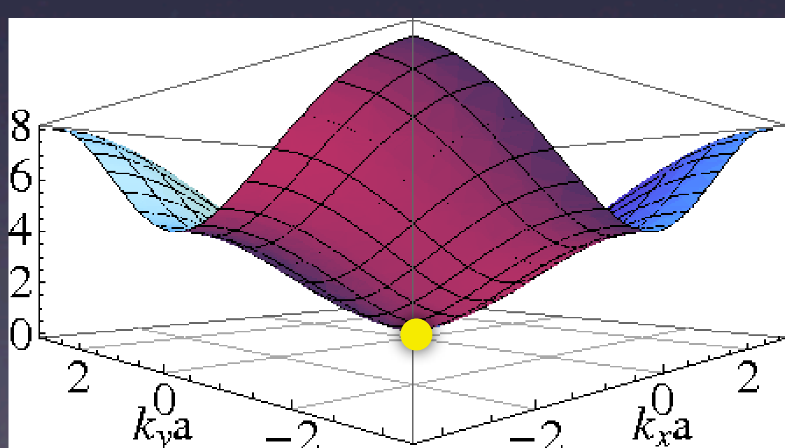
Lewenstein, Santos, Baranov & Fehrmann, PRL 2004

Spectrum on a lattice
(homogeneous system)

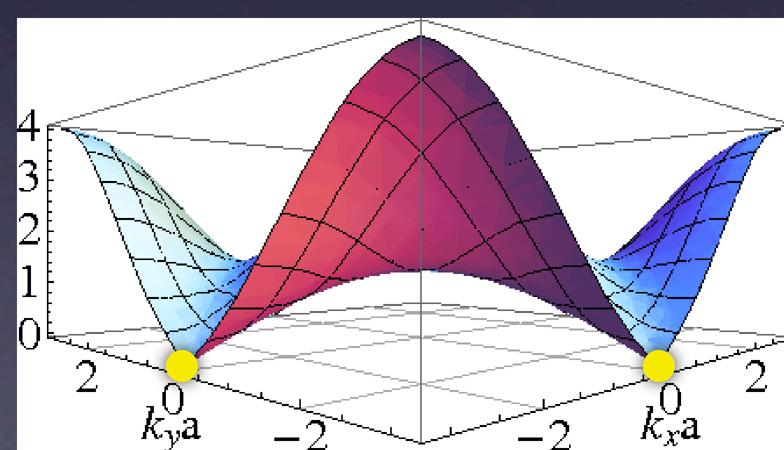
2D chiral ($p_x \pm ip_y$) SF: $E(\mathbf{k}) = \sqrt{\xi(\mathbf{k})^2 + \Delta_h(\mathbf{k})^2}$

with $\xi = -2t[\cos(k_x a) + \cos(k_y a)] - \mu$ and $\Delta_h^2 = \Delta_0[\sin^2(k_x a) + \sin^2(k_y a)]$

Linear dispersion at the Dirac cones



μ=-4t



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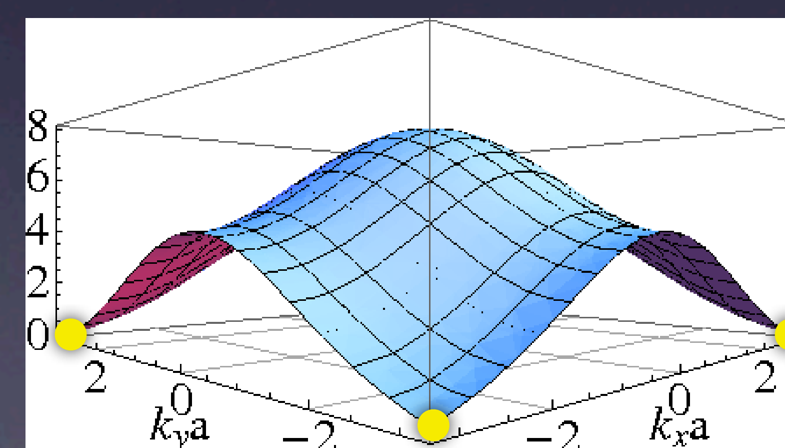
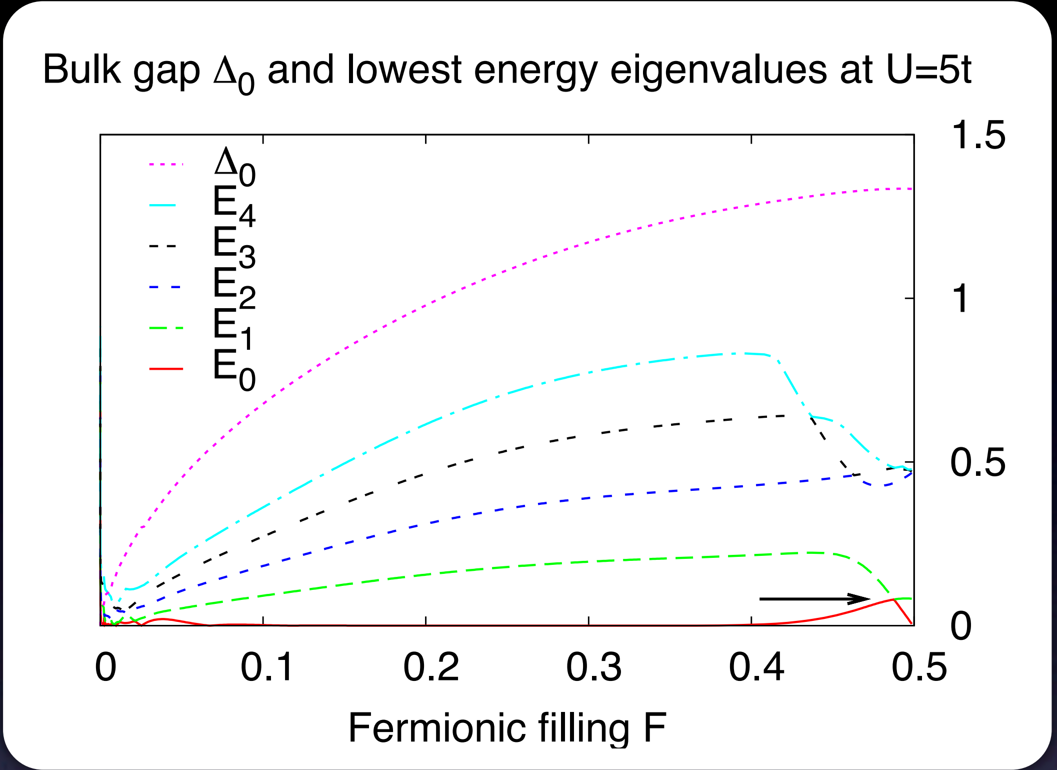


μ=4t

Two distinguishable topological phases for $-4t<\mu<0$ and $0<\mu<4t$

Spectrum with vortex



$$\text{Ansatz} : \Delta_{ij} = \chi_{ij} f_i \mathrm{e}^{\mathrm{i}w\theta_i}$$

$\chi_{ij} = \{1, \mathrm{i}, -1, -\mathrm{i}\}$: chirality
$w = \pm 1$: vortex direction of rotation
f_i : vortex amplitude at site i
θ_i : polar angle of site i

Δ_0~t~10nK (super-exch.)

Low-lying spectrum: $E_n \approx n\omega_0$ $_{n=0,1,2,\ldots}$

The eigenstate with $E_0 \ll \Delta_0$ is a Majorana fermion.

Particle-hole symmetry of the BdG eqs.: $\{E_n, \psi_n\} \leftrightarrow \{-E_n, \sigma_1 \psi_n^*\}$. Then, if $E_0 = 0$, $u_0 = v_0^*$

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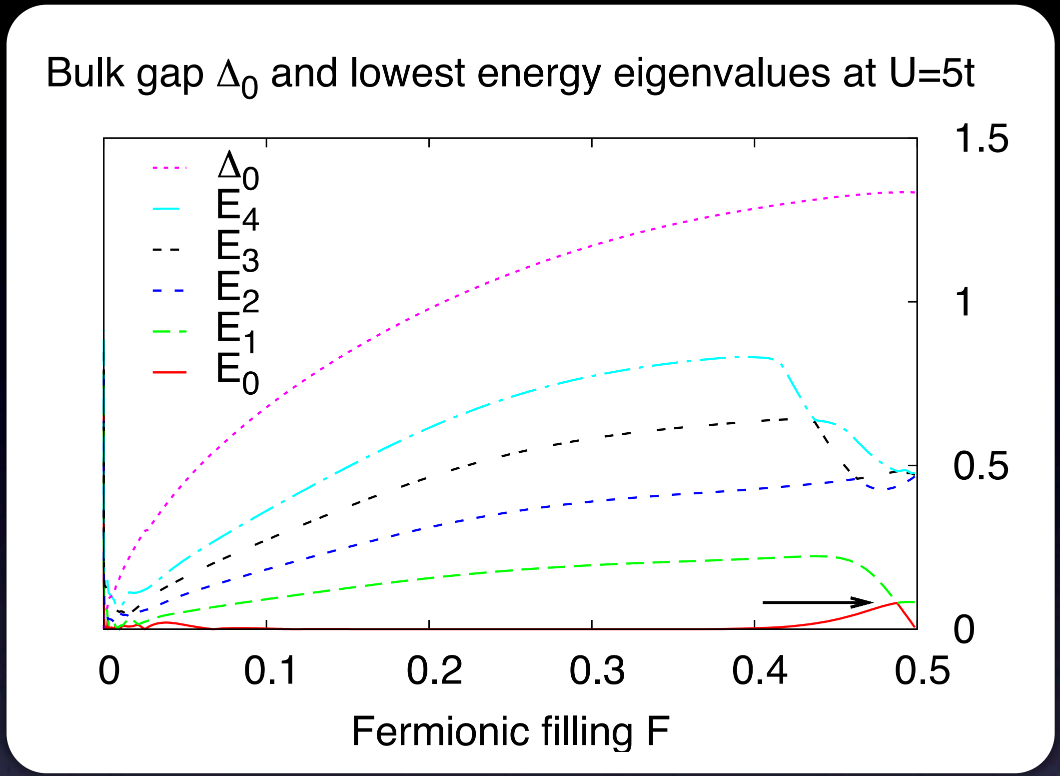

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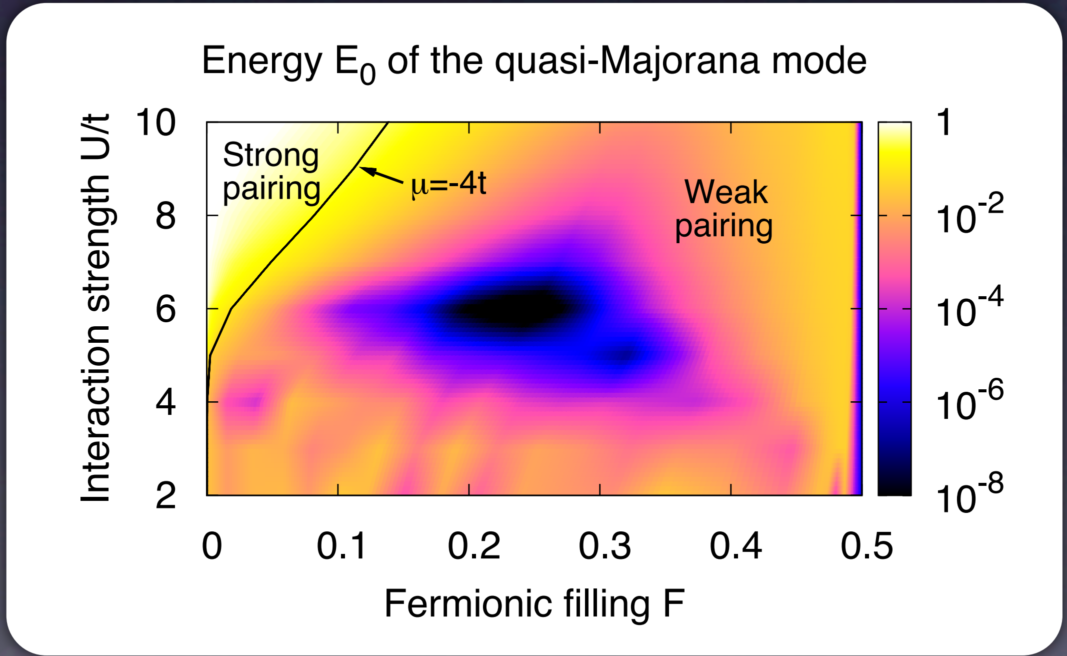

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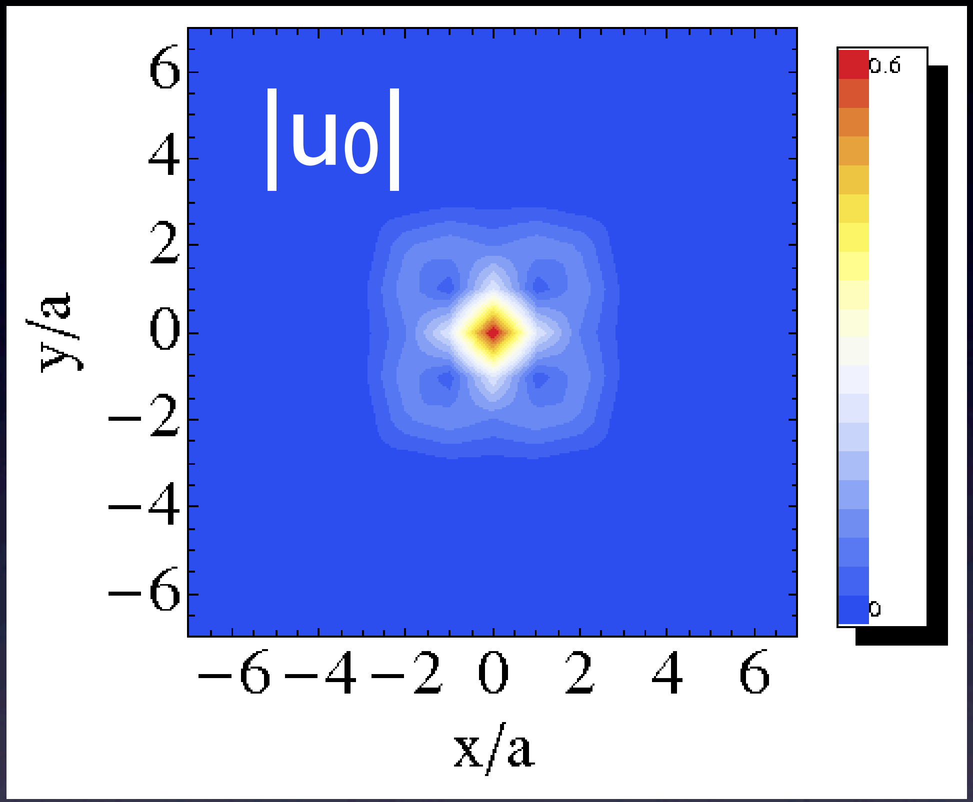
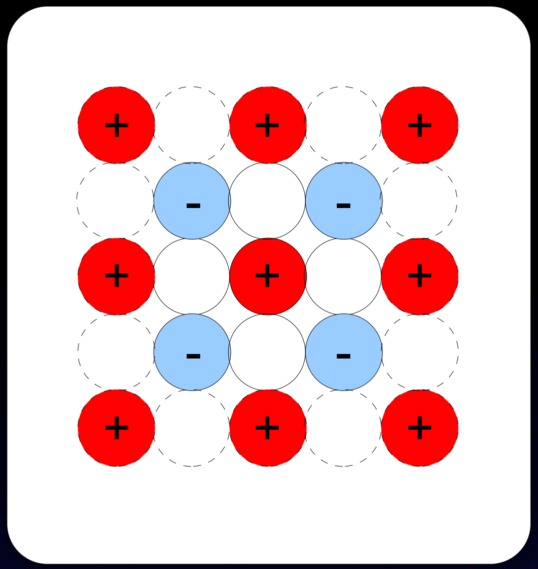
E=0 wavefunction



w=-1
U=5t

Oscillating wavefunction with exponentially decaying envelope
u_0 has a maximum (node) in the core for w=-1 (w=1)

Half filling



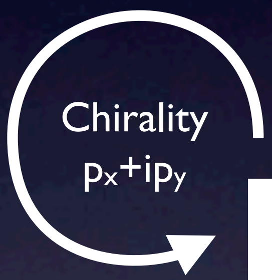
$\lambda_{latt}=\lambda_{wf}/2$

Zero-mode only on odd N*N lattices

d-wave checkerboard symmetry

w=1 state suddenly spreads close to the TPT

(Topological Phase Transition)



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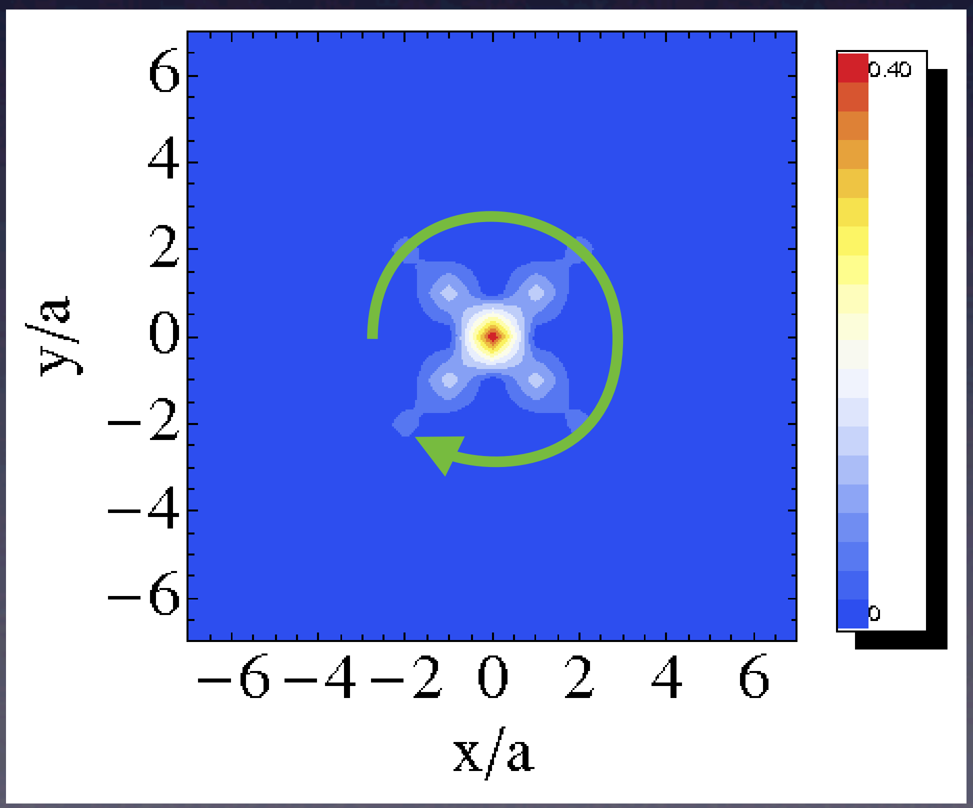

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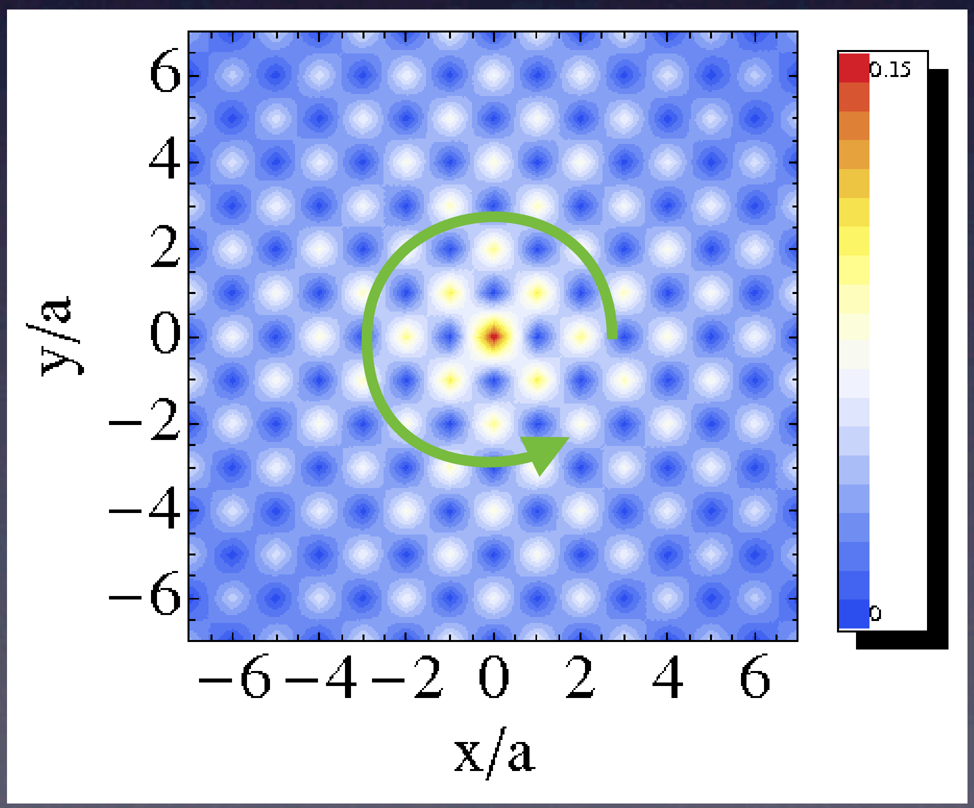
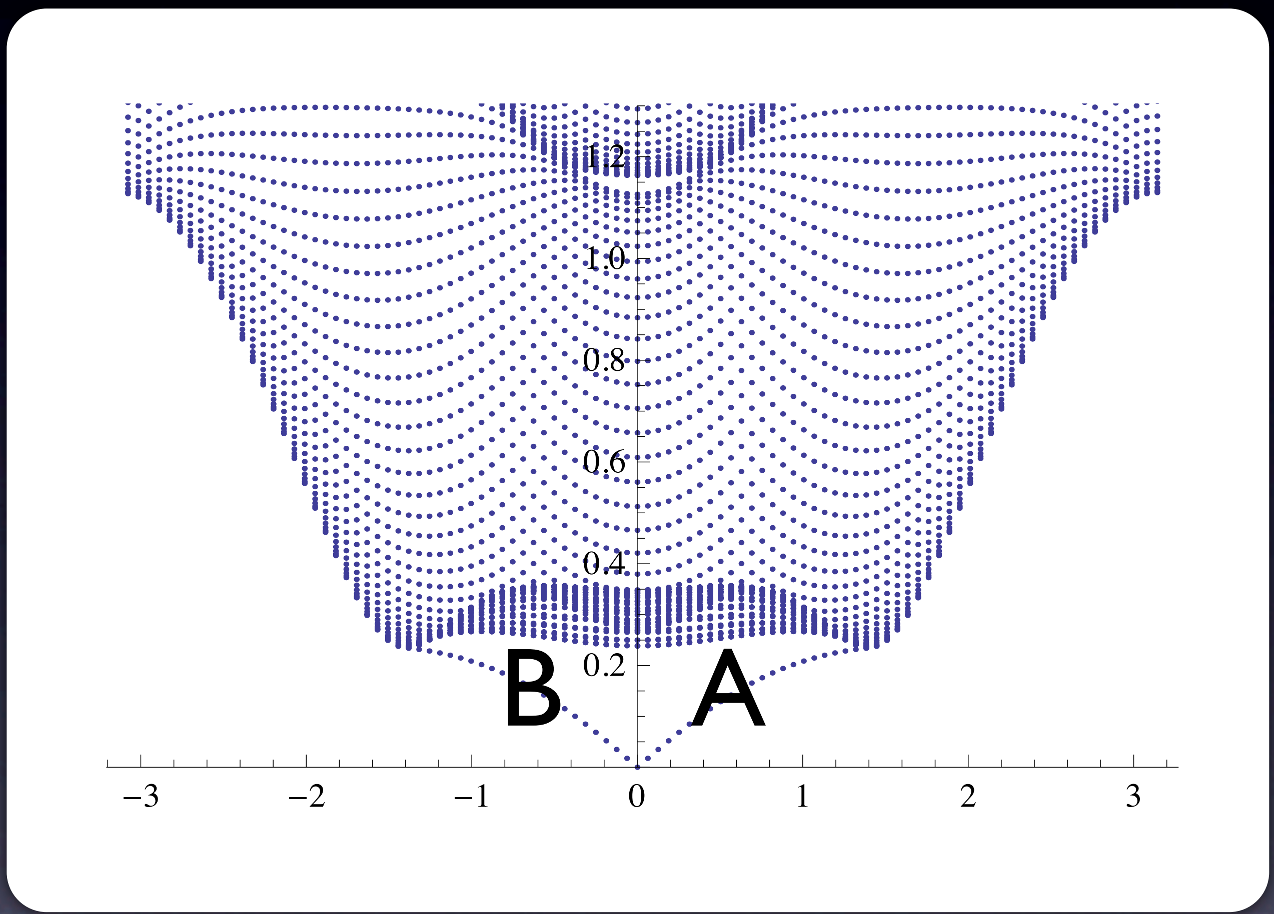
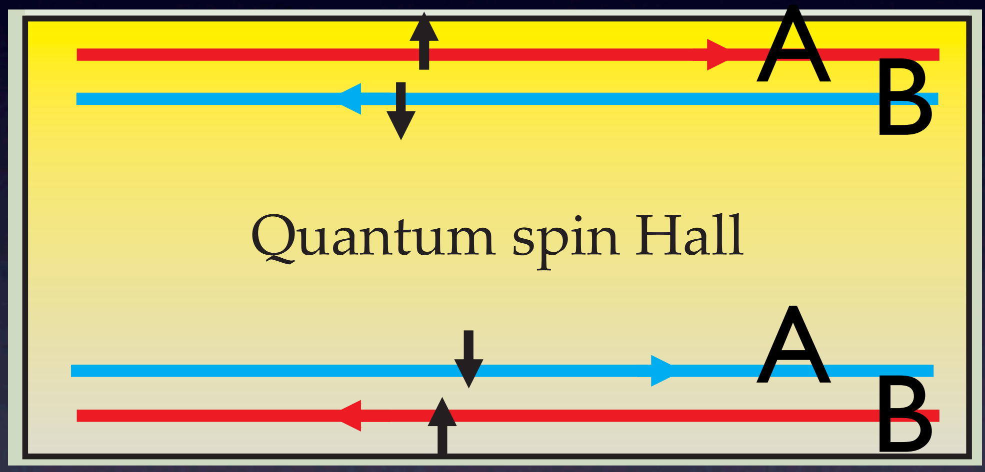


P. Massignan, A. Sanpera & M. Lewenstein, PRA 2010

Edge states





Degenerate pair of counter-propagating edge states with opposite spins