Dressed impurities in an ideal Fermi gas with narrow Feshbach resonances

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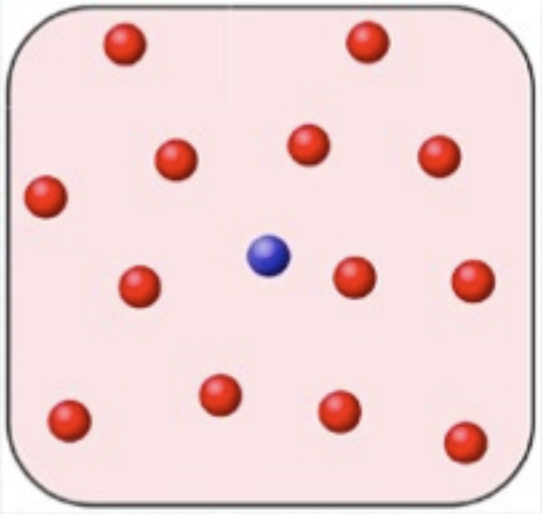
N>>1



normal Fermi liquid

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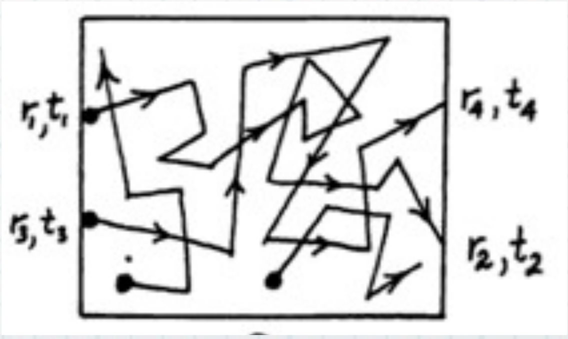
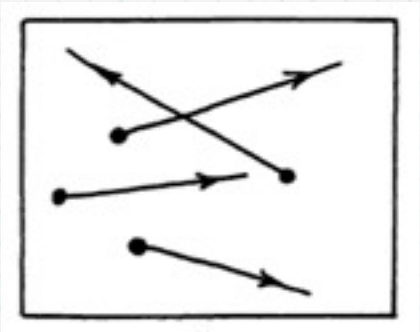
FerMix - EuroQuam

Quasi-Particles

Landau's idea:
who cares about real particles?



Of importance are the excitations,
which behave
as quasi-particles!



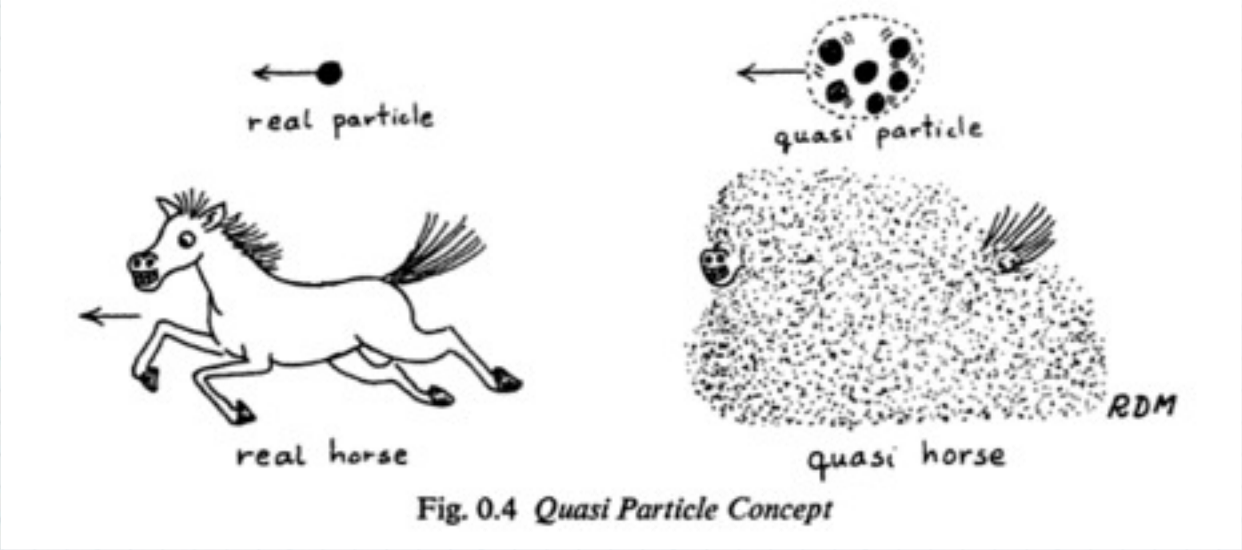


Fig. 0.4 *Quasi Particle Concept*

a QP is a "free particle" with:

- @ renormalized mass
- @ chemical potential
- @ shielded interactions
- @ q. numbers (charge, spin, ...)
- @ lifetime

Polaron: variational Ansatz

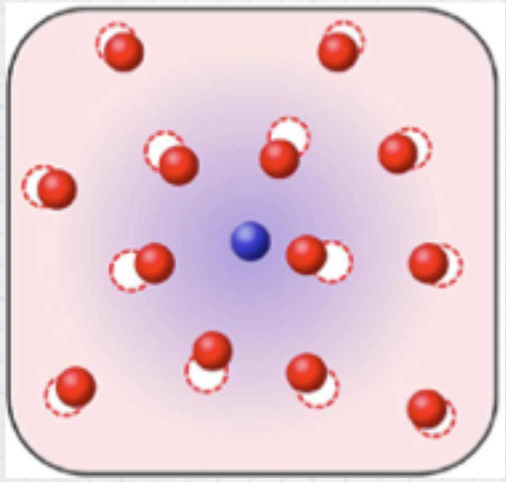
(F. Chevy, PRA 2006)

the $\downarrow$ impurity

$$|\psi_{\mathbf{p}}\rangle = \phi_0 c^\dagger_{\mathbf{p}\downarrow} |0\rangle_\uparrow + \sum_{q<k_F}^{k>k_F} \phi_{\mathbf{qk}}\, c^\dagger_{\mathbf{p}+\mathbf{q}-\mathbf{k}_\downarrow}\, c^\dagger_{\mathbf{k}_\uparrow}\, c_{\mathbf{q}_\uparrow}\, |0\rangle_\uparrow$$

non-interacting $\uparrow$ Fermi sea

Particle-Hole dressing



Very good agreement with QMC results for $\mu_\downarrow$ and m*

This variational Ansatz has a diagrammatic equivalent:
the forward scattering, or ladder, approximation.

(Combescot, Recati, Lobo and Chevy, PRL 2007)

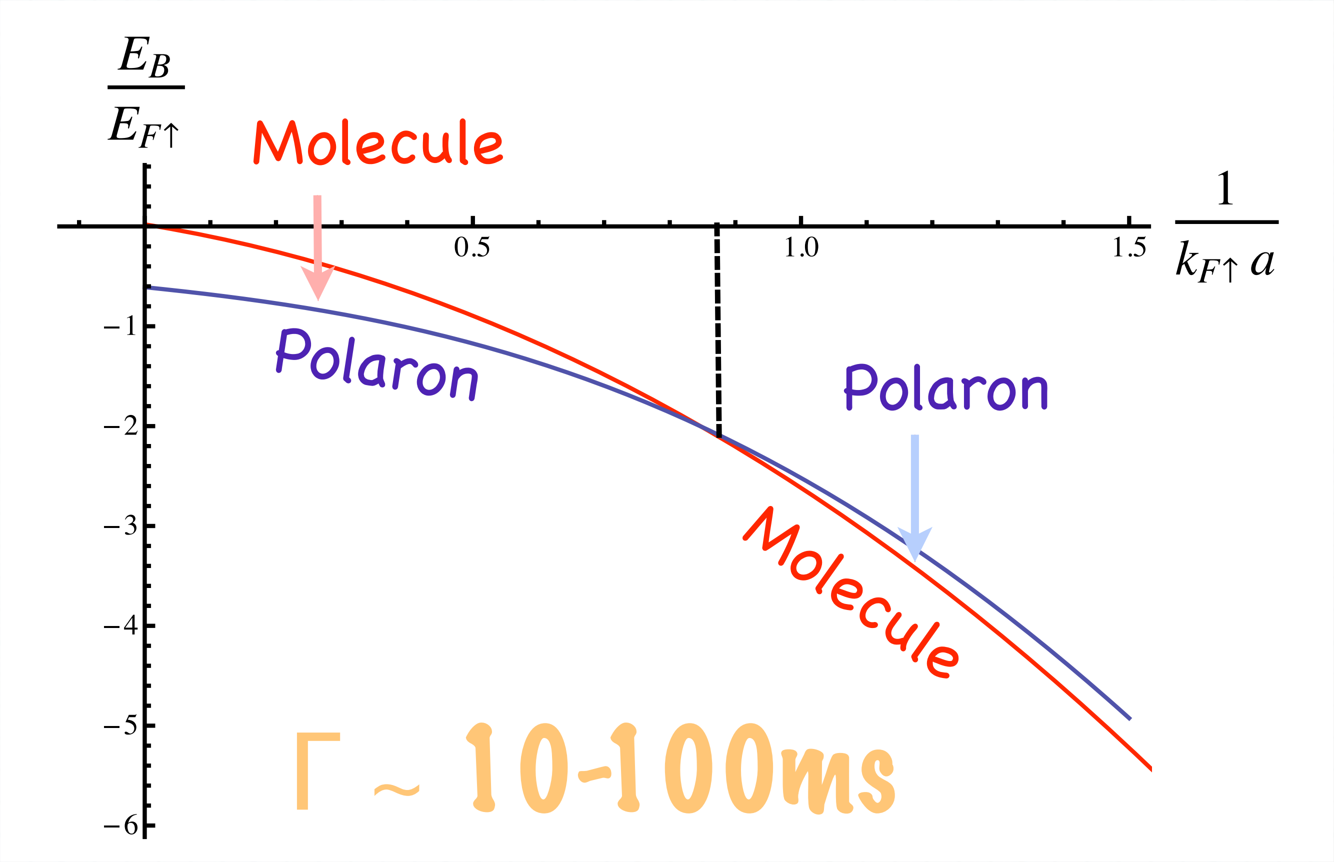
Very long QP lifetimes!

G. Bruun & PM, PRL 2010

$$\Gamma_P \sim Z_M \left(\Delta\omega\right)^{9/2}$$

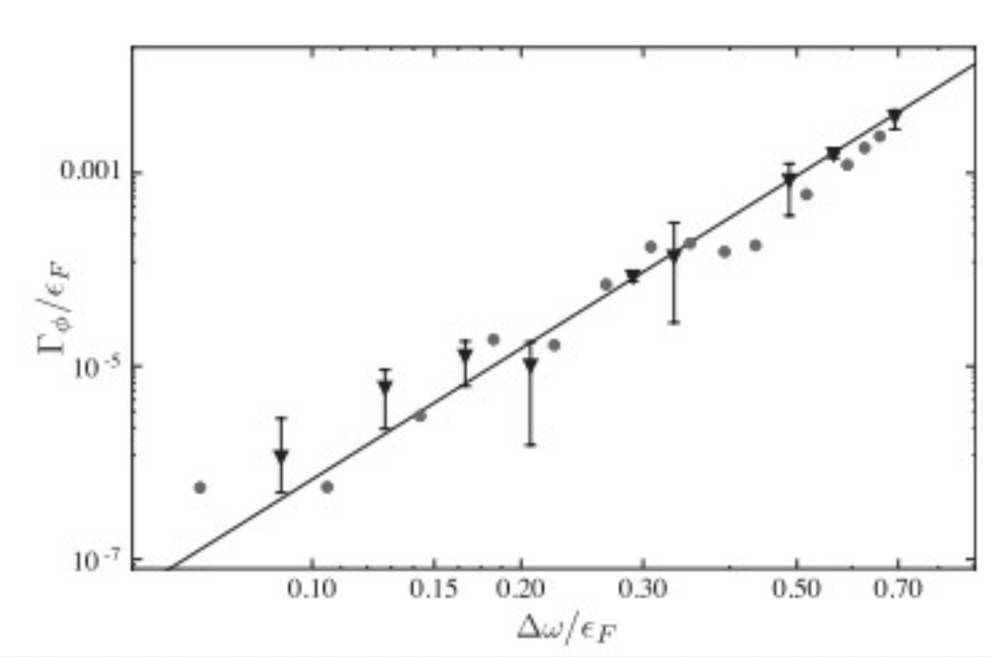
$$\Delta\omega = \omega_P - \omega_M$$

$$\Gamma_M \sim Z_P \left(-\Delta\omega\right)^{9/2}$$



Scaling confirmed by a RG calculation

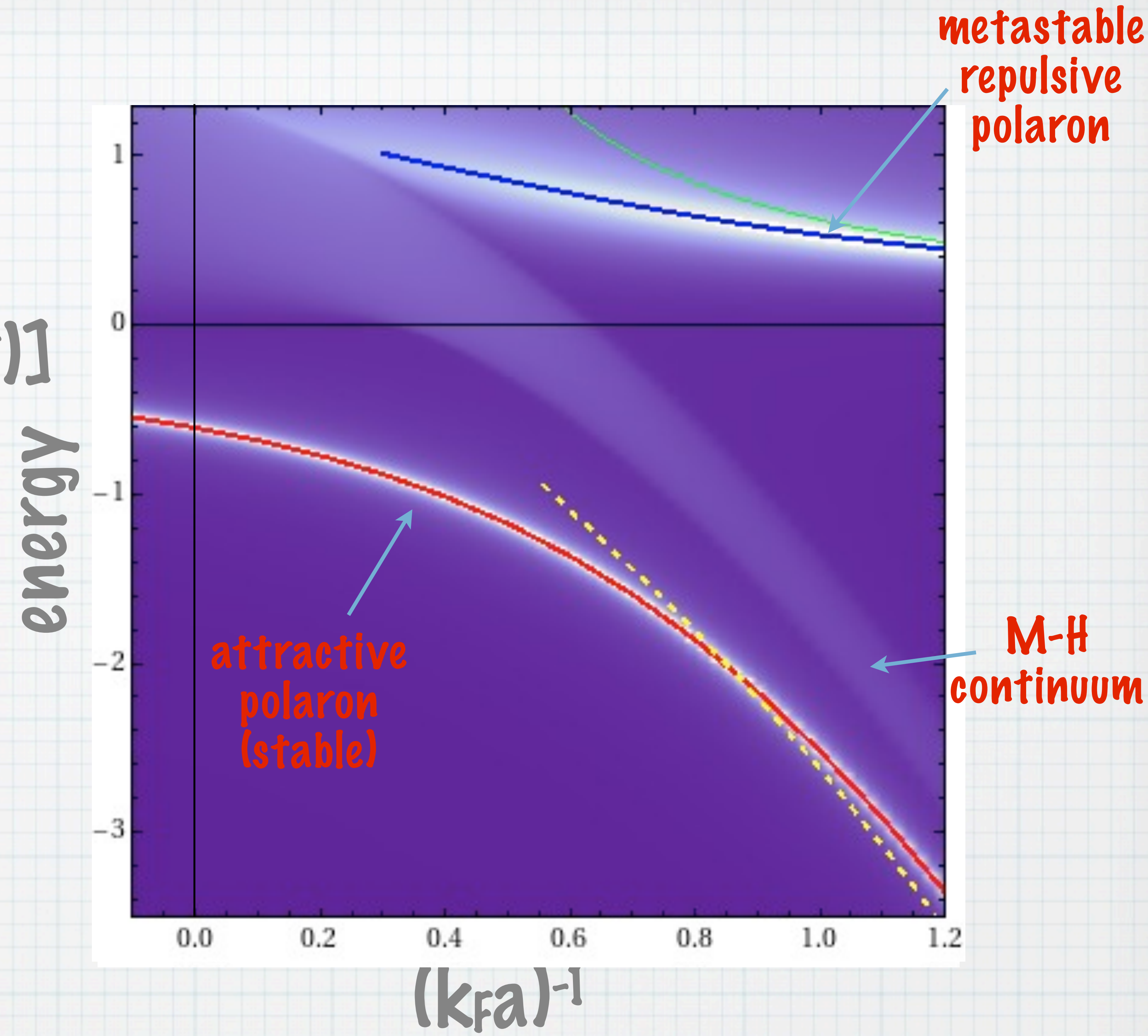
Schmidt & Enss, arXiv:1104.1379



spectral function

$A_\downarrow(\omega) = -\mathrm{Im}[G_\downarrow(k=0, \omega + i0^+)]$

$$E_M = -\frac{\hbar^2}{2m_r a^2} - \epsilon_F + \frac{2\pi\hbar^2 a_3}{m_3} n_\uparrow$$



$n_3 \approx 0$

RF spectrum: $$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] = -\frac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^3}\int \frac{d\epsilon}{2\pi}\left[f(\epsilon) - f(\epsilon+\omega)\right] A_2(\mathbf{k},\epsilon) A_3(\mathbf{k},\epsilon+\omega),$$

nonInt→Int: $$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] = -\frac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^3} f(\xi_{k,2}) A_\downarrow(\mathbf{k},\tilde{\omega}+\xi_{k,2}).$$

$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] \propto A_\downarrow(\mathbf{k},\tilde{\omega}+\xi_{k,2}).$$

a toy model with 3 FR

2-body bound states:

Polaronic states:

Decay rates:

• $a=0$

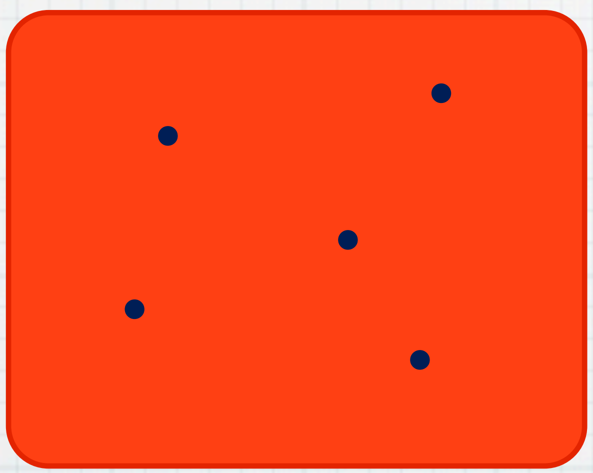
weak coupling:

$E \propto a$

$\Gamma \propto \Theta(a)$

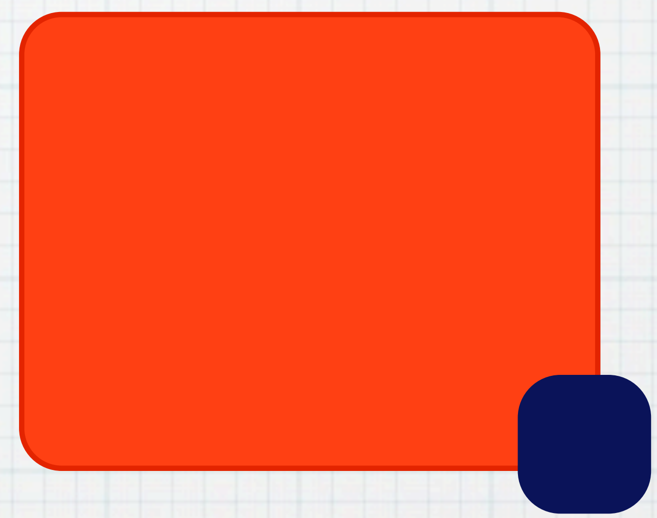
IFM
(Itinerant FerroMagnetism)

$\mu_\downarrow < E_F$



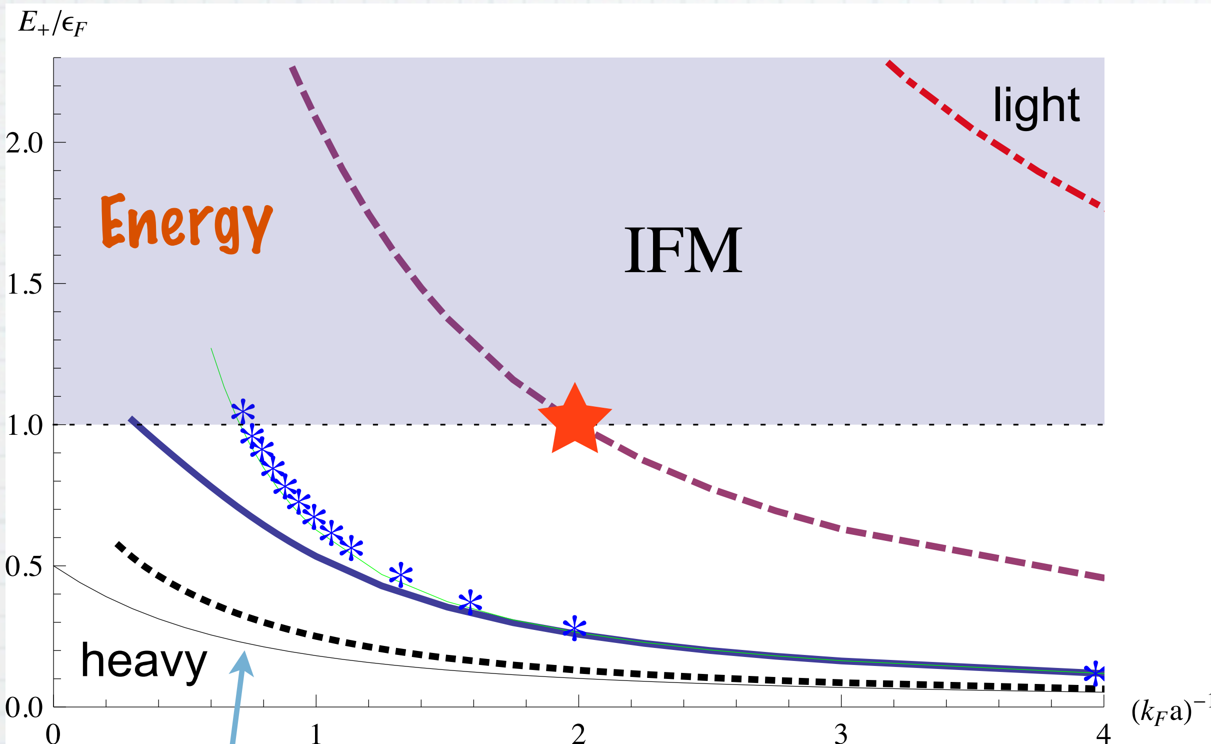
mixed state

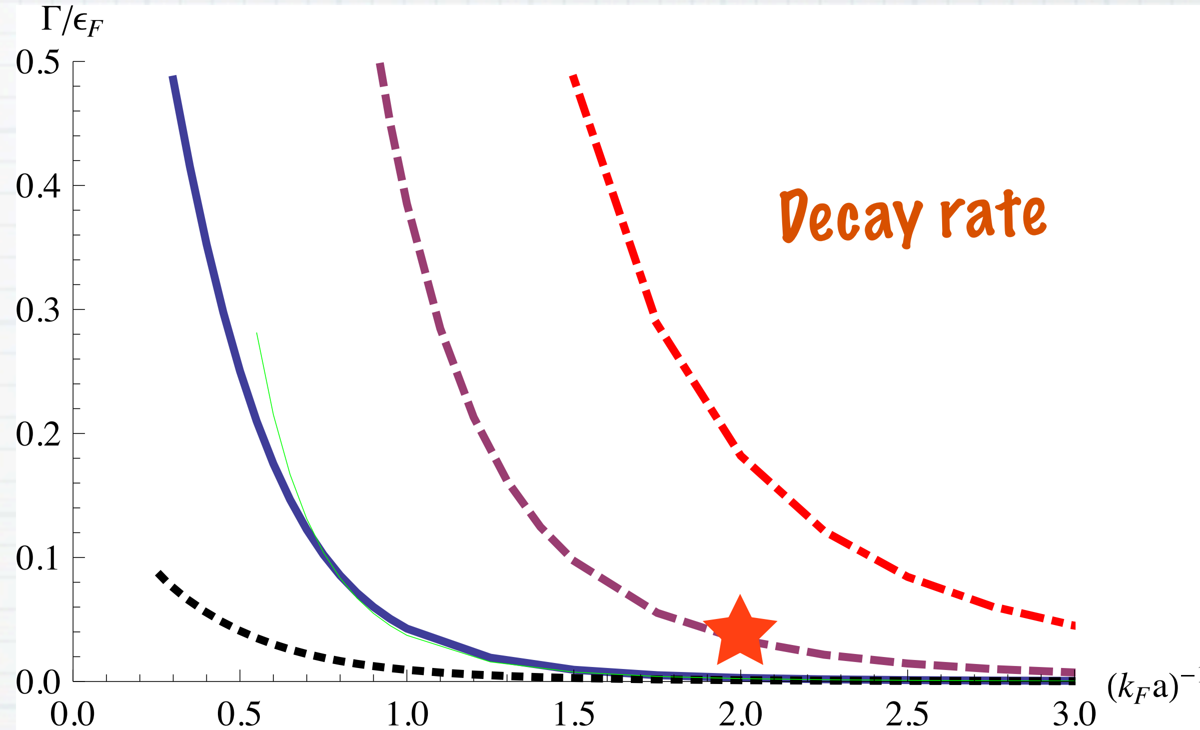
$\mu_\downarrow > E_F$



phase-separation

Repulsive polarons





Li impurities in a K gas:
IFM @ $k_F a=0.5$,
with $\Gamma/E_F=0.04$

$$m_\downarrow = \infty \; : \; \frac{E_\pm}{\epsilon_F} = -\frac{1}{\pi}\left\{(1+y^2)\left[\mp\frac{\pi}{2} + \arctan(y)\right] + y\right\}$$

	$m_\downarrow/m_\uparrow$
red dashed line	6/173
purple dashed line	6/40
solid blue line	1
black dashed line	40/6

Light impurities favour IFM

Narrow Feshbach Resonances

Scattering amplitude: $f = -\left[a^{-1} + ik + R^* k^2 + \ldots\right]^{-1}$

$$R^* = -\frac{r_e}{2} = \frac{\hbar^2}{2 m_r a_{\mathrm{bg}} \Delta B \delta\mu}$$

FR narrow if $R^* \gg R_{VdW}$

Most heteronuclear FR are (very) narrow!

$$E_M = -\frac{\hbar^2}{2 m_r a^*} \qquad \text{with} \quad a^* = \frac{2R^*}{\sqrt{1 + 4R^*/a} - 1}$$

$$a^* \left(\frac{R^*}{a} \ll 1\right) \sim a$$

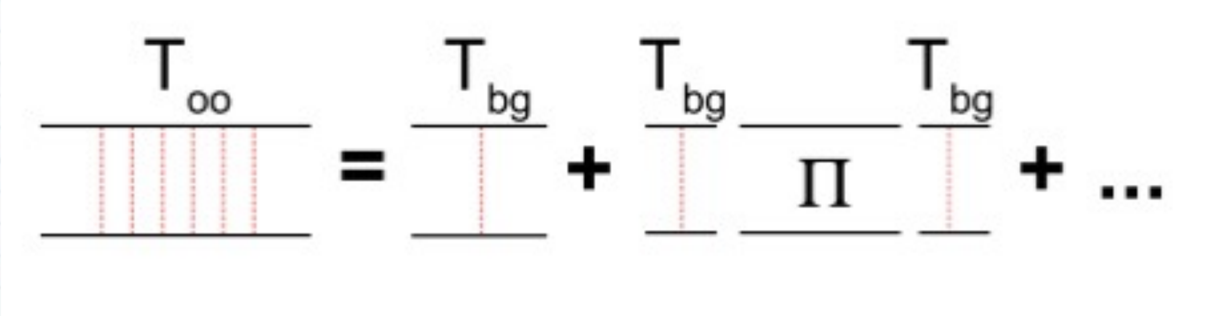
$$a^* \left(\frac{R^*}{a} \ll 1\right) \sim \sqrt{R^* a}$$

Many-body description of narrow FR

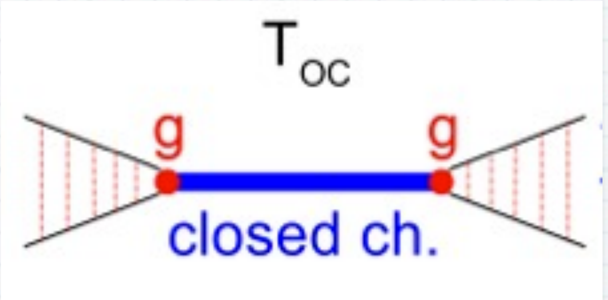
Bruun, Jackson & Kolomeitsev, PRA 2005
Massignan & Stoof, PRA 2008

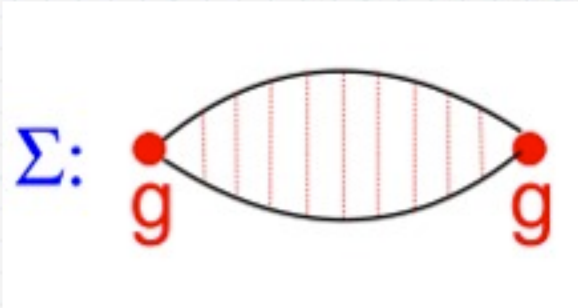
$$T = T_{\mathrm{OO}} + T_{\mathrm{OC}}$$

OO: open channel only



OC: involves coupling between open and closed channels





$$T_{OC} = \left(\frac{g}{1 - T_{bg}\Pi(E)}\right)^2 \frac{1}{E - \Delta\mu(B - B_0) - \Sigma(E)}$$

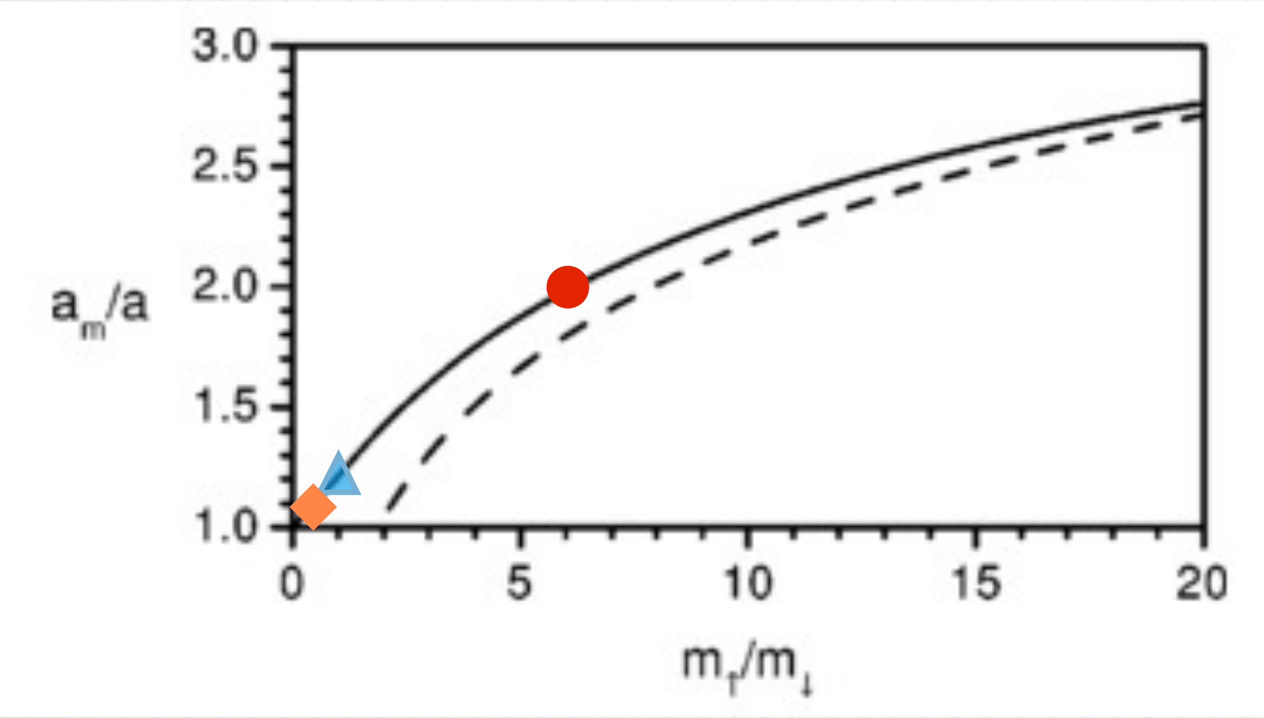
$$T = -\frac{2\pi\hbar^2}{m_r} f \quad \text{with} \quad f = -\left\{\left[a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0 - E_{\mathrm{CM}}/\delta\mu}\right)\right]^{-1} - \frac{2\pi\hbar^2}{m_r}\Pi(\mathbf{p}, E_{\mathrm{CM}})\right\}^{-1}$$

low energy expansion:

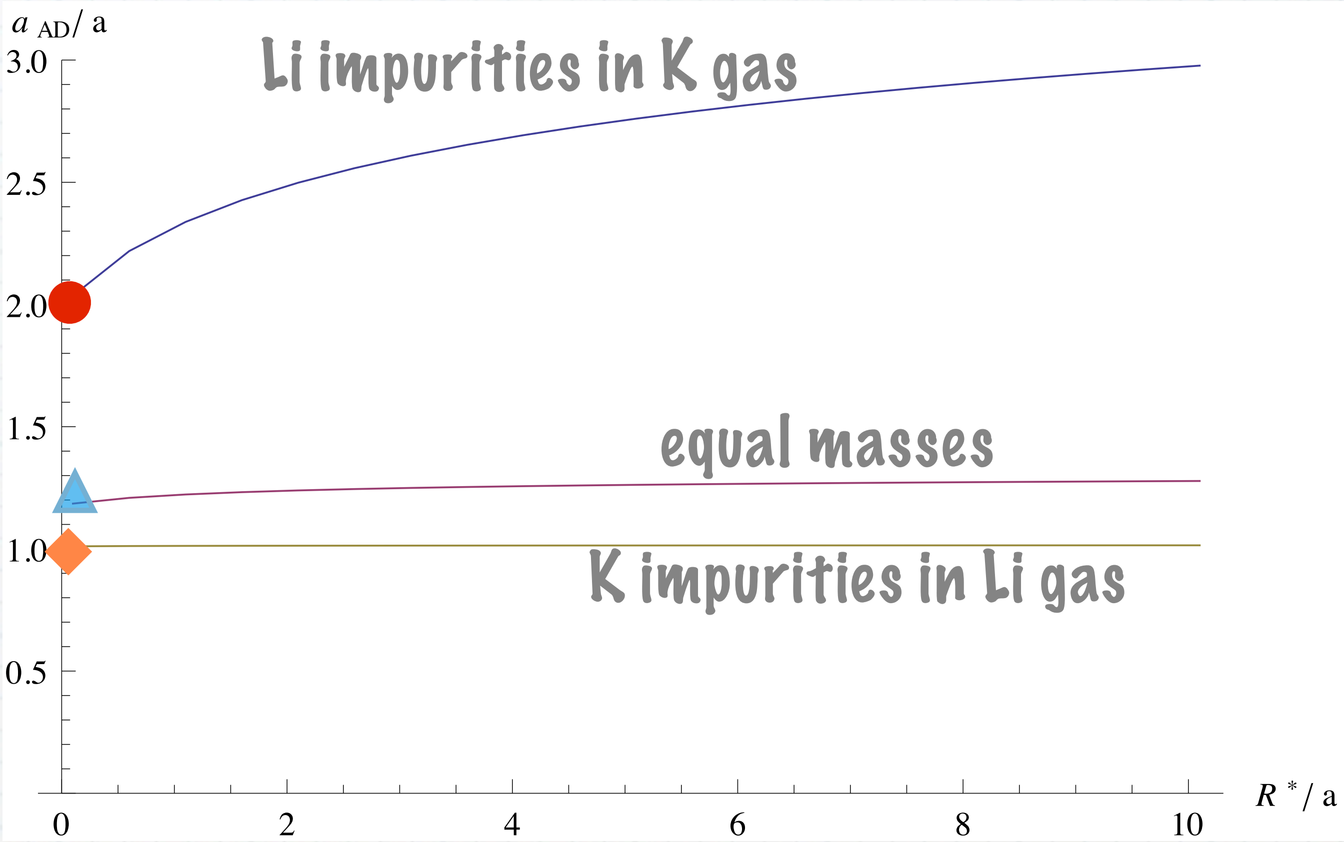
$$a^*(B) = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

$$R^*(B) = \frac{\hbar^2 \Delta B}{2 m_r a_{\mathrm{bg}} (B - B_0 - \Delta B)^2 \delta\mu}$$

Atom-Dimer scattering



Broad FR



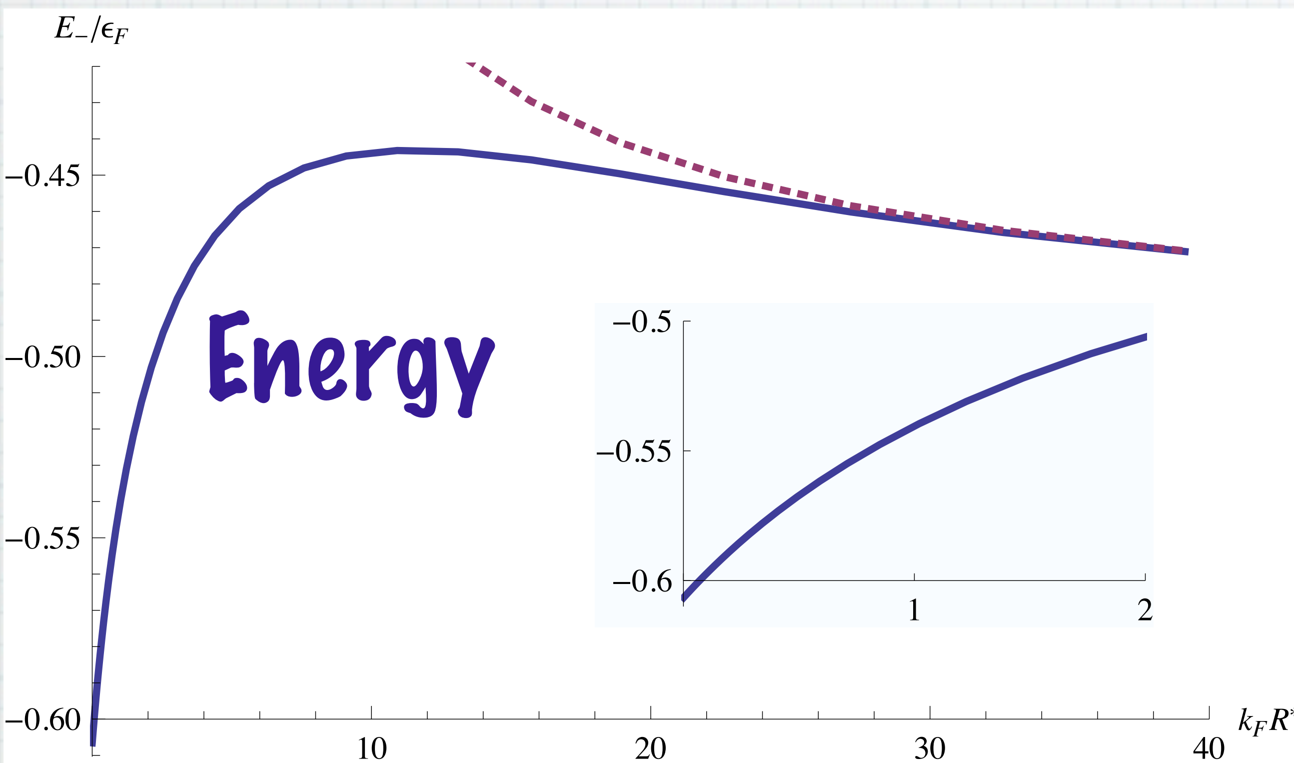
Narrow FR

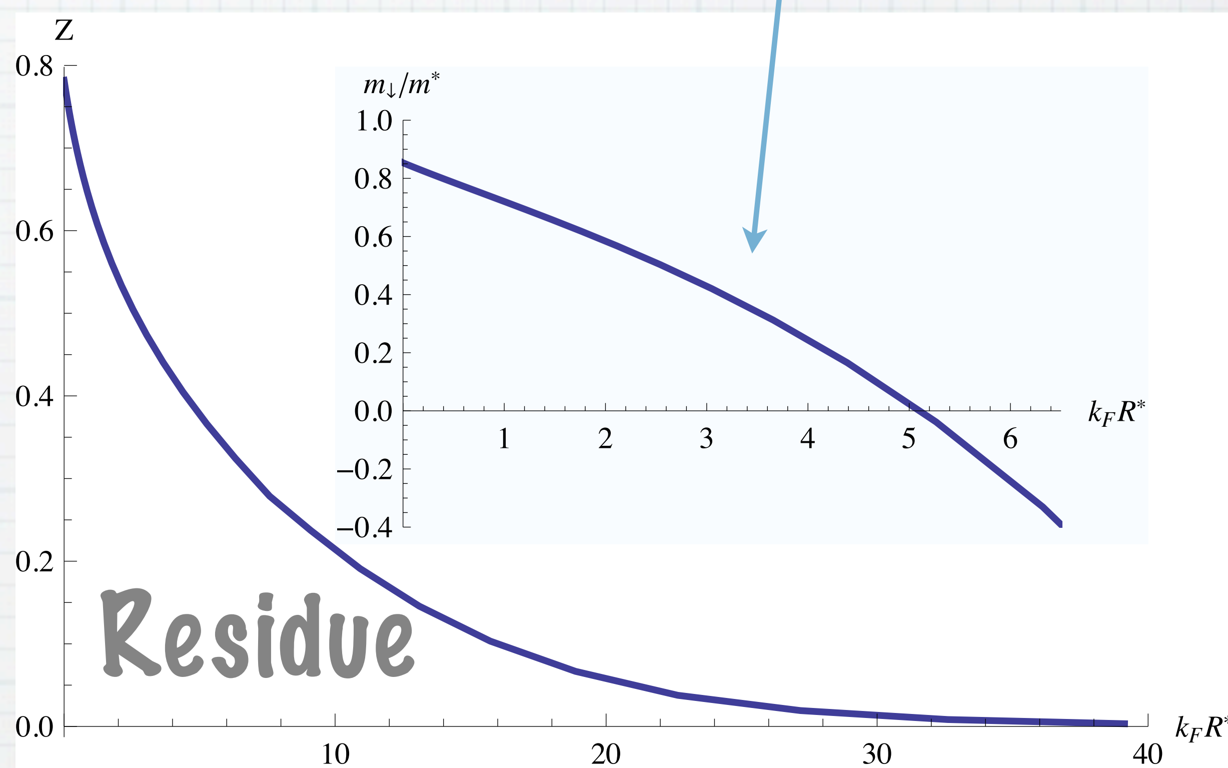
agrees with real-space calculation
(Petrov, PRA 2003; Petrov&Levinsen, arXiv: 1101.5979)

Attractive "narrow" polaron

at resonance ($a^{-1}=0$)
equal masses

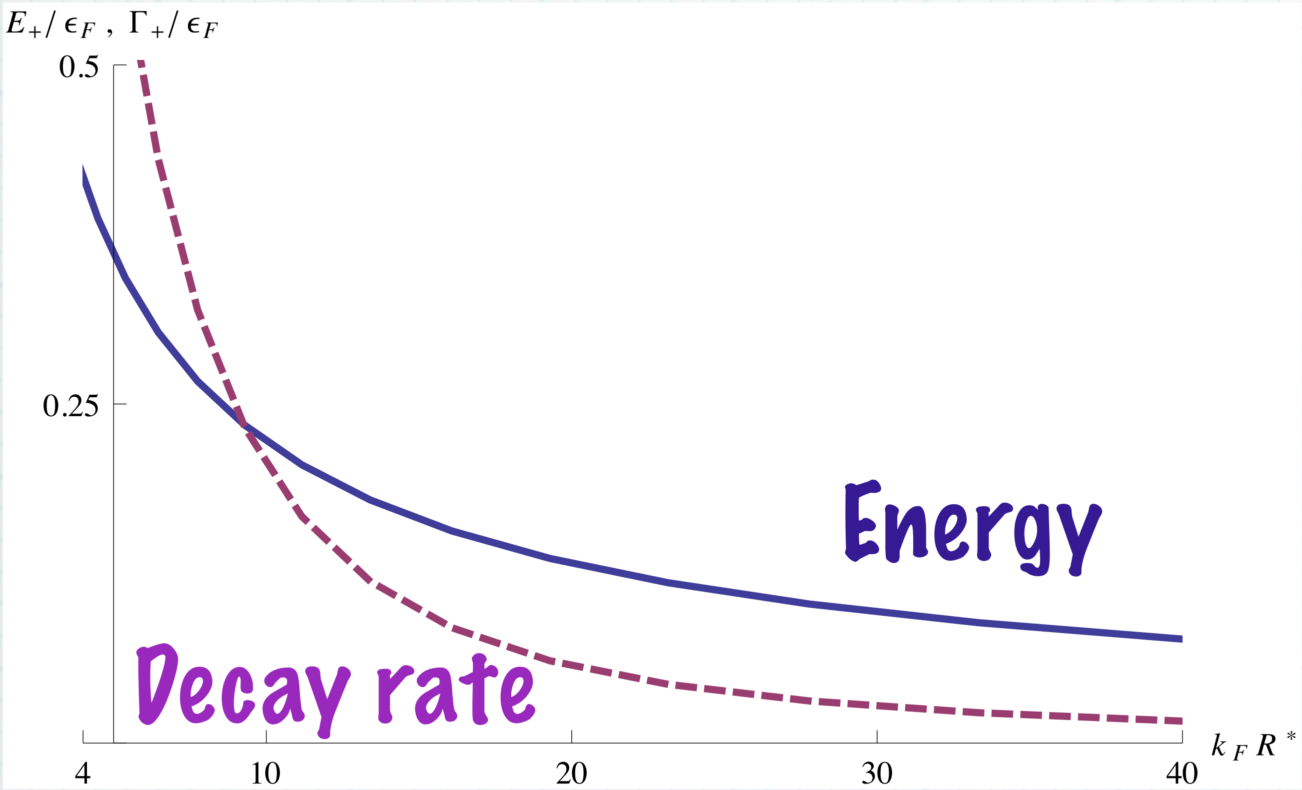
(inverse) effective mass

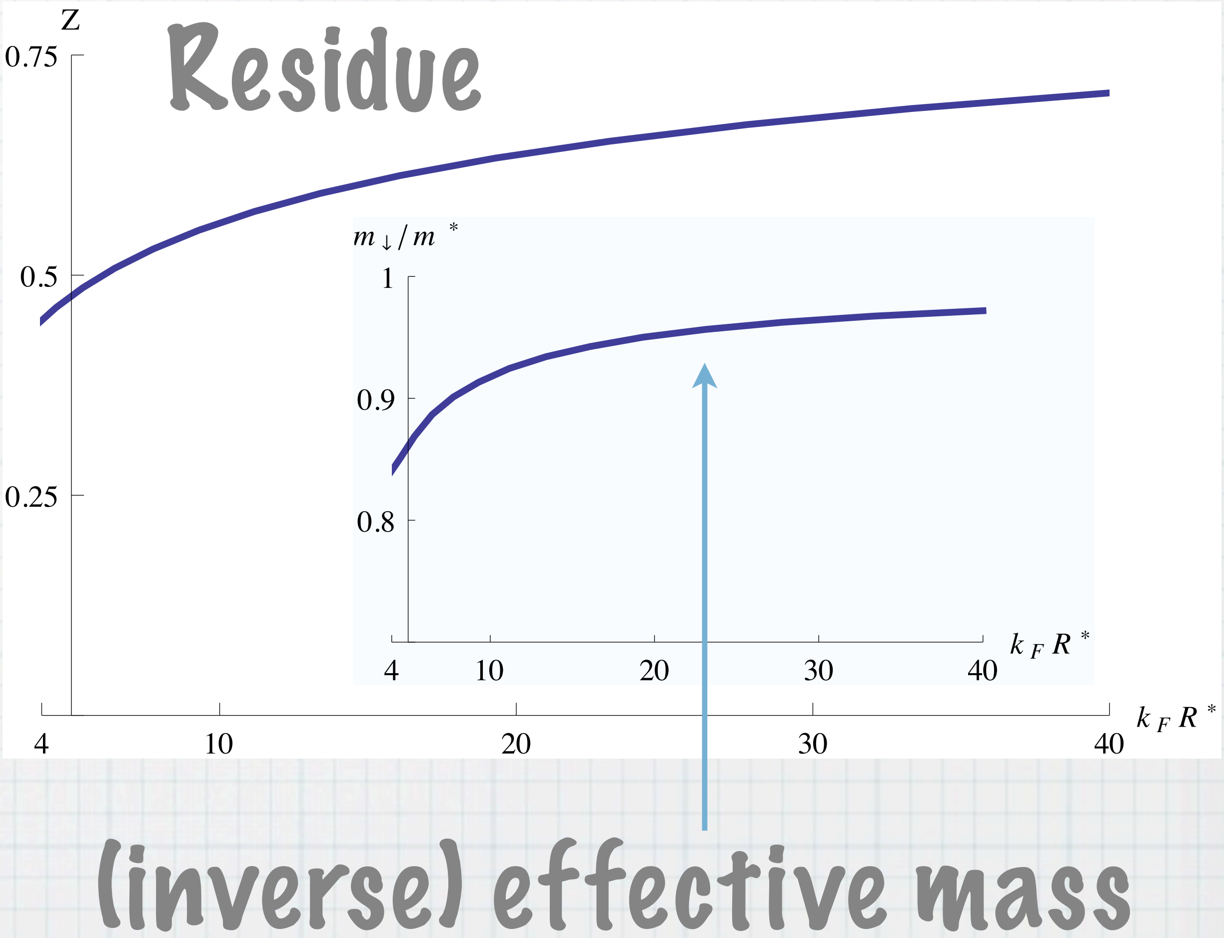




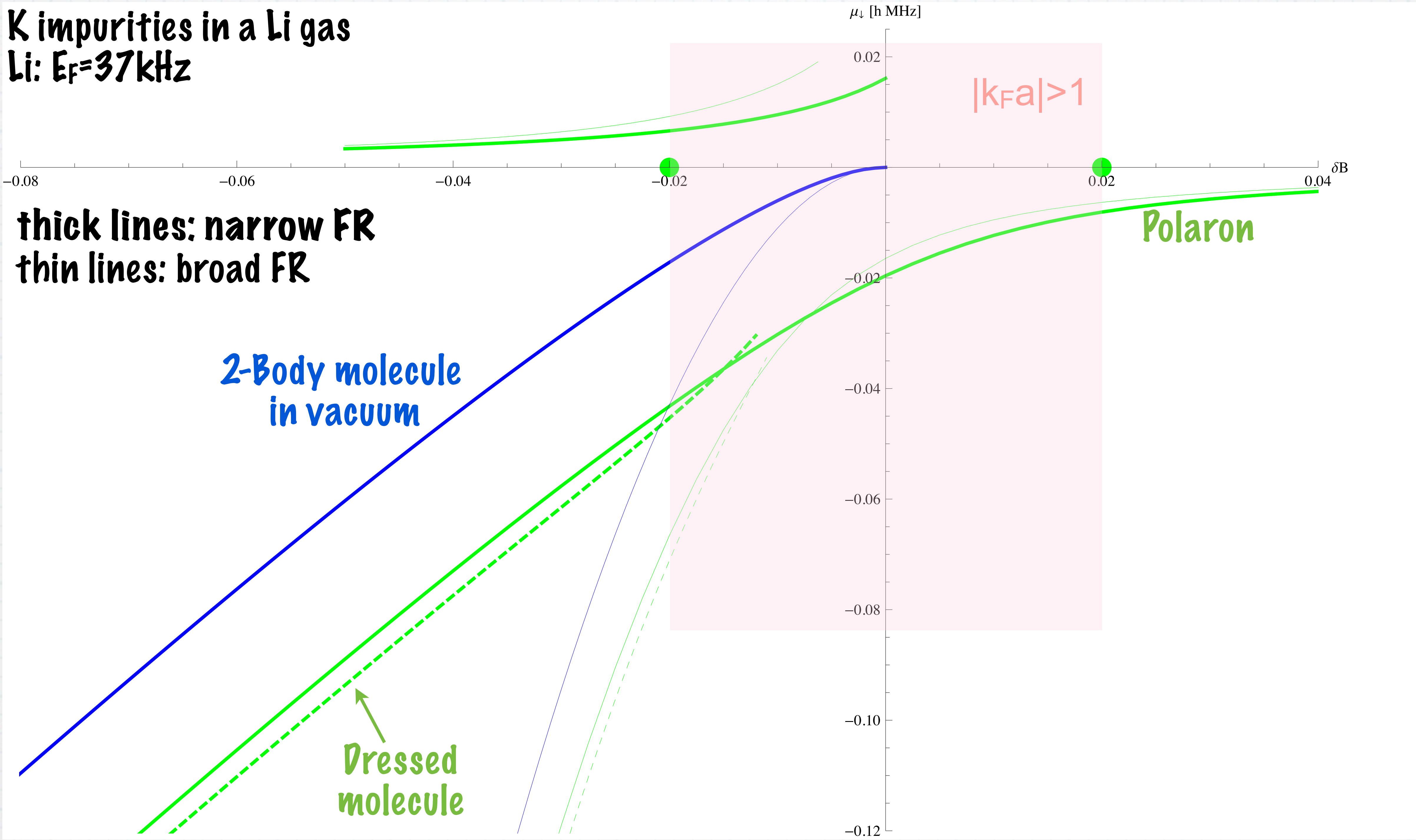
Repulsive "narrow" polaron

at resonance (a^{-1}=0)
equal masses

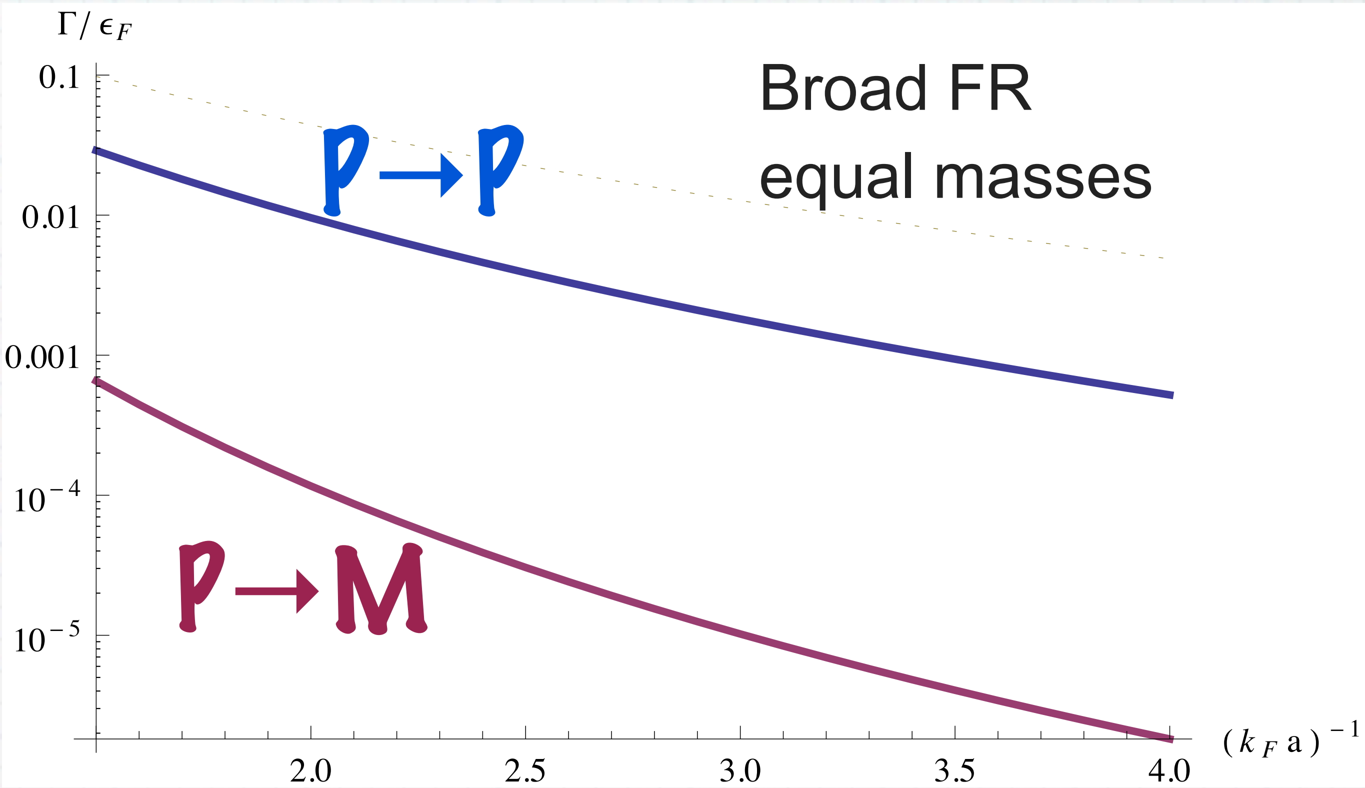




Dressed impurities at a narrow FR



Decay: analytics BEC



2-body: $$\Gamma_{(P_+ \to P_-)} \propto Z_- \frac{a^2}{a^*}$$

3-body: $$\Gamma_{(P_+ \to M)} \propto \frac{2\pi}{m_r^2 a^* \sqrt{1+4R^*/a}} T_{\rm vac}^2 (a^*)^5$$

Universal limit $\propto (k_F a)^6$

(agrees with Petrov, PRA 2003)

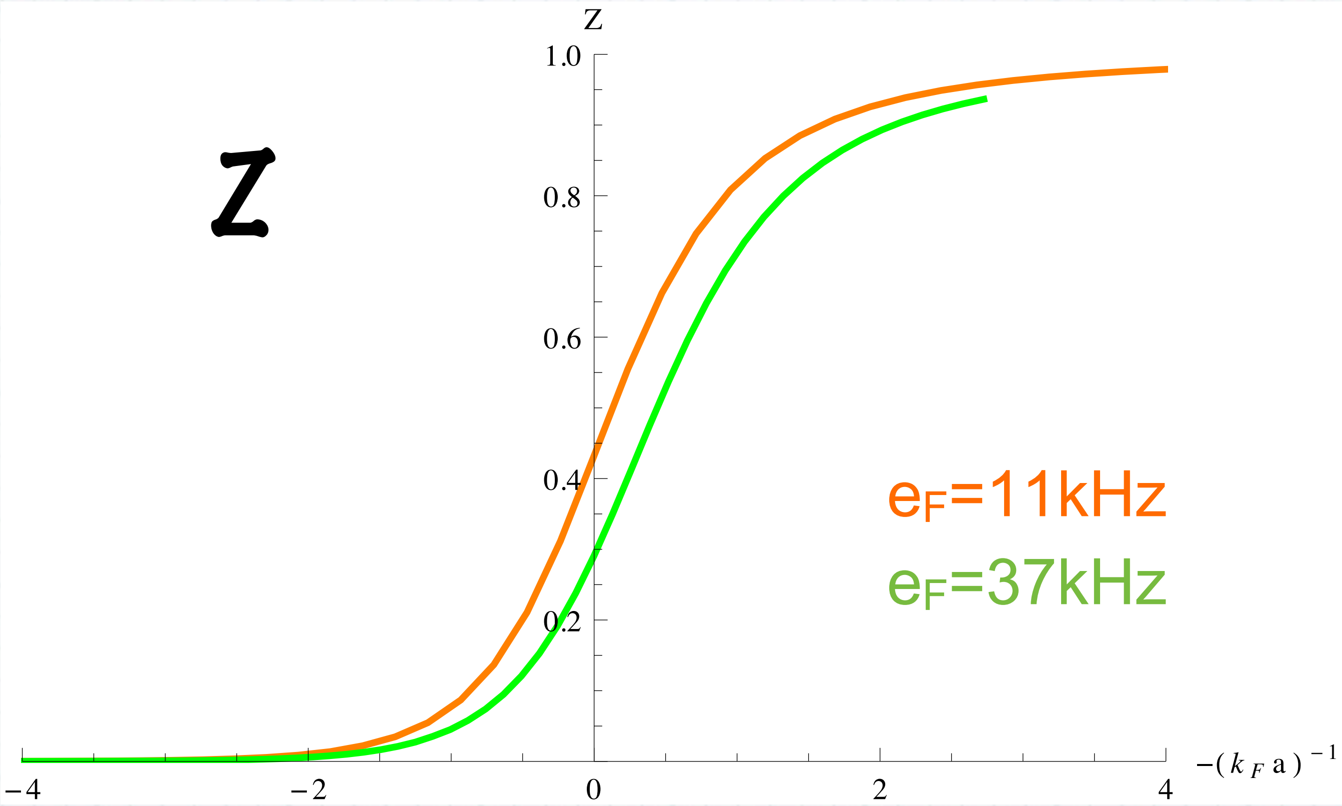
Narrow limit $\propto (a)^{9/2}(R^*)^{3/2}$

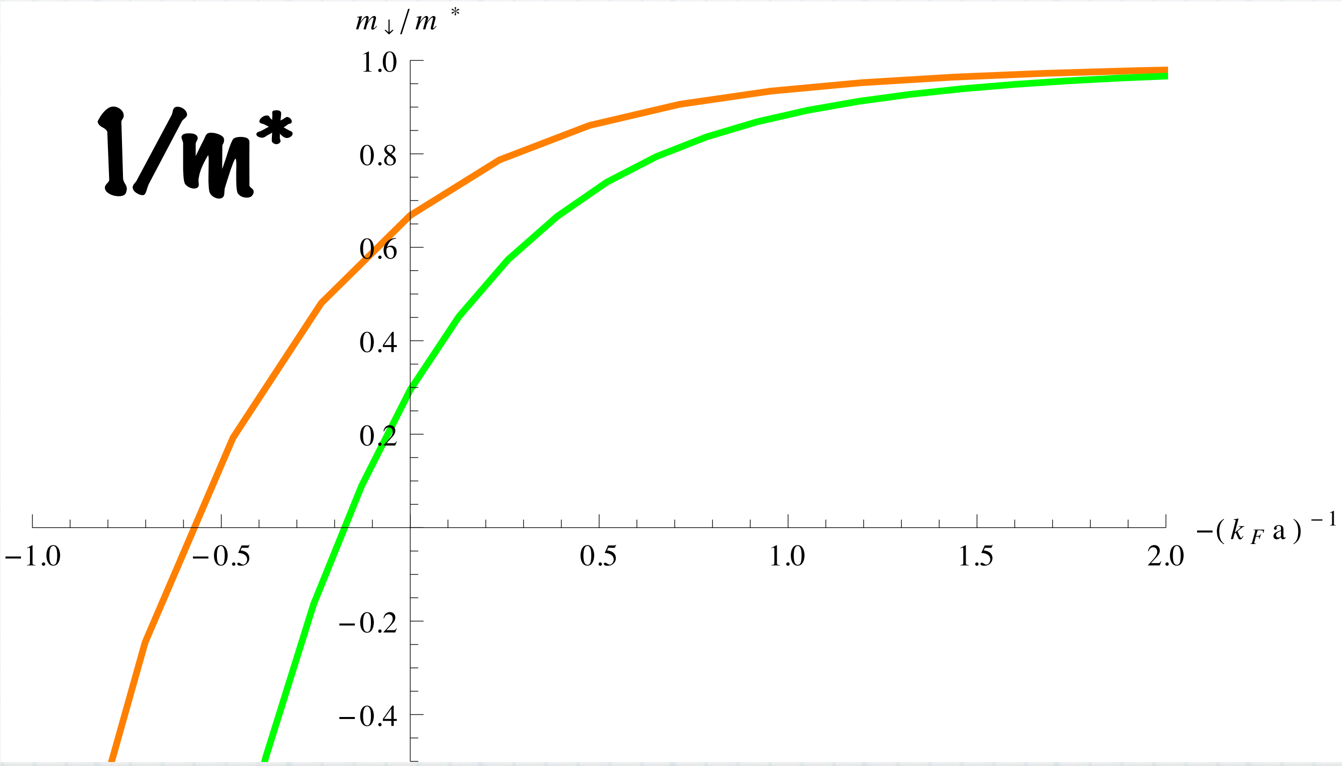
Conclusions

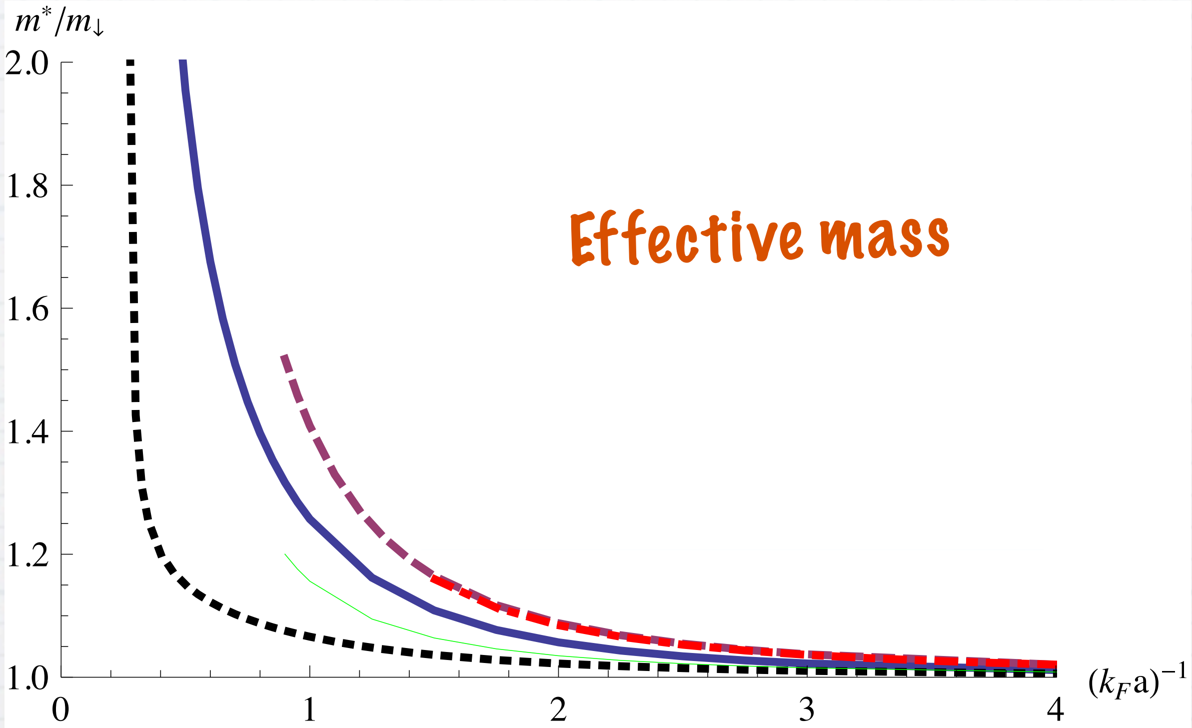
- Complete characterization of the repulsive branch: energy, residue, decay rate, m^*, ΔN, C_{imp}
- Itinerant FerroMagnetism easier to reach with light impurities
- RF spectra
- Many-body physics at narrow FR

I) G. Bruun and PM, PRL (2010)
II) K. Sadeghzadeh, G. Bruun, C. Lobo, PM, and A Recati, arXiv:Dec. 2010 (NJP in press)
III) PM and G. Bruun, arXiv:Feb 2011 (EPJD in press)
IV) more on narrow FR coming soon

K impurities in a Li gas







$m^*/m_\downarrow$
2.0
1.8
1.6
1.4
1.2
1.0
0
1
2
3
4
$(k_F a)^{-1}$
Effective mass

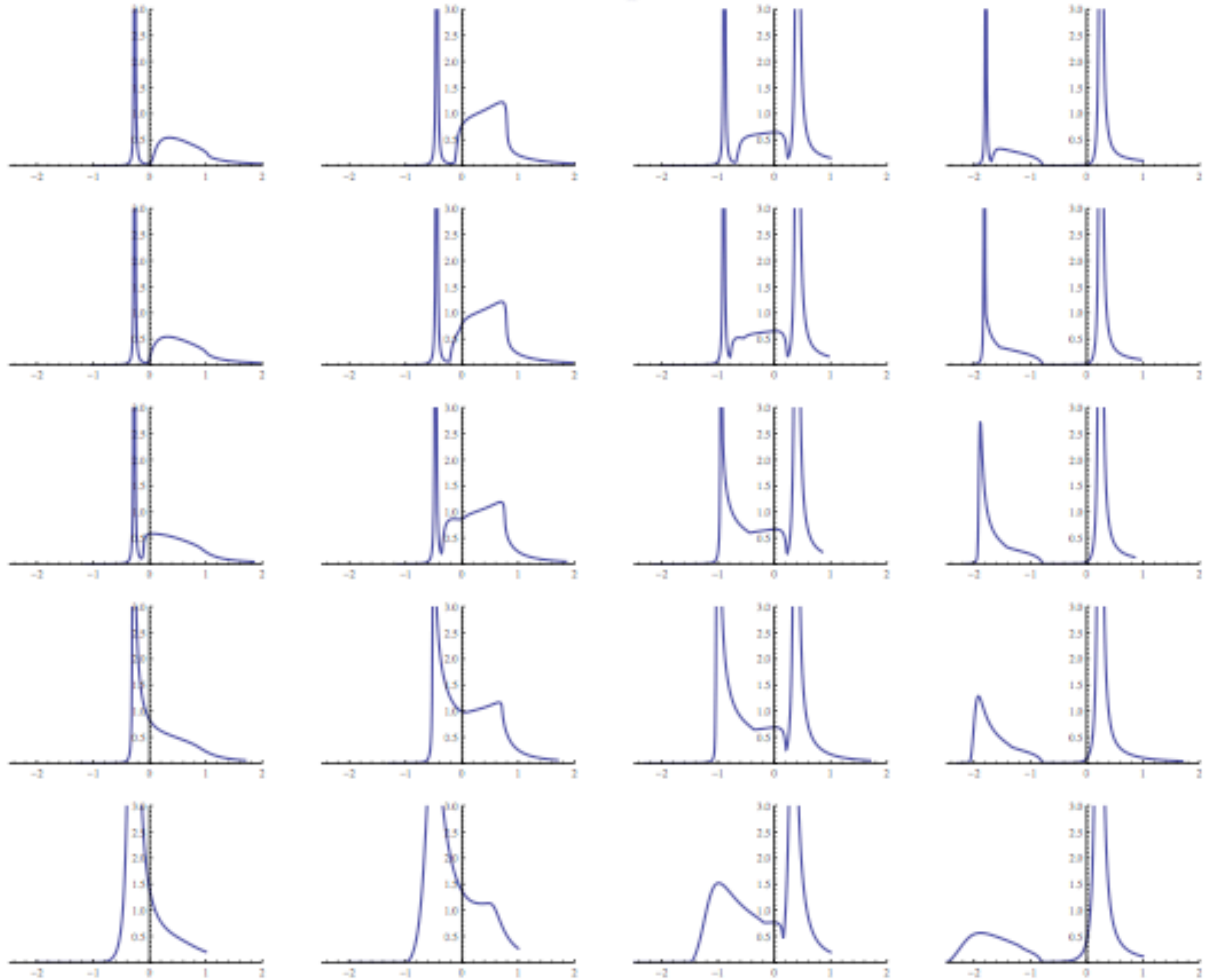


Figure 9: RF spectrum $A_\downarrow(\mathbf{k}, \tilde{\omega} + \xi_{k,2})$ of a single ^{40}K impurity with finite momentum, in a Fermi sea of ^{6}Li atoms. **From top to bottom**: $|\mathbf{p}|/k_{F\uparrow} = 0.1, 0.5, 1, \sqrt{2}, \sqrt{r}$.
From left to right: $(k_F a)^{-1} = -0.5, 0, 0.5, 1$.
The x-axis is the energy $\omega/E_{F\uparrow}$. For the fourth row, $p^2/(2m_\downarrow) = 3k_B T/2$, taking $k_B T = 0.2 E_{F\uparrow}$.
For the last row, $p^2/(2m_\downarrow) = E_{F\uparrow}$.

Pol→Mol decay

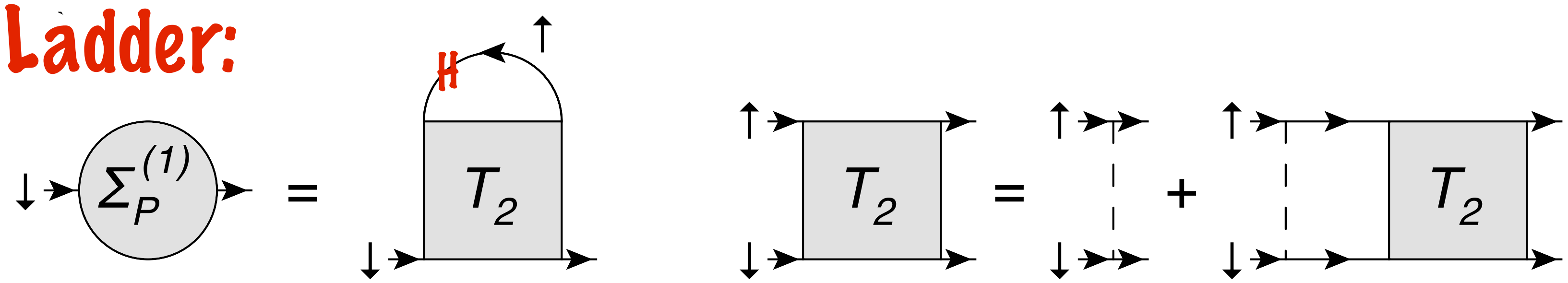
$$\Delta\omega = \omega_P - \omega_M > 0$$

Polaron: $G_\downarrow(\mathbf{p}, z)^{-1} = G^0_\downarrow(\mathbf{p}, z)^{-1} - \Sigma_P(\mathbf{p}, z)$

Decay rate: $\Gamma_P = -\mathrm{Im}\Sigma_P(p = 0, \omega_P)$

Hole expansion: $\Sigma_P(\mathbf{p}, z) = \Sigma_P^{(1)}(\mathbf{p}, z) + \Sigma_P^{(2)}(\mathbf{p}, z) + \dots$

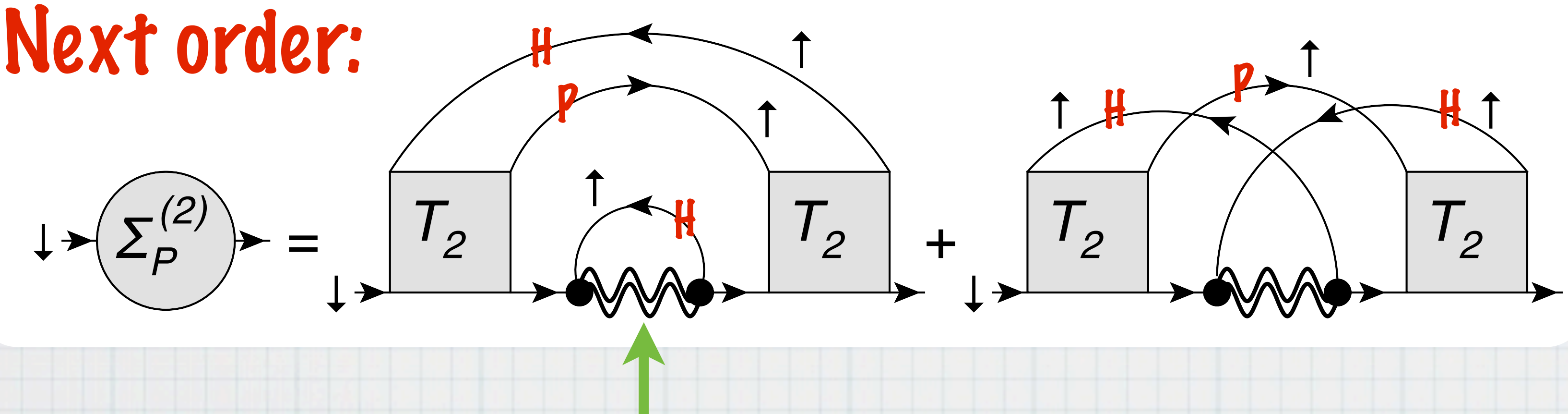
Ladder:



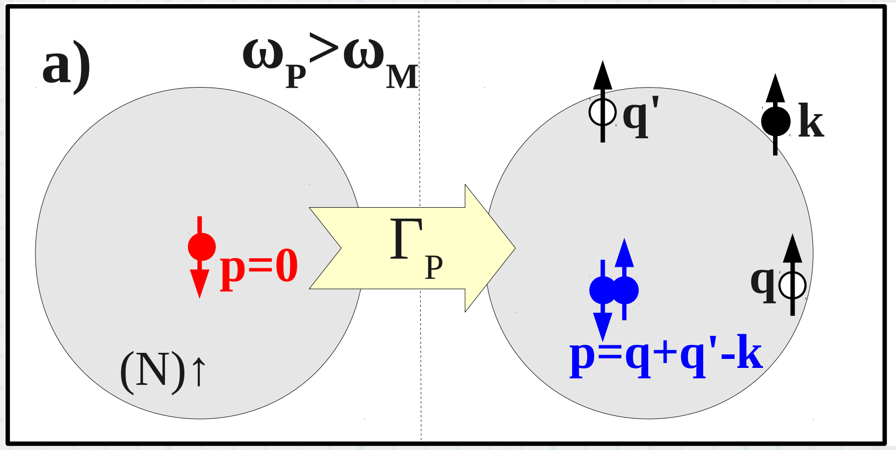
no damping
in the ladder approx.

3-body process

Next order:



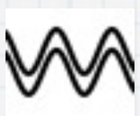
dressed molecule



molecule w.f. in vacuum:

$$\phi_q = \frac{\sqrt{8\pi a^3}}{1+q^2a^2} \quad \text{or} \quad \phi_r \propto \frac{\mathrm{e}^{-r/a}}{r}$$

dressed molecule:



$$D(\mathbf{p},\omega) \simeq \frac{Z_M}{\omega - \omega_M - p^2/2m_M^*}.$$

atom-molecule coupling:

$$\bullet = \frac{1}{g(\mathbf{p},z)} = \int \frac{d^3q}{(2\pi)^3}\phi_q \frac{1-f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z-\xi_{\mathbf{p}-\mathbf{q}\uparrow}-\xi_{q\downarrow}} \sim -\sqrt{\frac{m_r^2 a}{2\pi}}$$

(Bruun&Pethick, PRL 2004)

$$\Gamma_P = \frac{g^2 Z_M}{2}\int \frac{d^3k\; d^3q\; d^3q'}{(2\pi)^9}\left[F(\mathbf{q},\mathbf{k},\omega_P) - F(\mathbf{q}',\mathbf{k},\omega_P)\right]^2 \delta\left(\Delta\omega + \xi_{q\uparrow} + \xi_{q'\uparrow} - \xi_{k\uparrow} - \frac{(\mathbf{q}+\mathbf{q}'-\mathbf{k})^2}{2m_M^*}\right)$$

$$q, q' < k_F \;,\; k > k_F$$

$$F(\mathbf{q},\mathbf{k},\omega) = T_2(\mathbf{q},\omega+\xi_{q\uparrow})G_\downarrow^0(\mathbf{q}-\mathbf{k},\omega+\xi_{q\uparrow}-\xi_{k\uparrow})$$

Fermi's Golden rule

atom-molecule coupling

$$\Gamma_P = \frac{g^2 Z_M}{2} \int \frac{d^3k\ d^3q\ d^3q'}{(2\pi)^9} \left[F(\mathbf{q},\mathbf{k},\omega_P) - F(\mathbf{q}',\mathbf{k},\omega_P)\right]^2 \delta\left(\Delta\omega + \xi_{q\uparrow} + \xi_{q'\uparrow} - \xi_{k\uparrow} - \frac{(\mathbf{q}+\mathbf{q}'-\mathbf{k})^2}{2m_M^*}\right)$$

matrix element~$\Delta\omega$

density of final states~$\Delta\omega^{7/2}$

$$q, q' < k_F \ , \ k > k_F$$

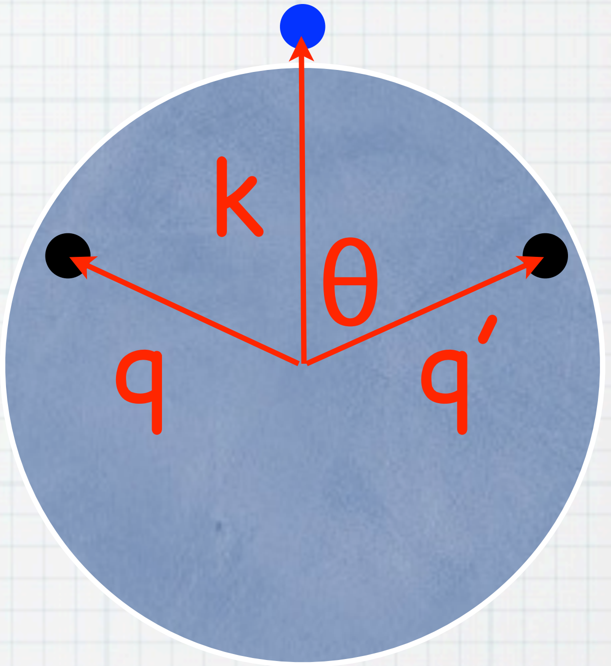
$$F(\mathbf{q},\mathbf{k},\omega) = T_2(\mathbf{q}, \omega + \xi_{q\uparrow}) G^0_\downarrow(\mathbf{q}-\mathbf{k}, \omega + \xi_{q\uparrow} - \xi_{k\uparrow})$$

In the neighborhood of the P-M crossing,

$\Delta\omega \ll \epsilon_F$

$q \simeq k \simeq k' \simeq k_F$

$$\int \frac{d^3k\; d^3q\; d^3q'}{(2\pi)^9}\delta\left(\ldots\right) \sim (m^*_M)^{3/2}(\Delta\omega)^{7/2}$$



the angular dependence of F is only on θ

The P+H+H form an equilateral triangle, since $q + q' - k \sim 0$

At the crossing, Fermi antisymmetry yields a vanishing of the matrix element:

$$F(\mathbf{q}, \mathbf{k}, \omega_P) - F(\mathbf{q}', \mathbf{k}, \omega_P)$$

Expand the difference to get an extra factor of Δω:

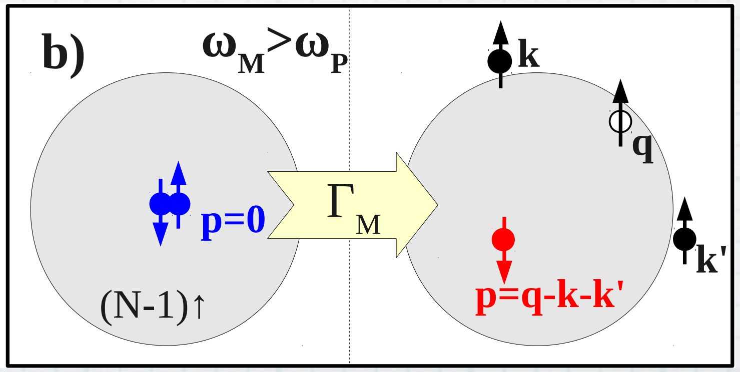
$$\Gamma_P \sim Z_M(k_F a)\,(m^*_M)^{3/2}\,(\Delta\omega)^{9/2}$$

1st order transition between the P&M states (no coupling at the crossing)

Mol→Pol decay

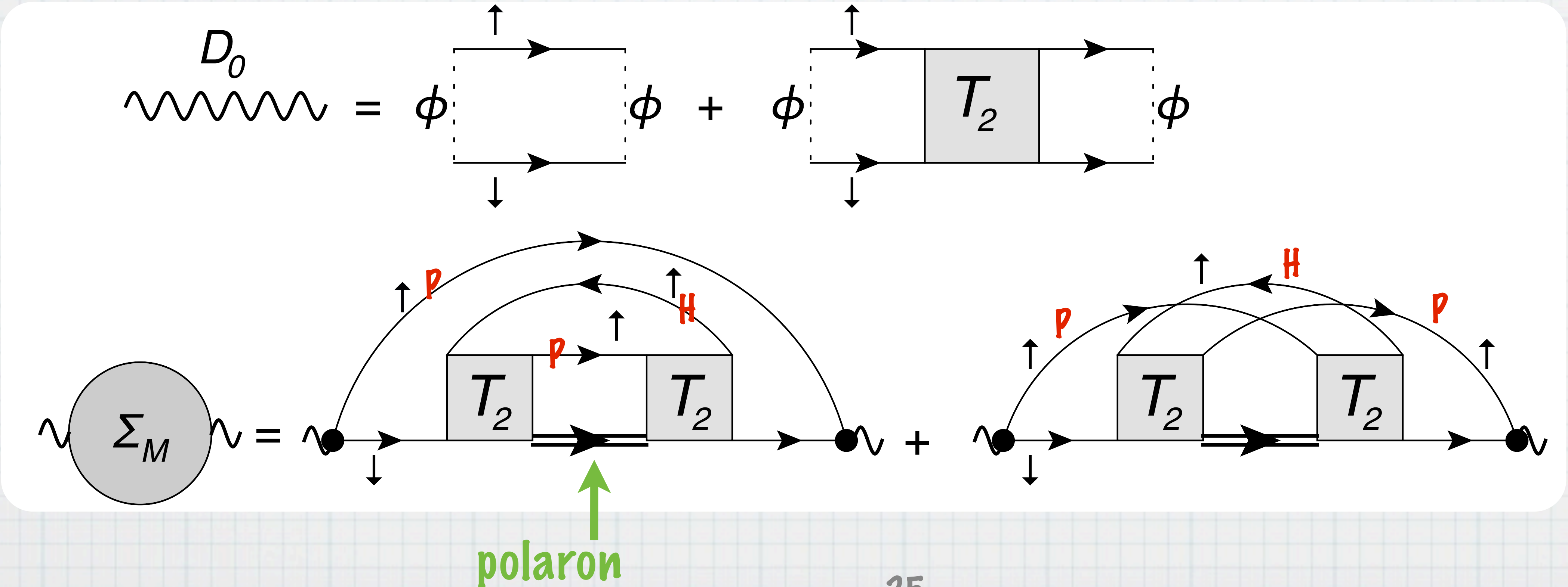
$$\Delta\omega = \omega_P - \omega_M < 0$$

Molecule: $D(\mathbf{p},z)^{-1} = D_0(\mathbf{p},z)^{-1} - \Sigma_M(\mathbf{p},z)$



Decay rate: $\Gamma_M = -\mathrm{Im}\Sigma_M(p=0,\omega_M)$

Vacuum: $$D_0(\mathbf{p},z) = \int d^3\check{q}\,\phi_q^2 \frac{1-f(\xi_{\mathbf{p-q}\uparrow})}{z-\xi_{\mathbf{p-q}\uparrow}-\xi_{q\downarrow}} + \frac{T_2(\mathbf{p},z)}{g(\mathbf{p},z)^2}$$



3-body process

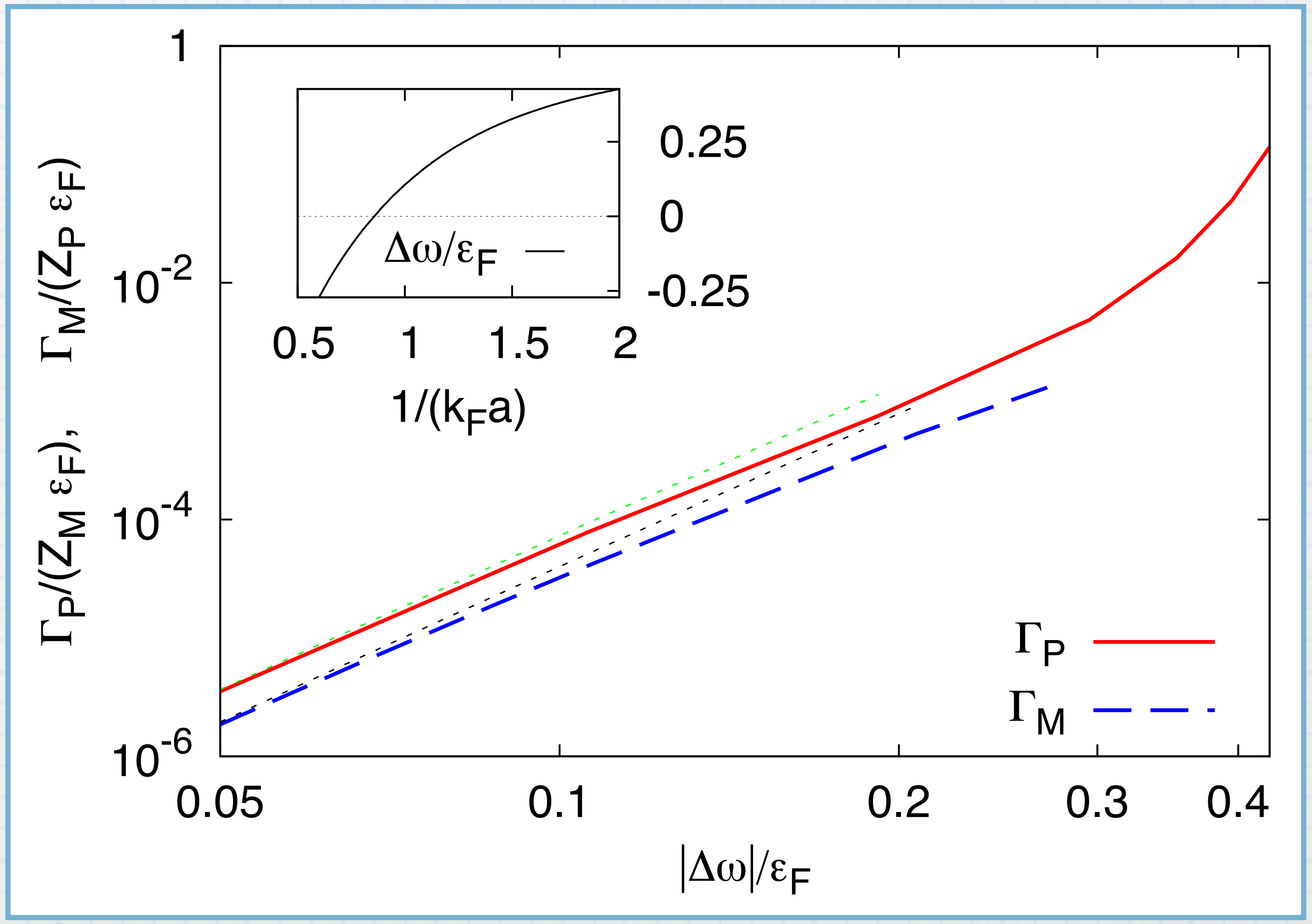
$$\Gamma_M = \frac{g^2 Z_P}{2} \int \frac{d^3k\ d^3k'\ d^3q}{(2\pi)^9} \left[C(\mathbf{q},\mathbf{k},\omega_M) - C(\mathbf{q},\mathbf{k}',\omega_M)\right]^2 \delta\left(|\Delta\omega| + \xi_{q\uparrow} - \xi_{k\uparrow} - \xi_{k'\uparrow} - \frac{(\mathbf{q}-\mathbf{k}-\mathbf{k}')^2}{2m_P^*}\right)$$

In the neighborhood of the M-P crossing, $\Gamma_M \sim Z_P(k_F a)\,(m_P^*)^{3/2}\,(-\Delta\omega)^{9/2}$

For both decay processes,
very **long lifetimes** are ensured by:

- limited phase-space
- Fermi antisymmetry

much longer than usual Fermi liquids



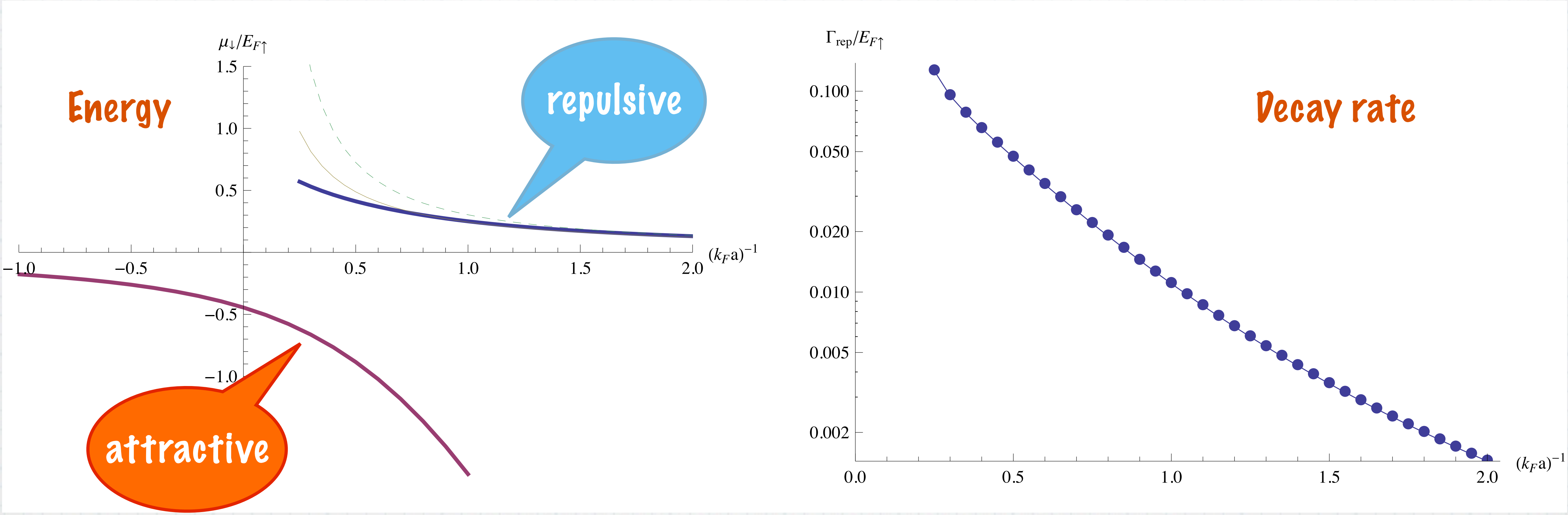
In the numerics:

$$\omega_M = -\frac{\hbar^2}{2m_r a^2} - \epsilon_F + g_3 n_\uparrow$$

$$a_3 = 1.18a$$

$$T_2(\mathbf{p},\omega) = \frac{2\pi a/m_r}{1 - \sqrt{2m_r a^2\left(\frac{p^2}{2m_M} - \omega - \epsilon_F + g_3 n_\uparrow\right)}}$$

Repulsive polaron



A ^{40}K impurity in a Fermi sea of ^{6}Li

atom-molecule coupling:

$$\bullet = \frac{1}{g(\mathbf{p}, z)} = \int \frac{d^3q}{(2\pi)^3} \phi_q \frac{1 - f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z - \xi_{\mathbf{p}-\mathbf{q}\uparrow} - \xi_{q\downarrow}} \sim -\sqrt{\frac{m_r^2 a}{2\pi}}$$

Vacuum:

$$D_0(\mathbf{p}, z) = \int d^3\check{q}\phi_q^2 \frac{1 - f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z - \xi_{\mathbf{p}-\mathbf{q}\uparrow} - \xi_{q\downarrow}} + \frac{T_2(\mathbf{p}, z)}{g(\mathbf{p}, z)^2}$$

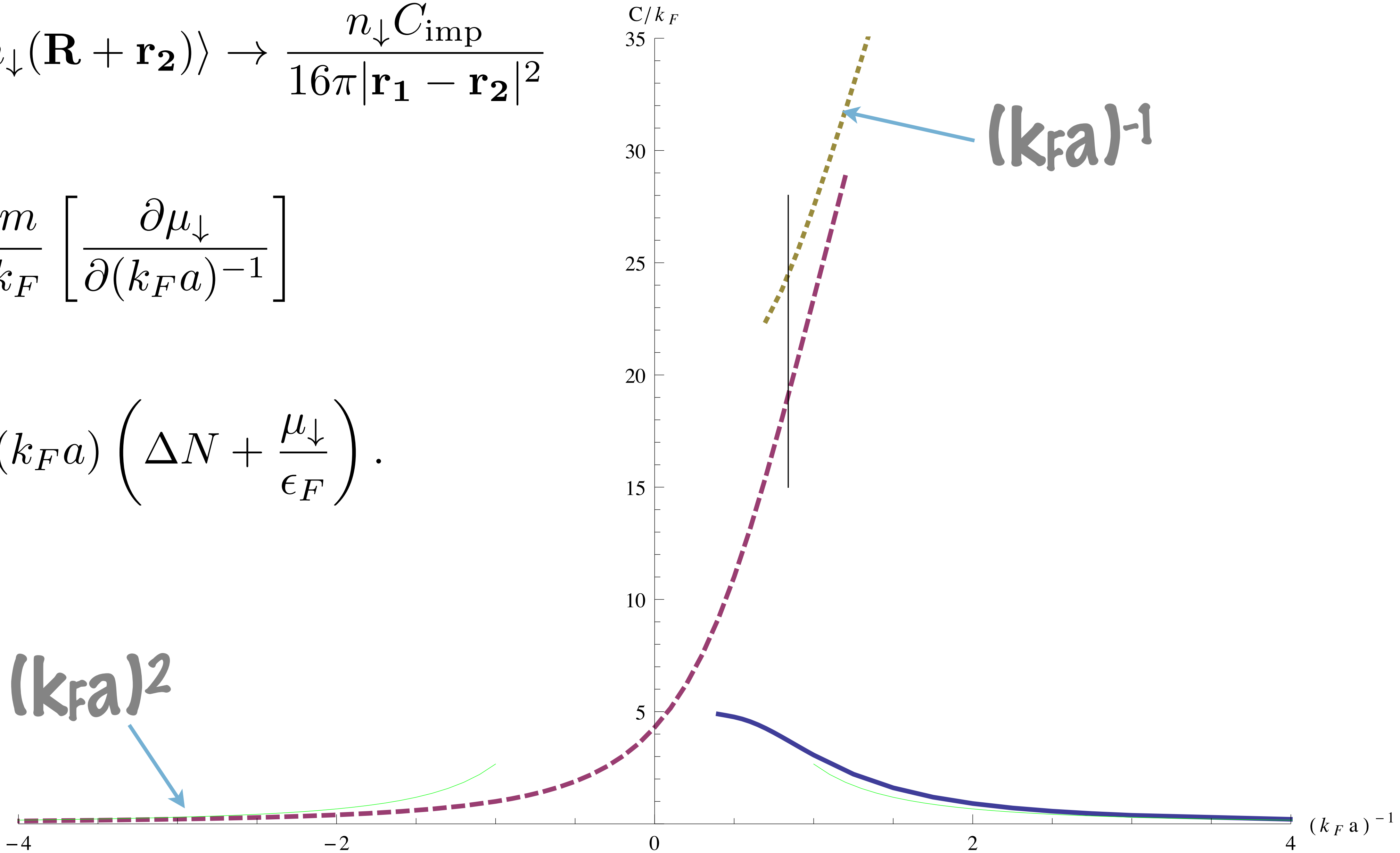
Tan's contact

C_{imp}: contact density per impurity

$$\langle n_\uparrow(\mathbf{R}+\mathbf{r_1})n_\downarrow(\mathbf{R}+\mathbf{r_2})\rangle \to \frac{n_\downarrow C_{\mathrm{imp}}}{16\pi|\mathbf{r_1}-\mathbf{r_2}|^2}$$

$$C_{\mathrm{imp}} = -\frac{4\pi m}{\hbar^2 k_F}\left[\frac{\partial \mu_\downarrow}{\partial (k_F a)^{-1}}\right]$$

$$\frac{C_{\mathrm{imp}}}{k_F} = -4\pi(k_F a)\left(\Delta N + \frac{\mu_\downarrow}{\epsilon_F}\right).$$



RF spectra

↑: 1

↓: 2,3

$\omega_0 = \varepsilon_3 - \varepsilon_2 > 0$

$n_3 \approx 0$

$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] = -\frac{1}{2}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}\int \frac{\mathrm{d}\epsilon}{2\pi}\left[f(\epsilon) - f(\epsilon+\omega)\right] A_2(\mathbf{k},\epsilon) A_3(\mathbf{k},\epsilon+\omega),$$

MIT: Int➔nonInt

$$A_3(\mathbf{k},\omega) = 2\pi\delta(\omega - \xi_{k,3})$$

$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] = -\frac{1}{2}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3} f(\tilde{\omega} - \xi_{k,2}) A_\downarrow(\mathbf{k}, \tilde{\omega} - \xi_{k,2})$$

$$\tilde{\omega} = \omega - \mu_2 + \mu_3$$

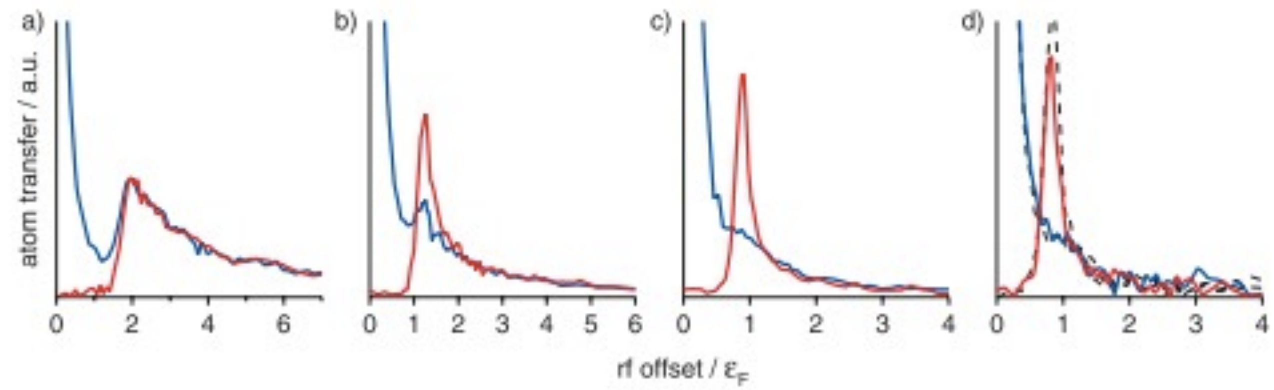


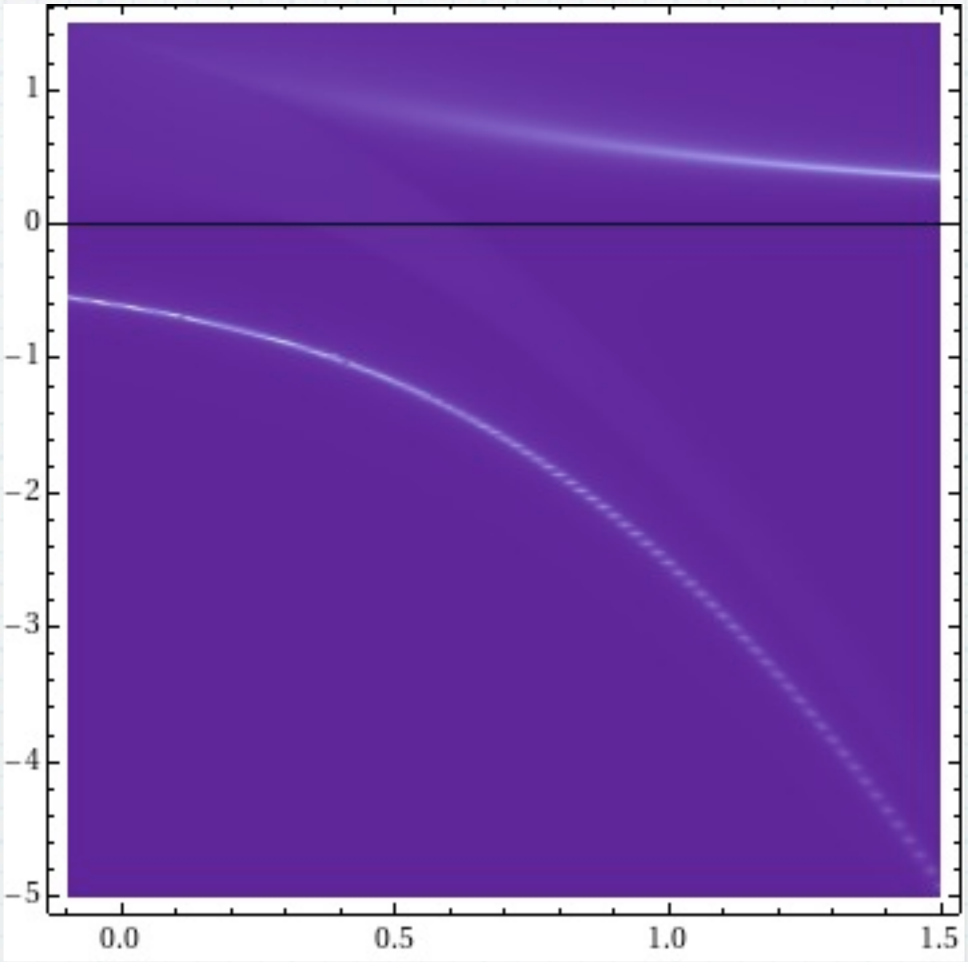
FIG. 2 (color online). rf spectroscopy on polarons. Shown are spatially resolved, 3D reconstructed rf spectra of the environment (blue, state $|1\rangle$) and impurity (red, state $|3\rangle$) component in a highly imbalanced spin-mixture. (a) Molecular limit; (b),(c) Emergence of the polaron, a distinct peak exclusively in the minority component. (d) At unitarity, the polaron peak is the dominant feature in the impurity spectrum, which becomes even more pronounced for $1/k_F a < 0$ (not shown). For the spectra shown as dashed lines in (d) the roles of states $|1\rangle$ and $|3\rangle$ are exchanged. The local impurity concentration was $x = 5(2)\%$ for all spectra, the interaction strengths $1/k_F a$ were (a) 0.76(2), (b) 0.43(1), (c) 0.20(1), and (d) 0 (unitarity).

Innsbruck: nonInt→Int

$$A_2(\mathbf{k},\omega) = 2\pi\delta(\omega - \xi_{k,2})$$

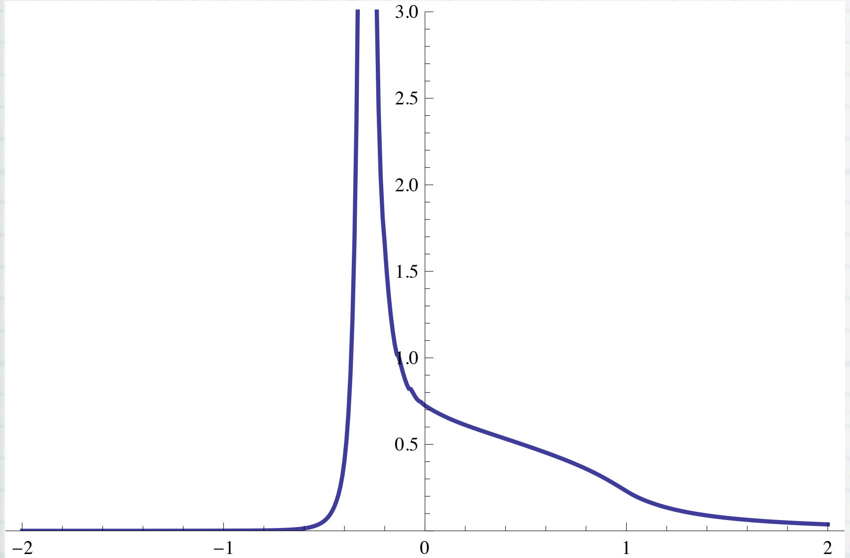
$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] = -\frac{1}{2}\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3} f(\xi_{k,2}) A_{\downarrow}(\mathbf{k},\tilde{\omega}+\xi_{k,2}).$$

$$\mathrm{Im}[\chi(\mathbf{q}=0,\omega)] \propto A_{\downarrow}(\mathbf{k},\tilde{\omega}+\xi_{k,2}).$$

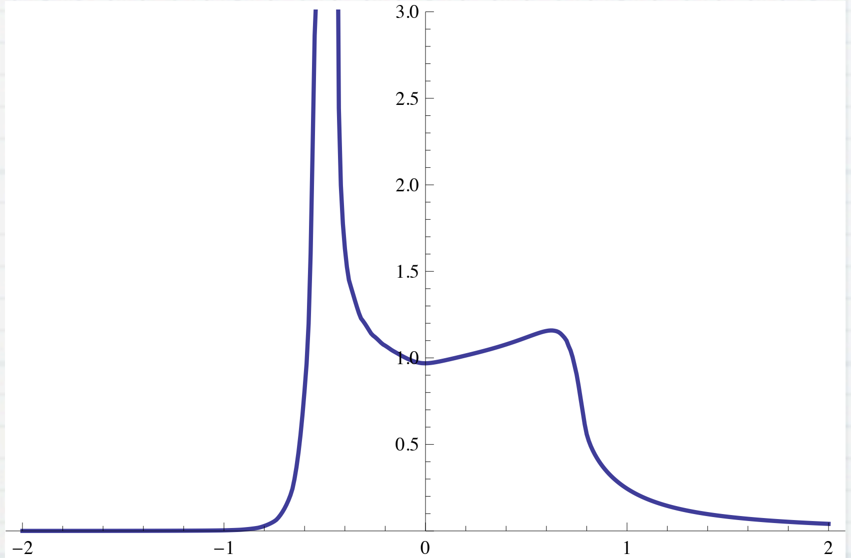


Thermal average

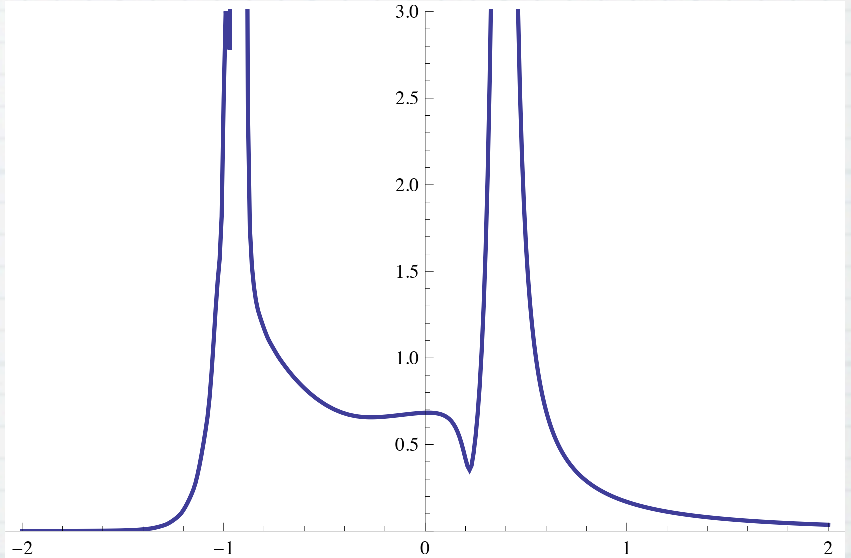
Degenerate gas of Li at $T=0.2T_F$
with few thermal K atoms



$(k_F a)^{-1}=-0.5$



$(k_F a)^{-1}=0$

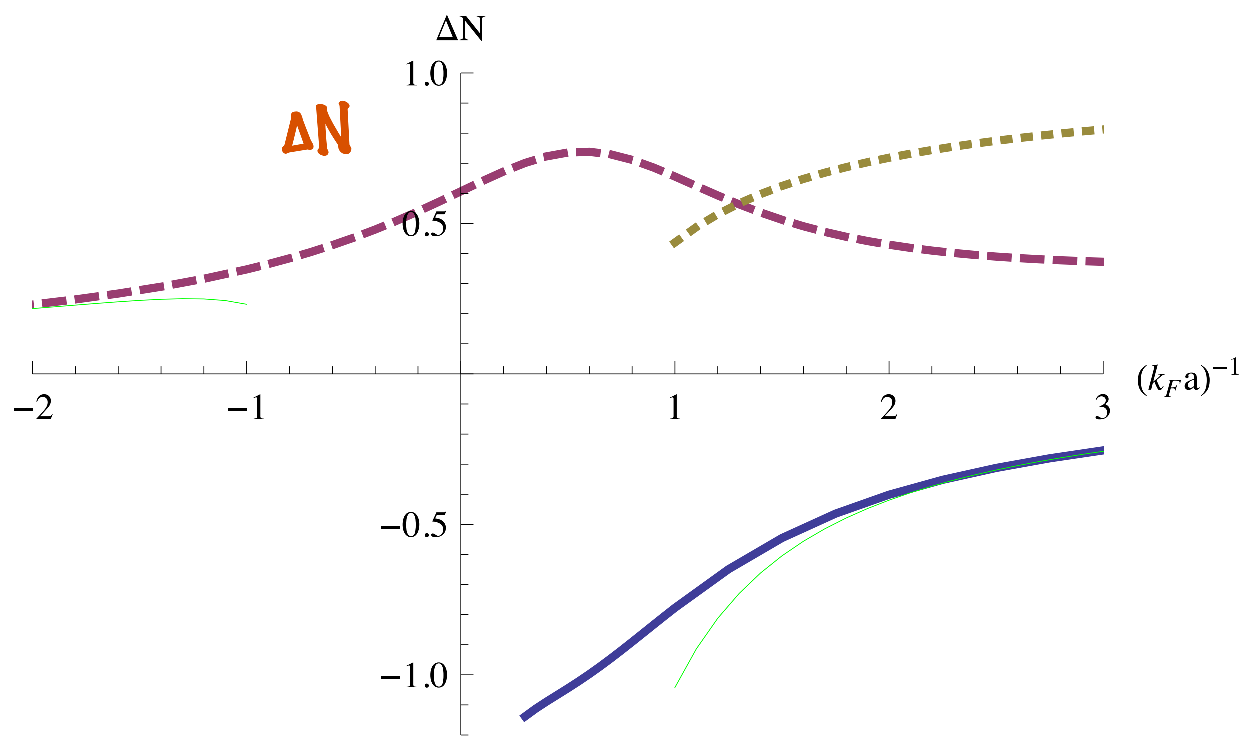


$(k_F a)^{-1}=0.5$

of particles in the dressing cloud

$$\delta\mu_\uparrow = \frac{\partial^2 \varepsilon}{\partial n_\uparrow \partial n_\downarrow} + \frac{\partial^2 \varepsilon}{(\partial n_\uparrow)^2}\Delta N = 0$$

$$\Delta N = -\left(\frac{\partial \mu_\downarrow}{\partial n_\uparrow}\right)_{n_\downarrow} \Big/ \left(\frac{\partial \mu_\uparrow}{\partial n_\uparrow}\right)_{n_\downarrow} \approx -\left(\frac{\partial \mu_\downarrow}{\partial \epsilon_F}\right)_{n_\downarrow}$$



weak coupling: $\Delta N = -\frac{2}{\pi} k_F a - \frac{4}{\pi^2}(k_F a)^2 + \ldots$