

# SYNTHETIC GAUGE FIELDS FOR ULTRACOLD ATOMS IN SYNTHETIC DIMENSIONS

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# Outlook

Synthetic gauge fields

Synthetic dimensions

# Synthetic gauge fields for neutral atoms

Theory: Jaksch&Zoller, NJP 2003  
Osterloh et al., PRL 2005  
Gerbier&Dalibard, NJP 2010  
Bermudez et al., PRL 2010 (TRI Top. Ins.)  
...  
...  
...

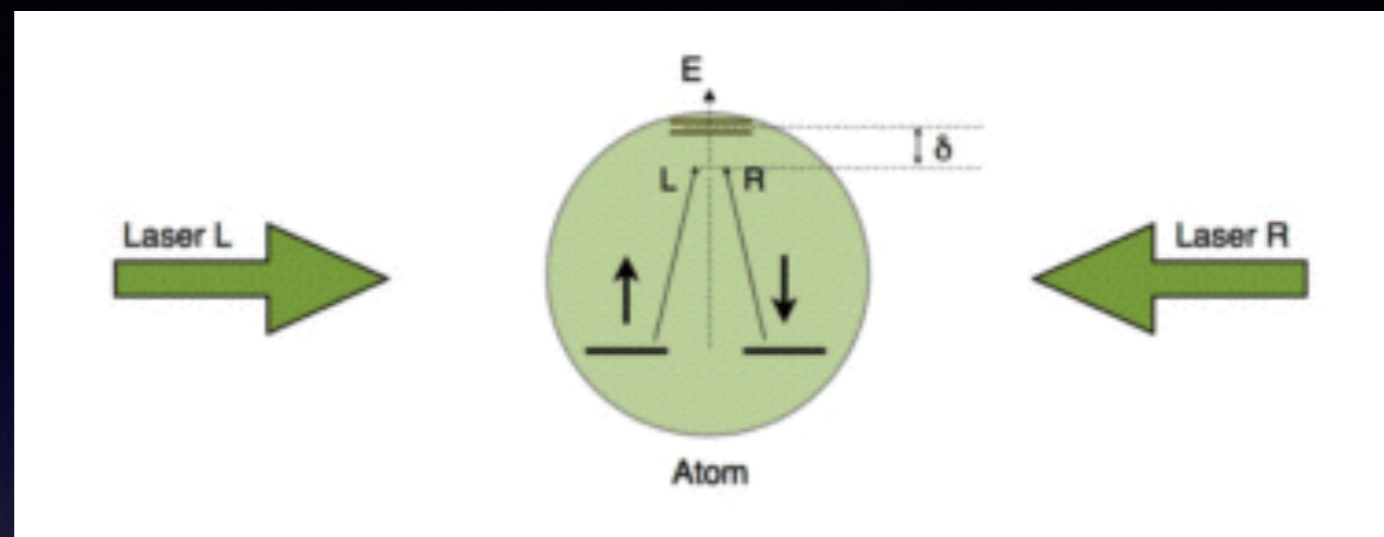
- adiabatic Raman passage
- adiabatic control of superpositions of degenerate dark states
- spatially varying Raman coupling
- Raman-induced transitions to auxiliary states in optical lattices

## REVIEWS:

*Artificial gauge potentials for neutral atoms*  
J. Dalibard, F. Gerbier, G. Juzeliunas, and P. Öhberg, RMP 2011

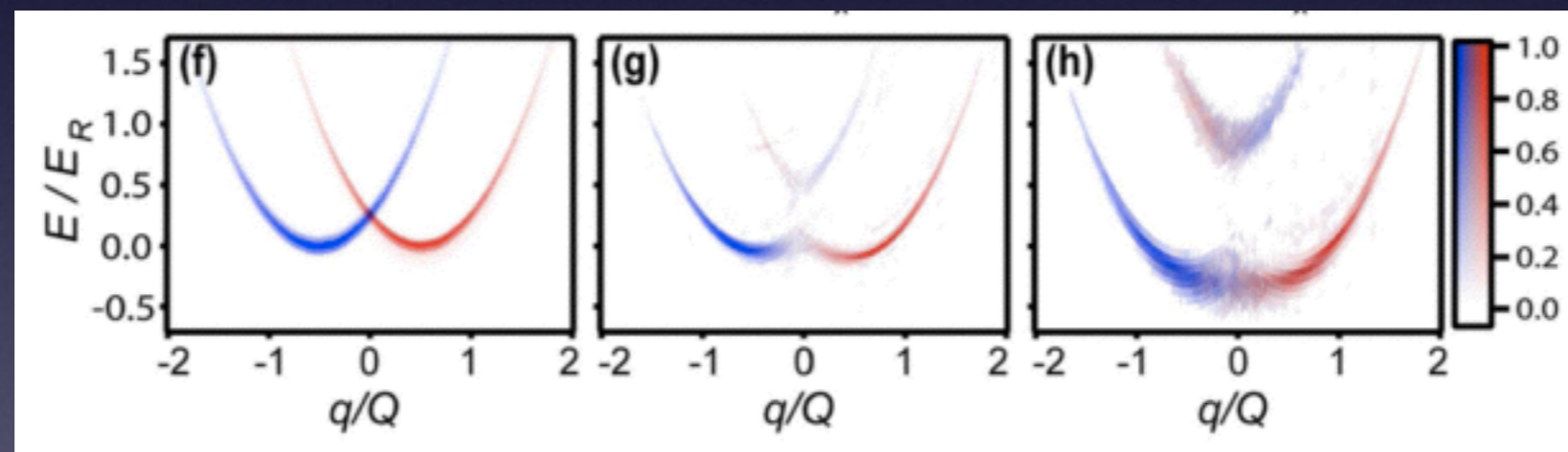
*Light-induced gauge fields for ultracold atoms*  
N. Goldman, G. Juzeliunas, P. Öhberg, and I. Spielman, arXiv:1308.6553

# Synthetic gauge fields for neutral atoms



$$|\uparrow, q=k_x - Q/2\rangle$$

$$|\downarrow, q=k_x + Q/2\rangle$$



spin-orbit gap

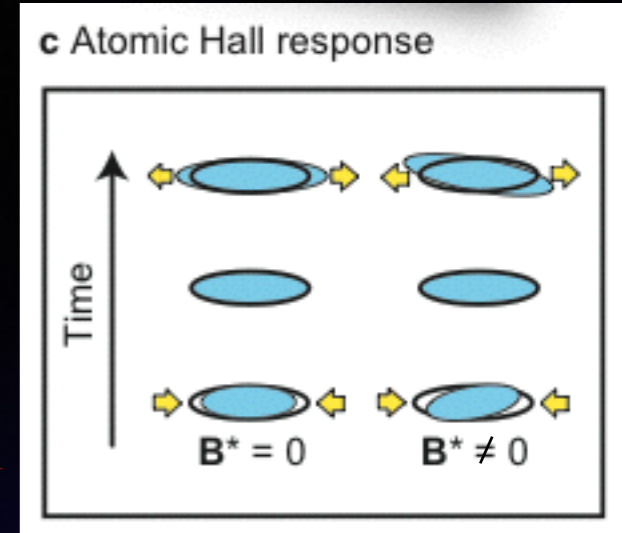
increasing intensity of Raman lasers

spin flip  $\leftrightarrow$  momentum kick,  
i.e., spin-orbit coupling

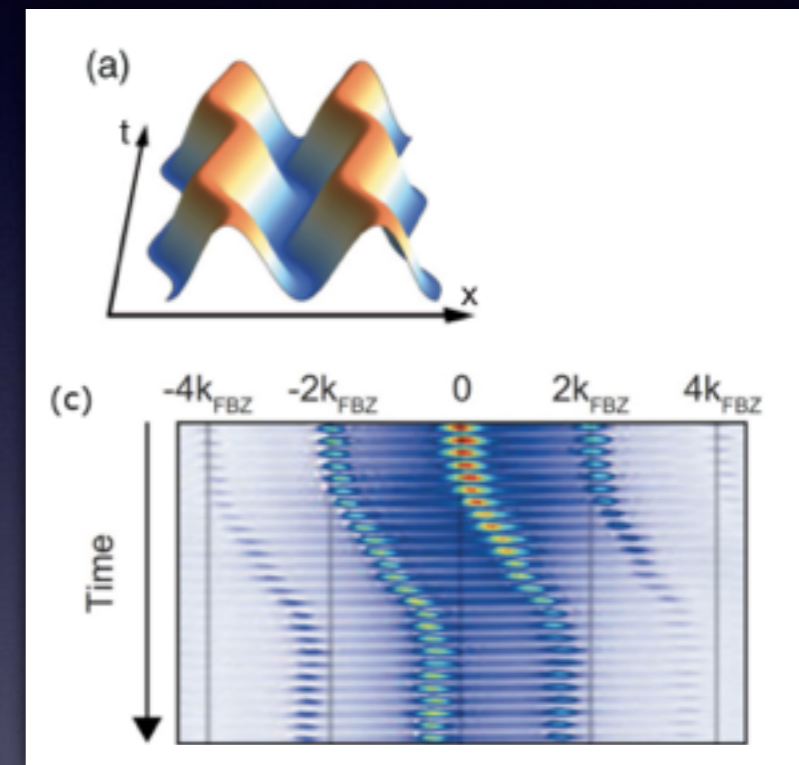
P.Wang et al., PRL 2012 (Shanxi U.)  
L.W. Cheuk et al., PRL 2012 (MIT)

# a field moving fast..

- NIST: *Synthetic magnetic fields for ultracold neutral atoms*, Nature (2009)
- A synthetic electric force acting on neutral atoms*, Nature Phys. (2011)
- Spin-orbit-coupled Bose-Einstein condensates*, Nature (2011)
- Observation of a superfluid Hall effect*, PNAS (2012)
- Peierls Substitution in an Engineered Lattice Potential*, PRL (2012)

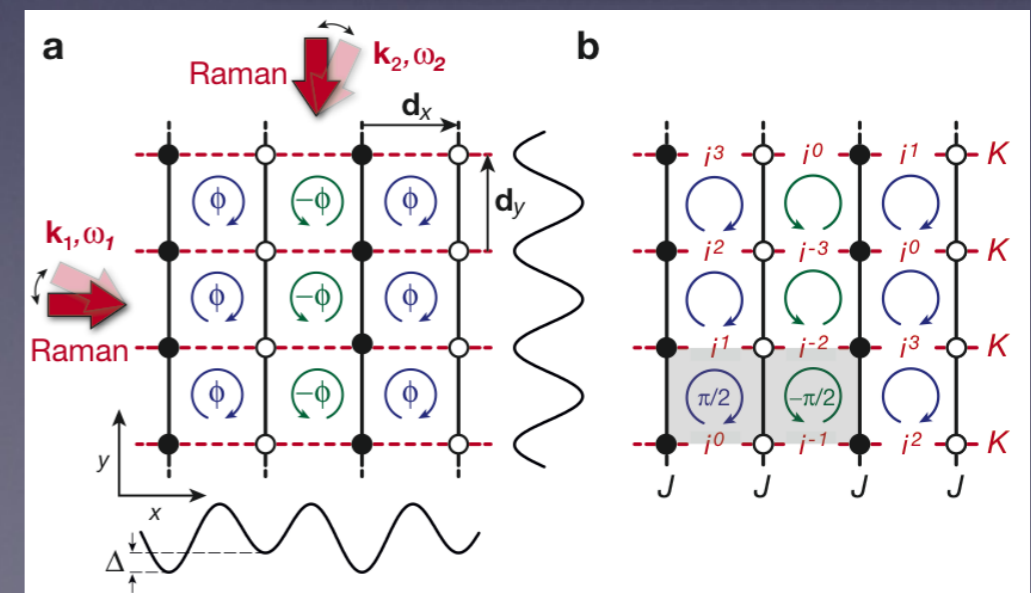


- ICFO & Hamburg & Dresden:  
*Tunable Gauge Potential for Neutral Spinless Particles in Driven Optical Lattices*, PRL (2012)  
 (method independent of the internal structure of the atoms!!)



- Munich: *Experimental realization of strong staggered magnetic fields in an optical lattice*, PRL (2011)

.....



# Synthetic dimensions

# Q. Sim. & Extra Dimensions

Quantum simulation with ultracold atoms:

- Hubbard model (SF-MI transition, ...)
- synthetic gauge fields (relativistic dispersions, ...)
- strongly-correlated states (QH, spin liquids, ...)

Extra (=non-spatial) dimensions:

- attempts to unify gravitation with electro-weak forces (Kaluza-Klein, Yang-Mills, ...)
- thermal QFT: compactification of euclidean time

leads to Matsubara frequencies

(extra-dim is usually discrete and compact)

*quantum simulation of an extra dimension?*



# The main idea

- use a system with  $D$  spatial dimensions
- encode the  $(D+1)^{\text{th}}$  dimension in a different degree of freedom (e.g., the spin)

$$\tilde{\mathbf{r}} = (\mathbf{r}, \sigma)$$

$$H = - \sum_{d=1}^{D+1} \sum_{\tilde{\mathbf{r}}} J_d \hat{a}_{\tilde{\mathbf{r}}+\mathbf{u}_d}^\dagger \hat{a}_{\tilde{\mathbf{r}}} + \text{h.c.}$$

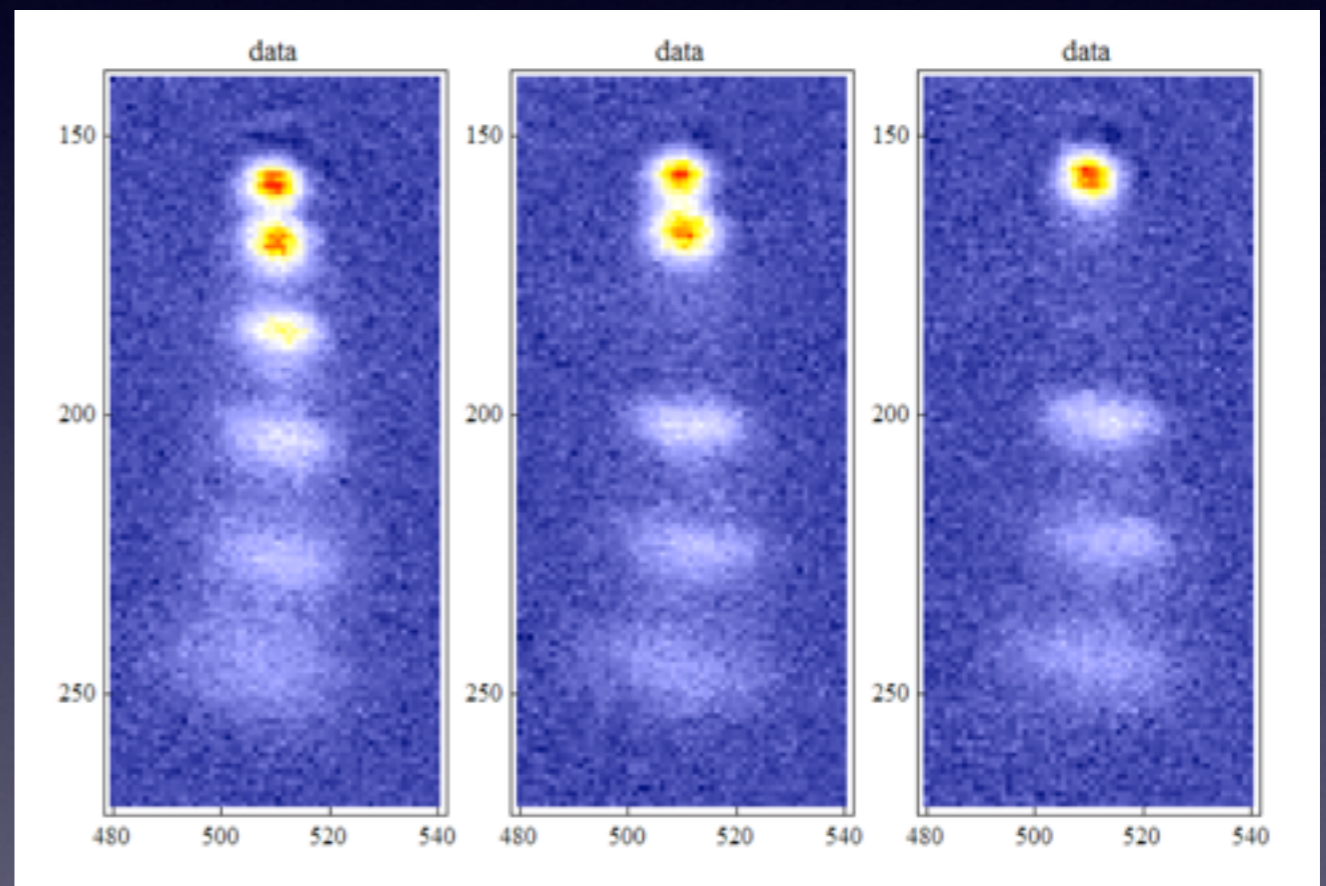
$$= - \sum_{\sigma=1}^N \left[ \sum_{d=1}^D \sum_{\mathbf{r}} J_d \hat{a}_{\mathbf{r}+\mathbf{u}_d}^{(\sigma)\dagger} \hat{a}_{\mathbf{r}}^{(\sigma)} + \underbrace{J_\sigma \hat{a}_{\mathbf{r}}^{(\sigma+1)\dagger} \hat{a}_{\mathbf{r}}^{(\sigma)}} \right] + \text{h.c.}$$

only nearest-neighbor  
“spin-tunneling”

# Large N systems

species	N
Li	2,3,...
87	3
173	6
40	10
87	10
161-163	22
165	128

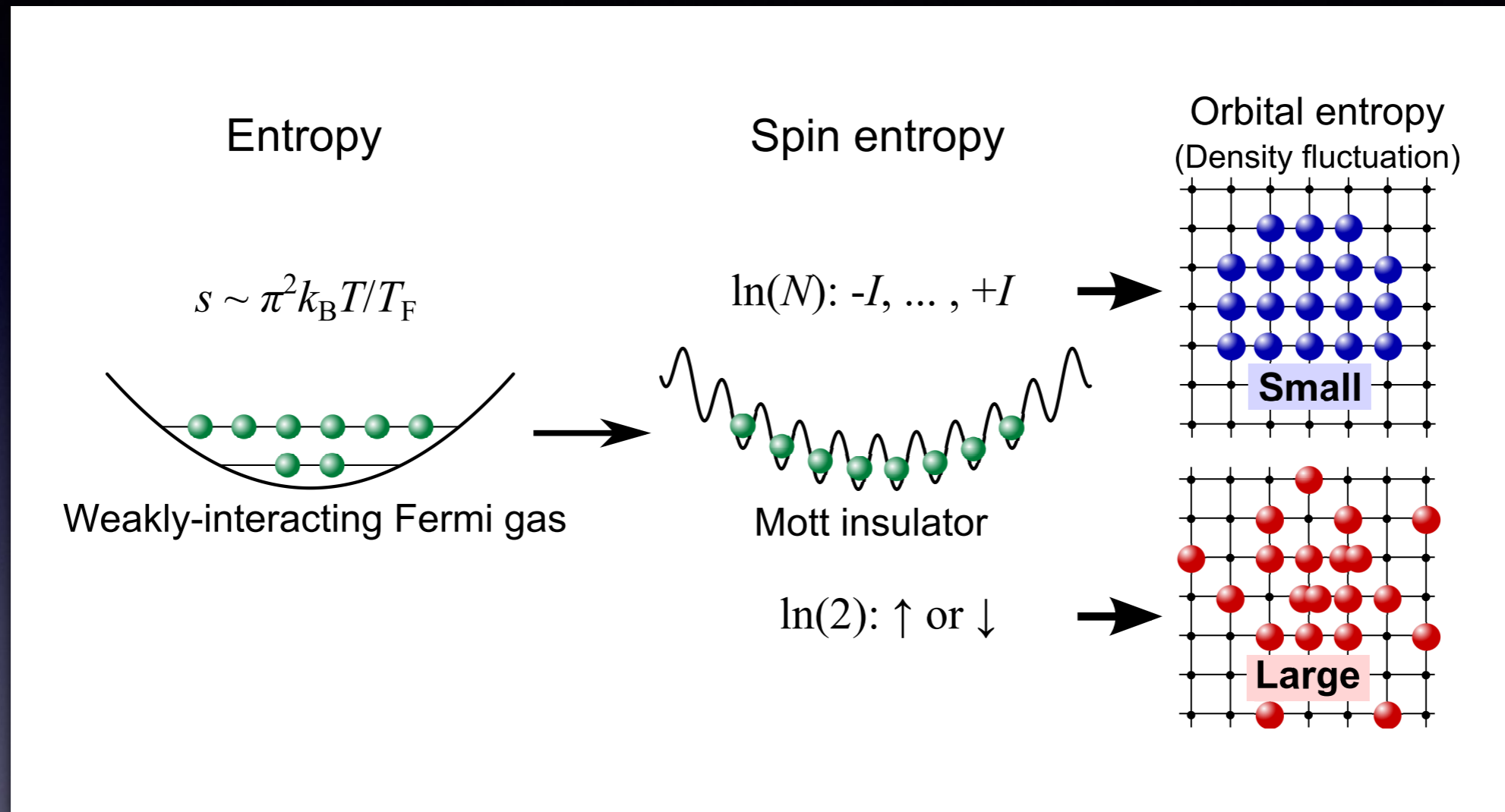
$^{173}\text{Yb}$  at LENS:



†M. Saffman and K. Mølmer, PRA 2008

interactions in earth-alkali atoms are  $SU(N)$  invariant!

# SU(6) Mott insulator



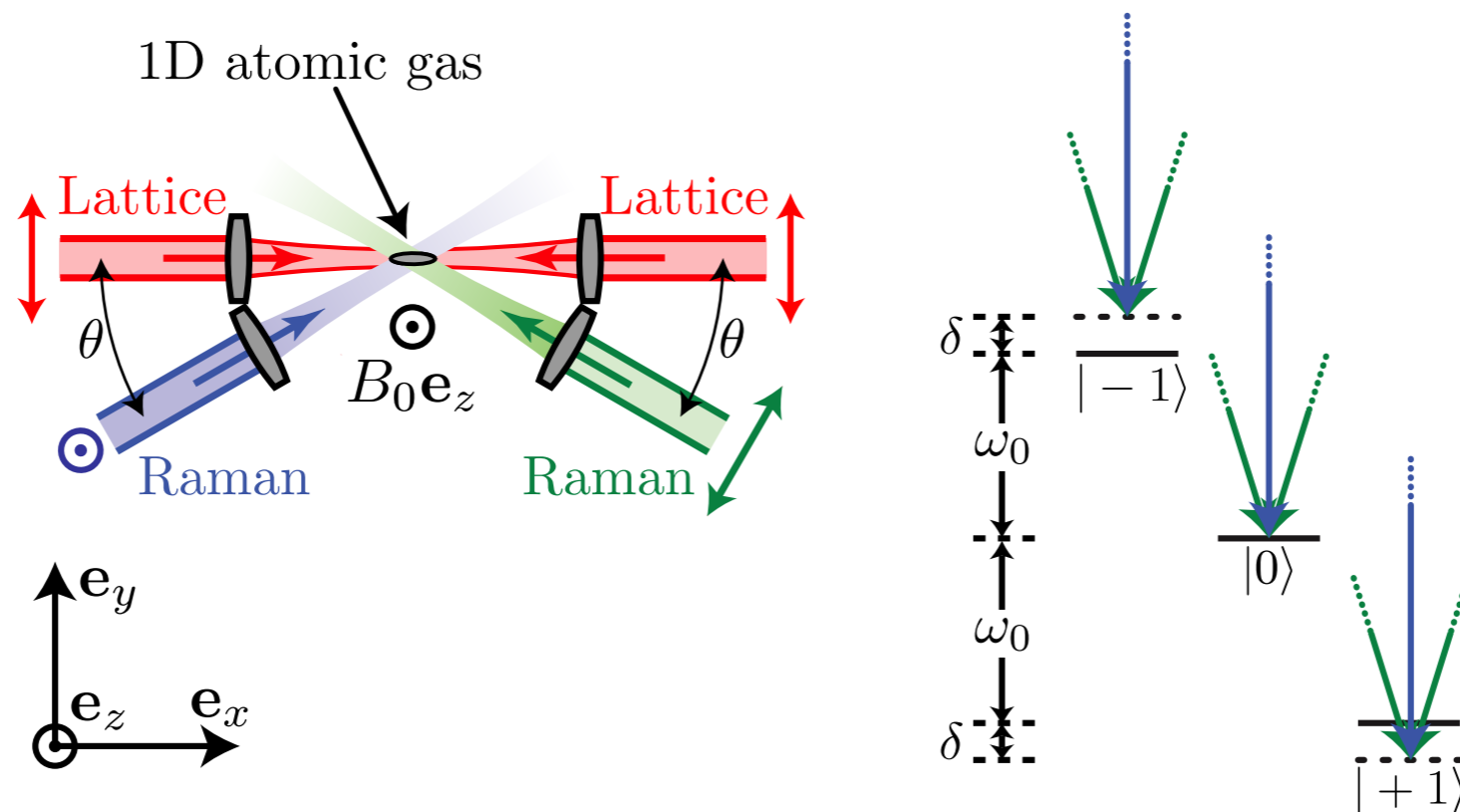
S.Taie, R.Yamazaki, S.Sugawa, and Y.Takahashi, Nature Phys. 2012

Novel cooling mechanisms?

# Implementation

Proposal:

- $F=1$   $^{87}\text{Rb}$  atoms in a 1D optical lattice (deep, but not in the Mott regime)
- two linearly polarized  $\lambda_R$  Raman beams, providing recoil  $k_R = 2\pi \cos \theta / \lambda_R$
- uniform magnetic field in the orthogonal direction



Effective magnetic field:  $\boldsymbol{\Omega}_T = \delta \mathbf{e}_z + \Omega_R [\cos(2k_R x) \mathbf{e}_x - \sin(2k_R x) \mathbf{e}_y]$

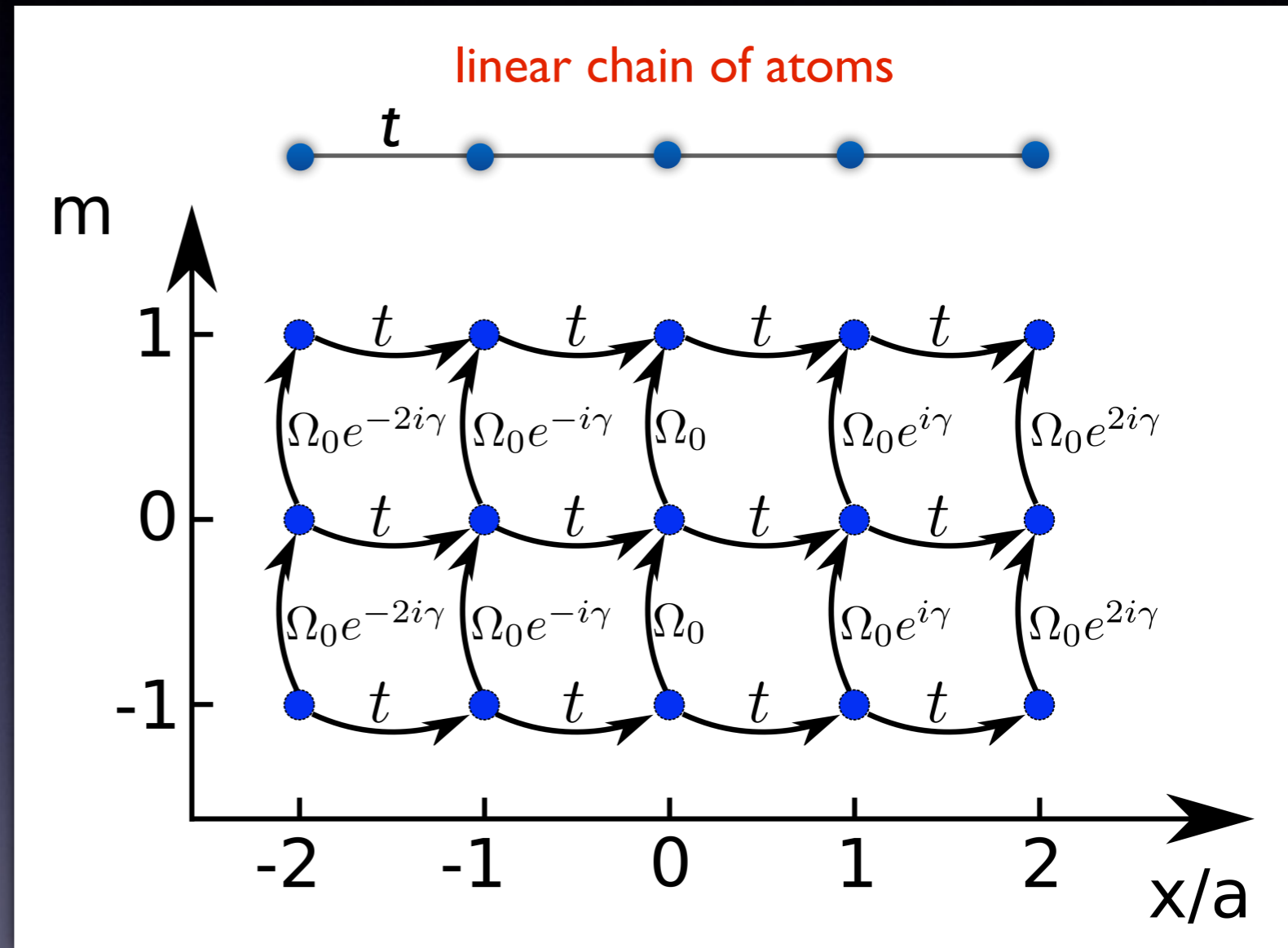
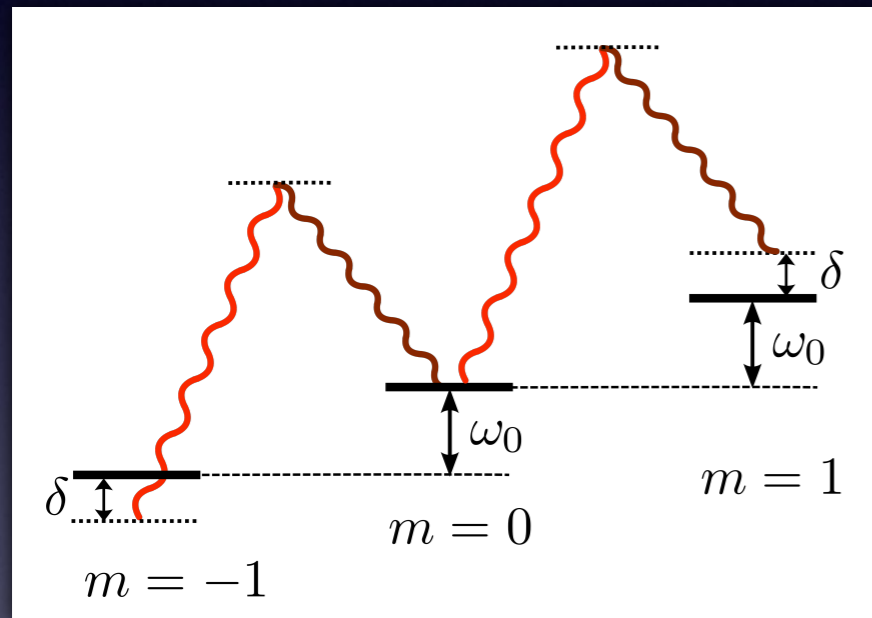
Atom-light coupling:  $H_{\text{al}} = \boldsymbol{\Omega}_T \cdot \mathbf{F} = \delta F_z + (F_+ e^{ik_R x} + F_- e^{-ik_R x}) \Omega_R / 2$

Raising (or *spin-hopping*) operator:  $F_+ |m\rangle = g_{F,m} |m+1\rangle$

$$g_{F,m} = \sqrt{F(F+1) - m(m+1)}$$

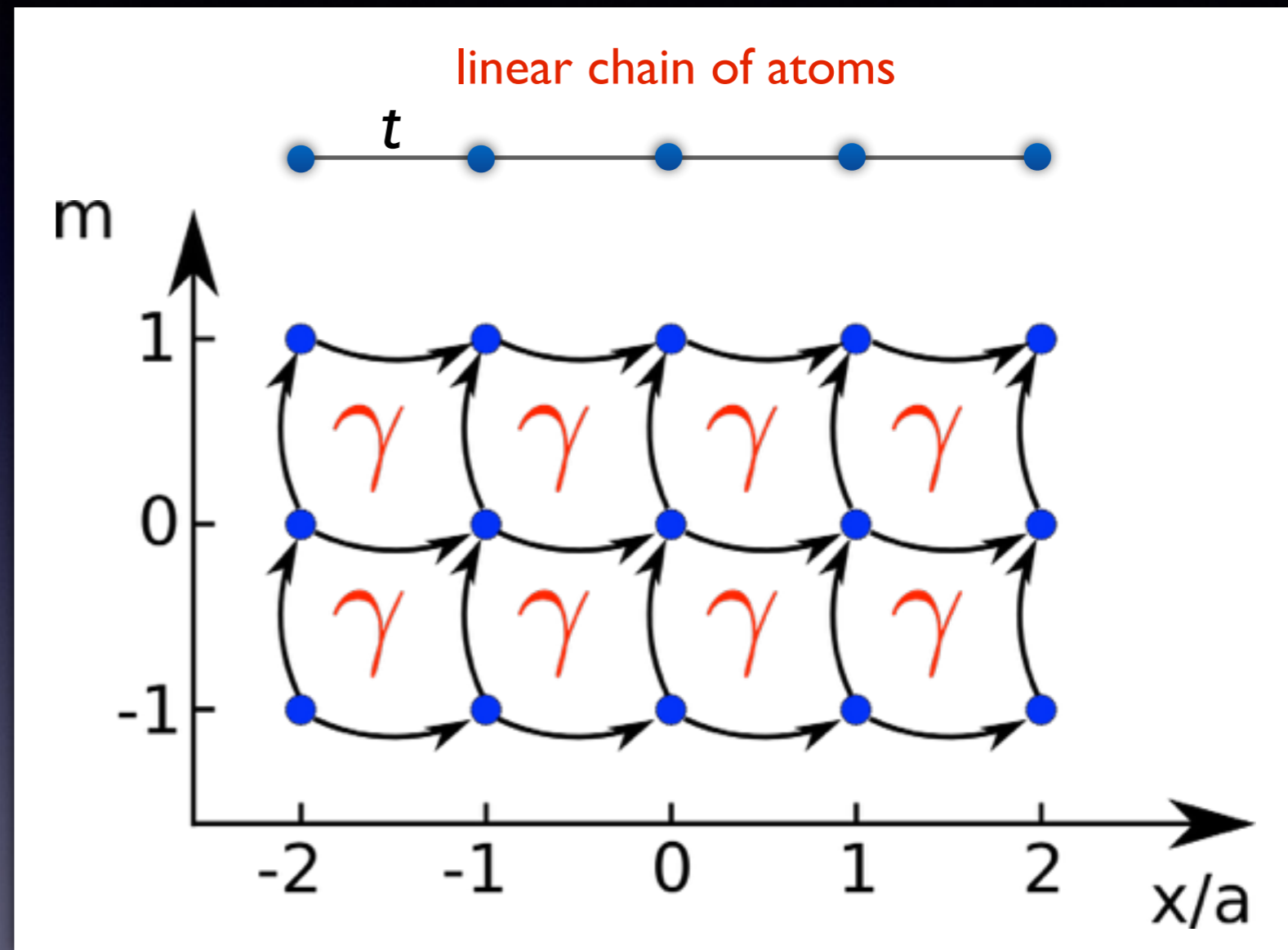
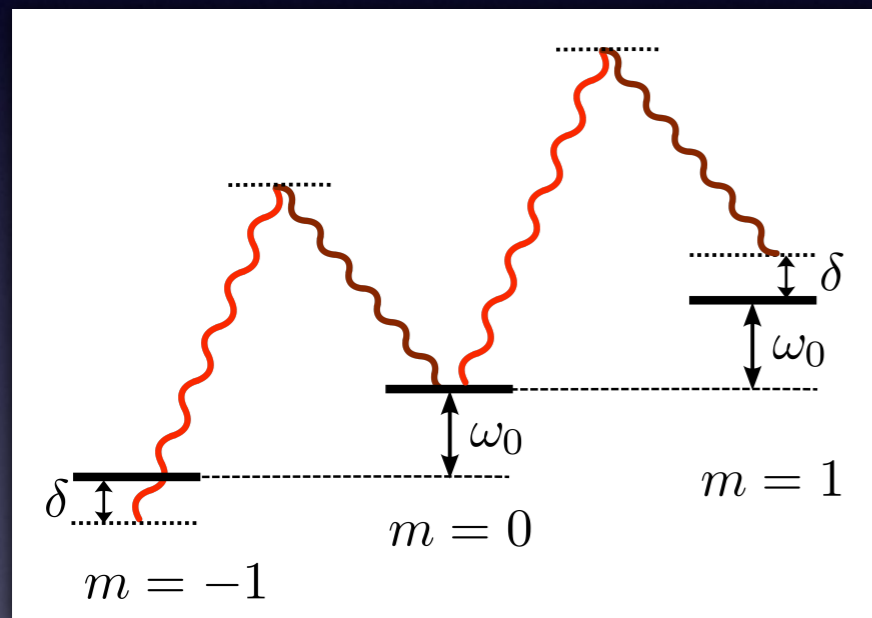
# Implementation

laser-assisted  
position-dependent  
spin-tunneling



# Implementation

laser-assisted  
position-dependent  
spin-tunneling

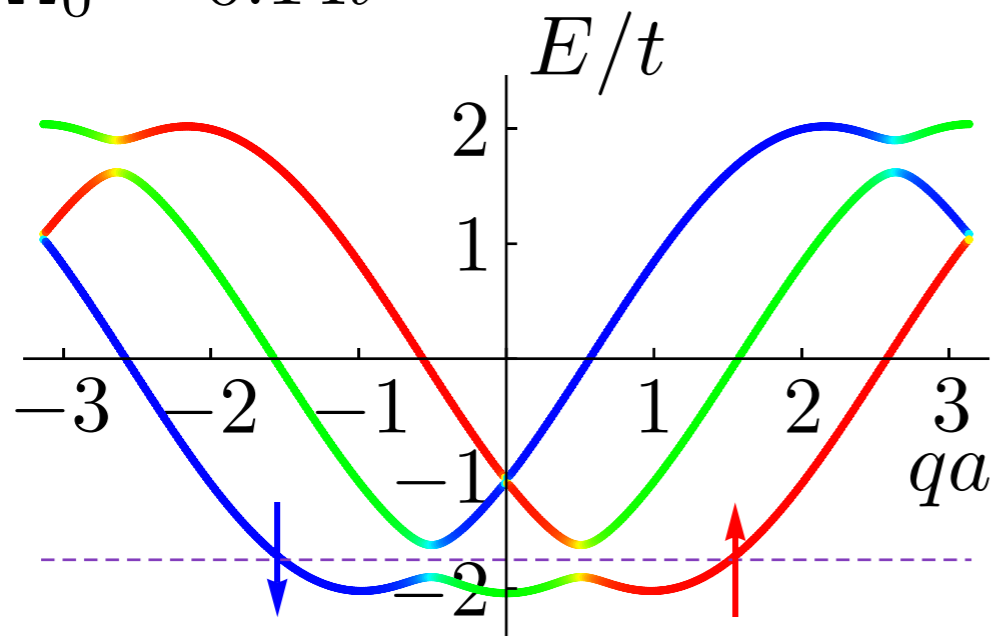


yields strong and non-staggered magnetic fluxes  
and “ $\infty$ -ranged interactions”

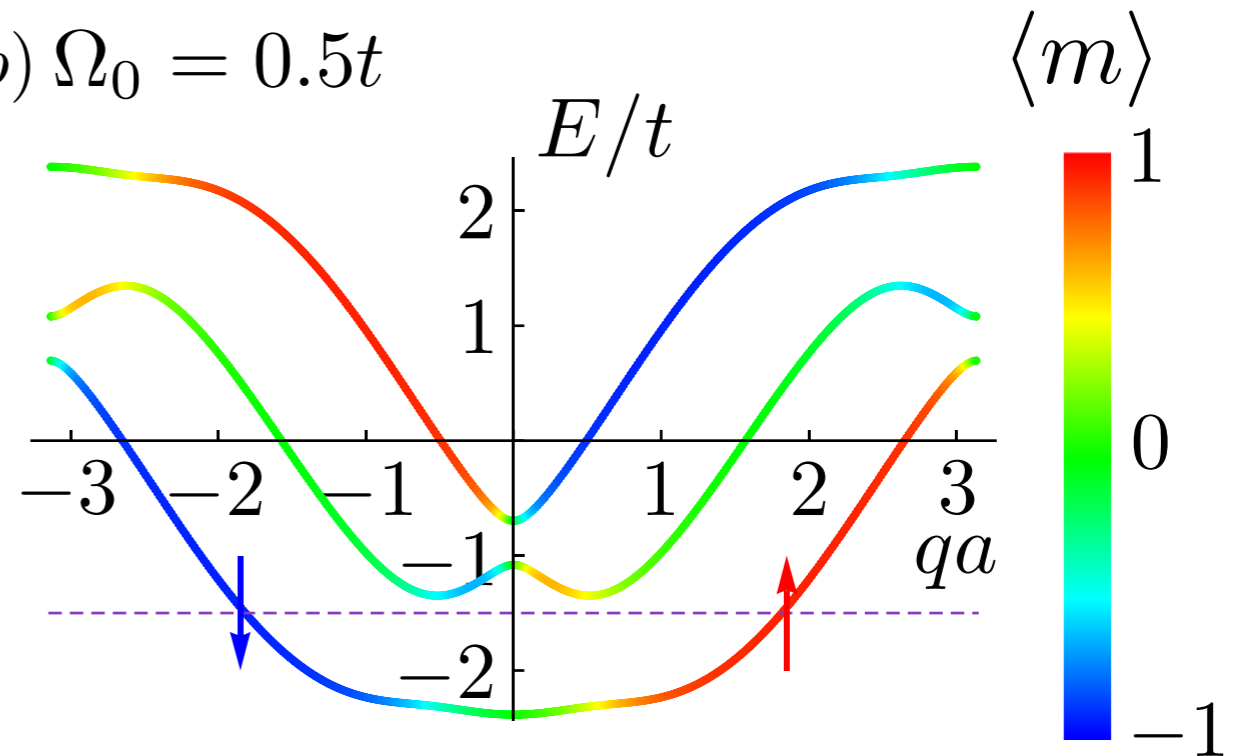
A. Celi, PM, J. Ruseckas, N. Goldman, I. Spielman, G. Juzeliunas, and M. Lewenstein, arXiv:1307.8349

# Spectrum

(a)  $\Omega_0 = 0.14t$



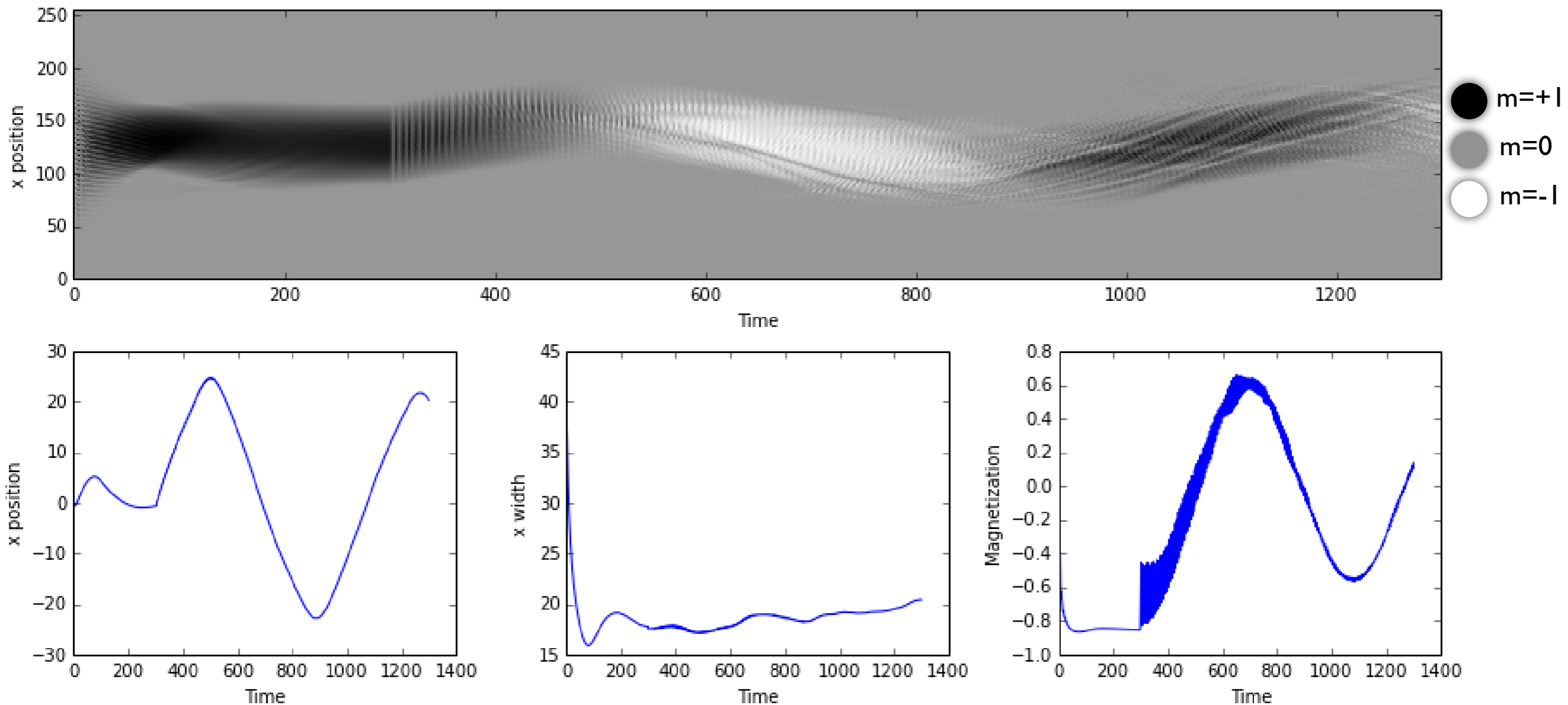
(b)  $\Omega_0 = 0.5t$



States inside the Raman-gap have  $|\langle m \rangle| \approx F$ ,  
i.e., are **chiral edge states on the synthetic lattice**

# Edge state dynamics

An  $F=1$   $^{87}\text{Rb}$  gas in a harmonic trap  
for  $t < 300$ , an energy detuning keeps the gas in  $m=+1$   
the detuning is set to zero for  $t > 300$ , starting out of phase oscillations in  $x$ - and spin-directions

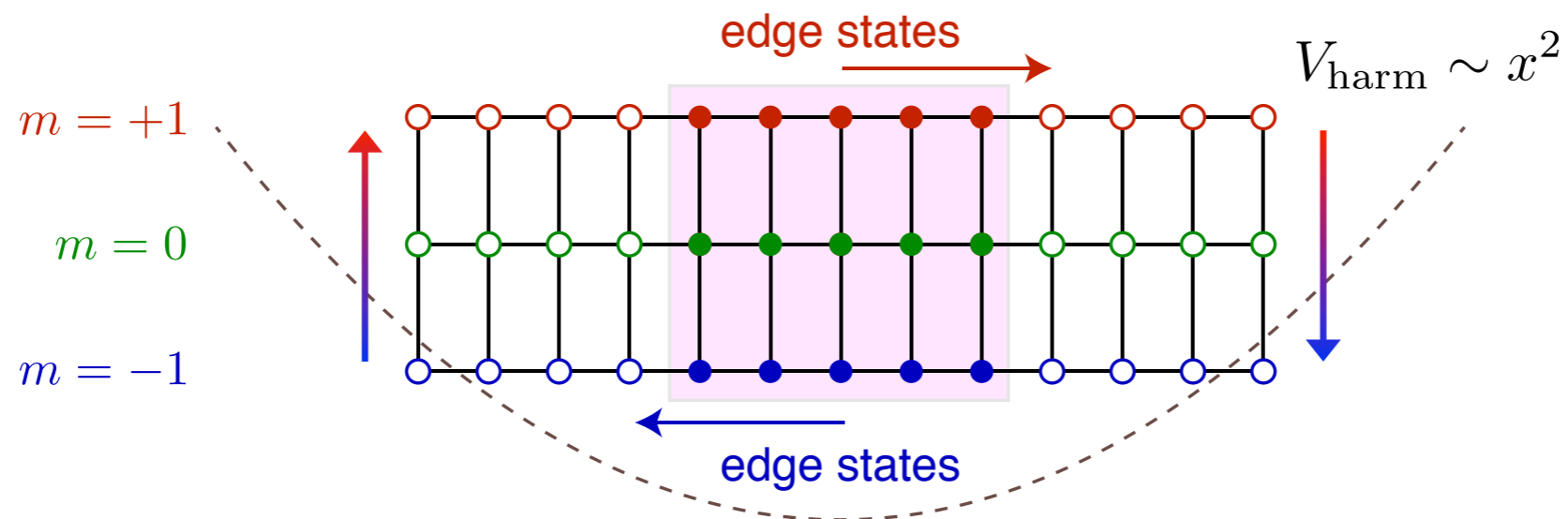




# Edge state dynamics

An “F=1 Fermi gas” in a harmonic trap

for  $t < 0$ , the gas is confined by hard walls in the shaded region

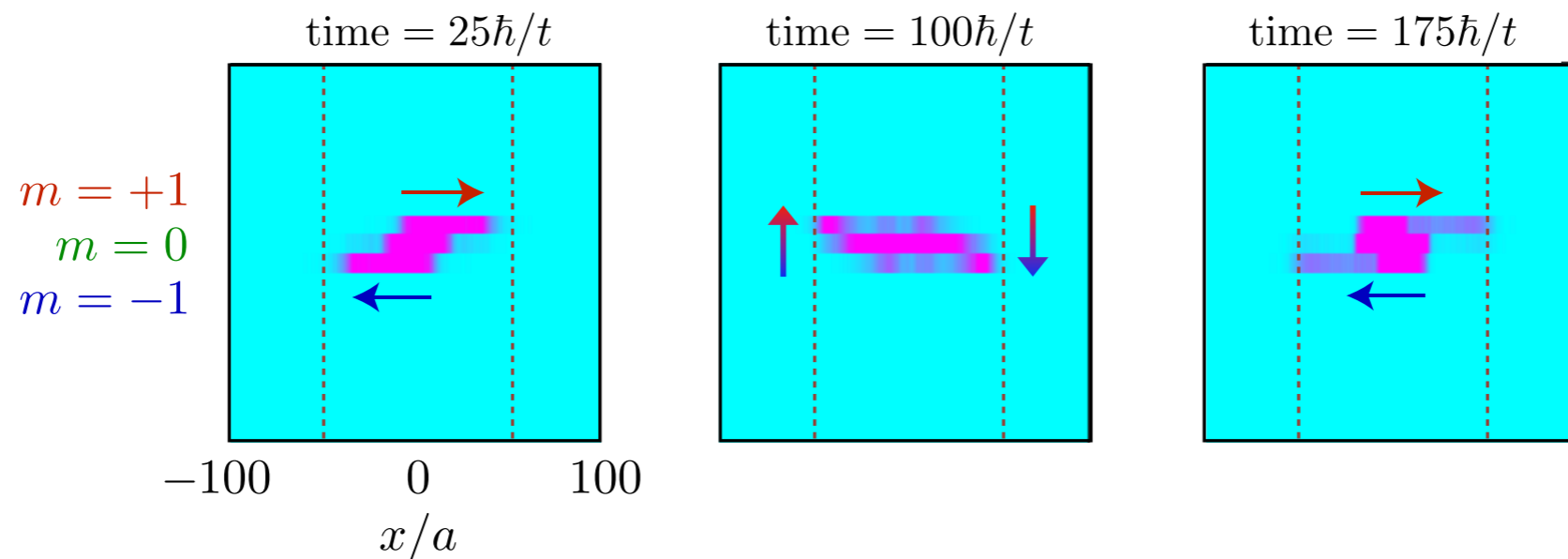
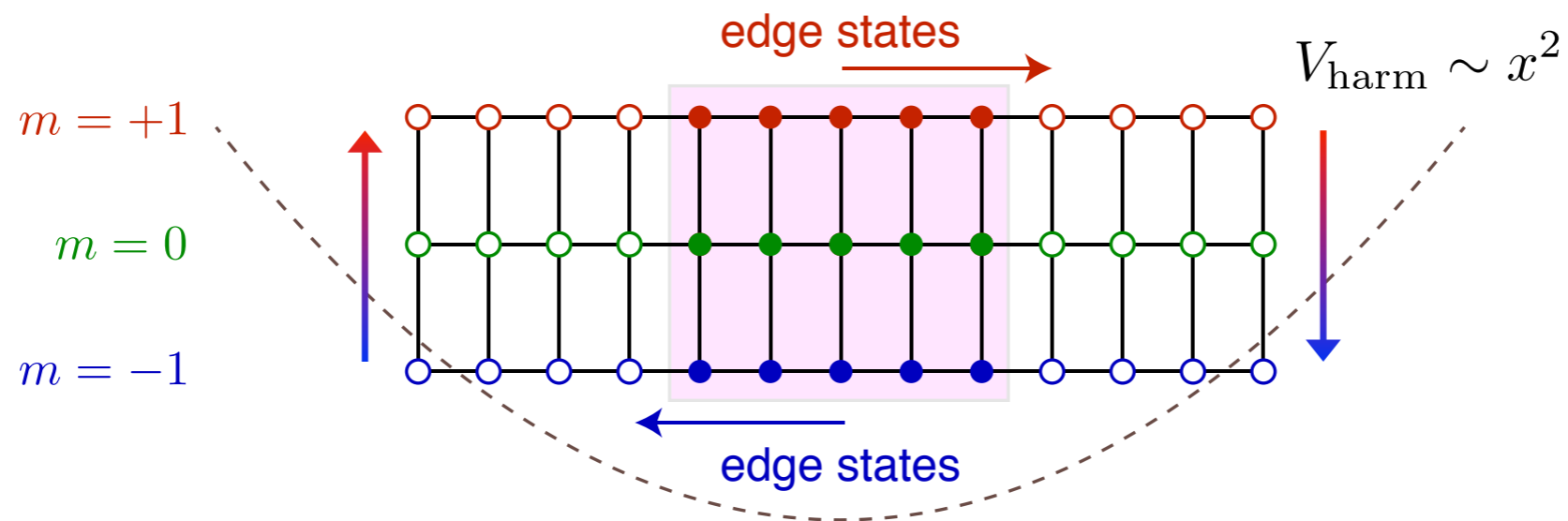


$$\begin{aligned}\Omega_0 &= 0.5t \\ \Phi &= 1/2\pi \\ E_F &= -1.4t\end{aligned}$$

# Edge state dynamics

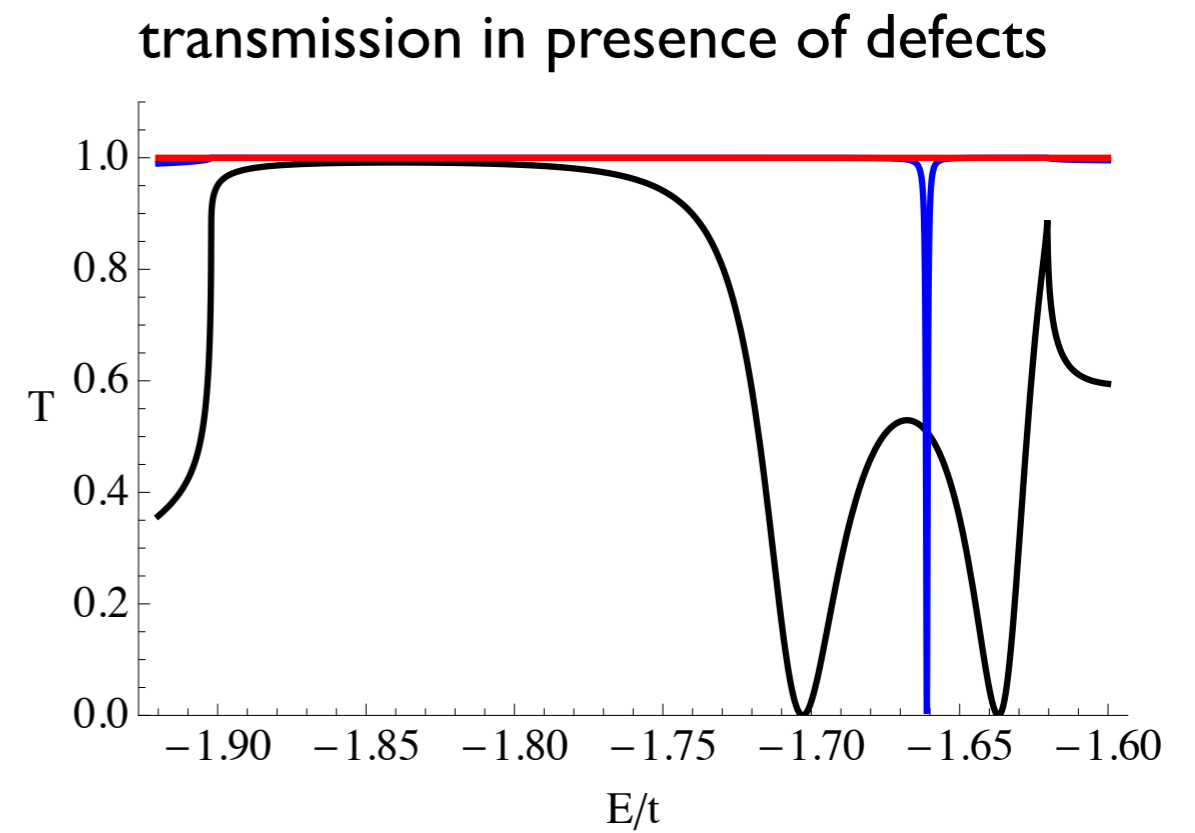
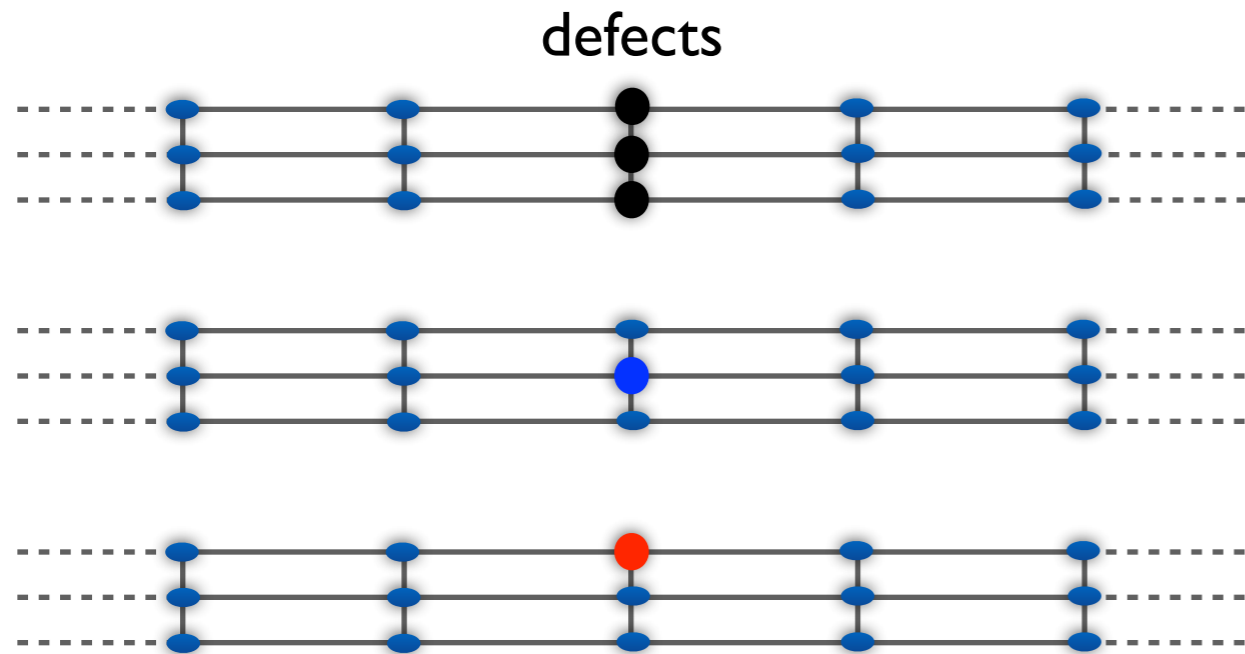
An “F=1 Fermi gas” in a harmonic trap

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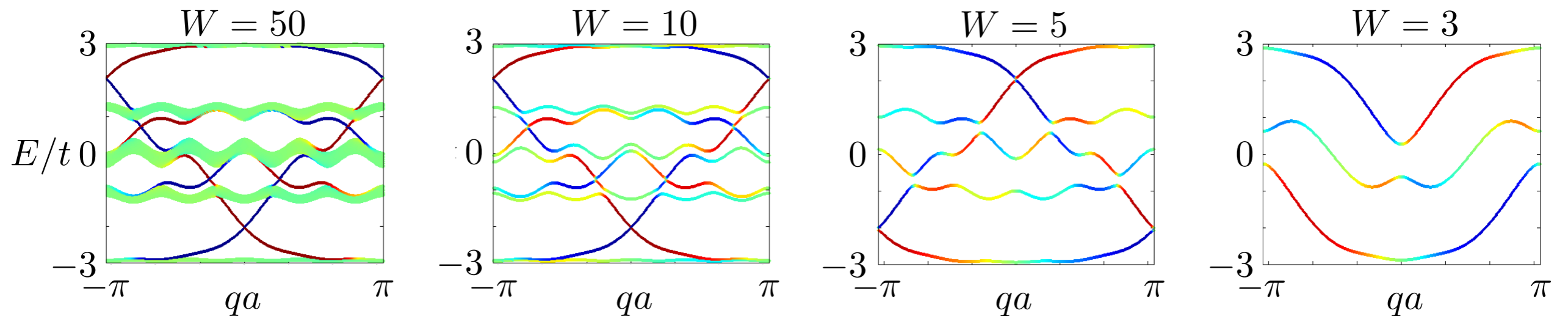
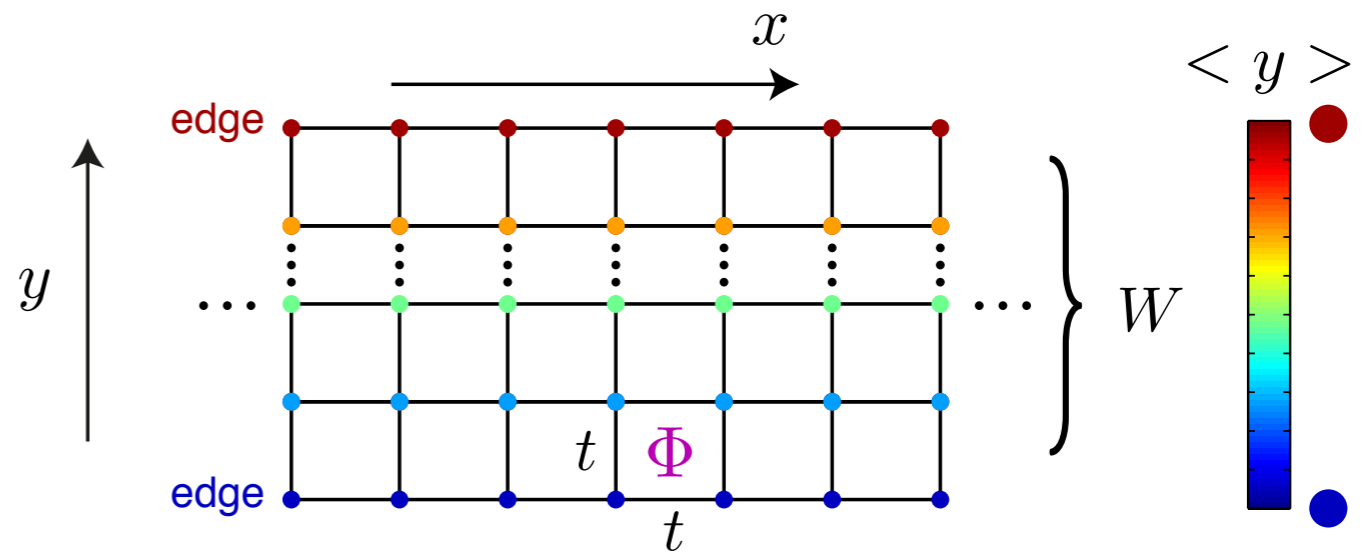
$$\begin{aligned}\Omega_0 &= 0.5t \\ \Phi &= 1/2\pi \\ E_F &= -1.4t\end{aligned}$$

# Transmission across defects



# Hofstadter strip

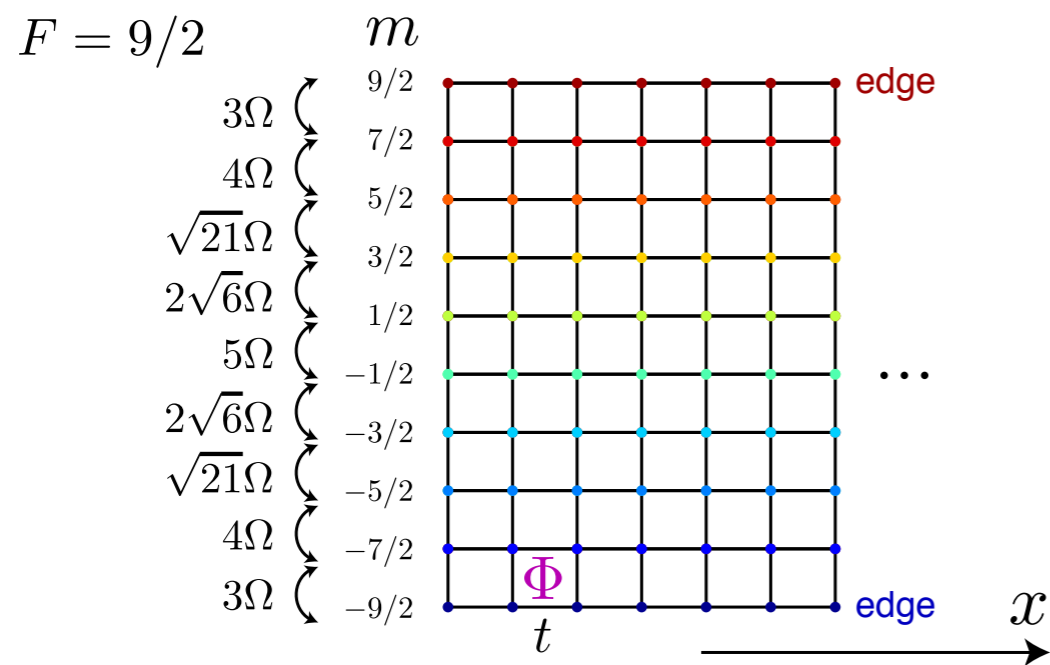
Regular 2D square  $N \times W$  lattice,  
pierced by a uniform magnetic field:



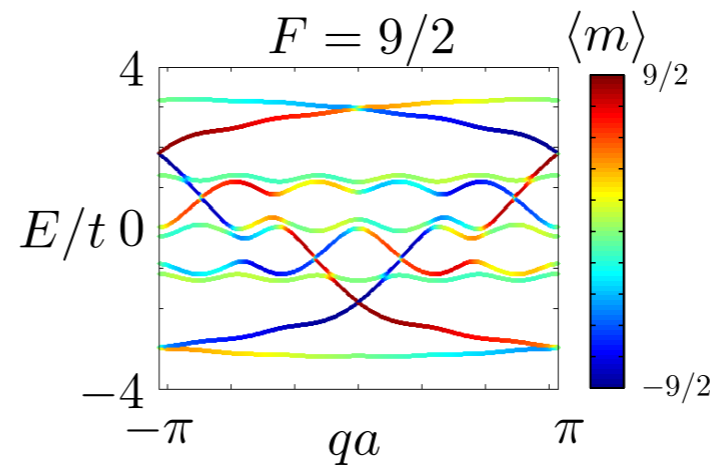
$$\Phi = 1/5$$

for a detailed discussion of the case  $W=2$ , see also: D. Hugel and B. Paredes, arXiv:1306.1190

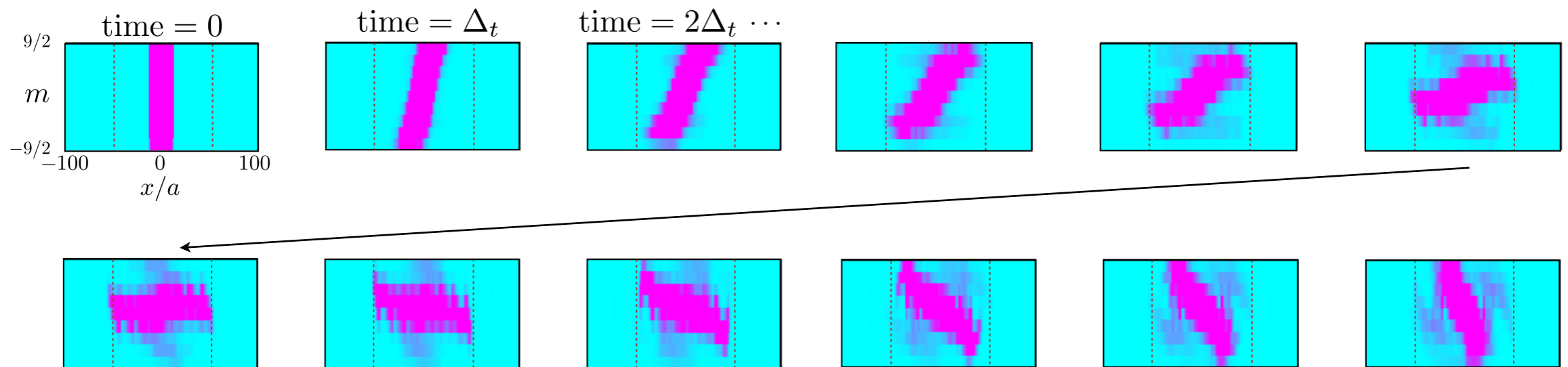
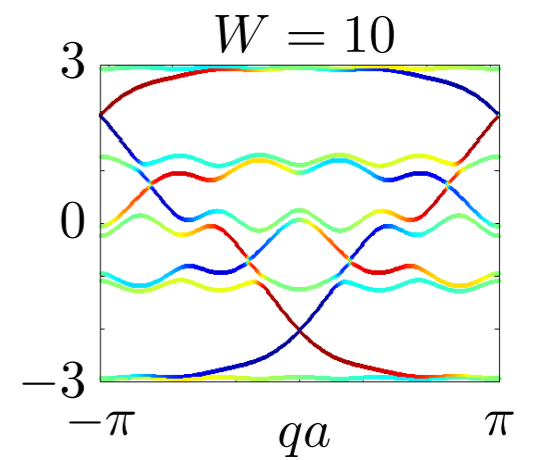
# Edge state dynamics for $^{40}\text{K}$



realistic hoppings



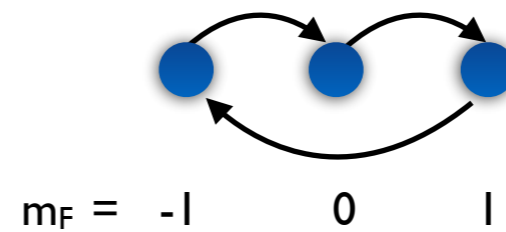
uniform case



# From open to closed b.c.



+



=

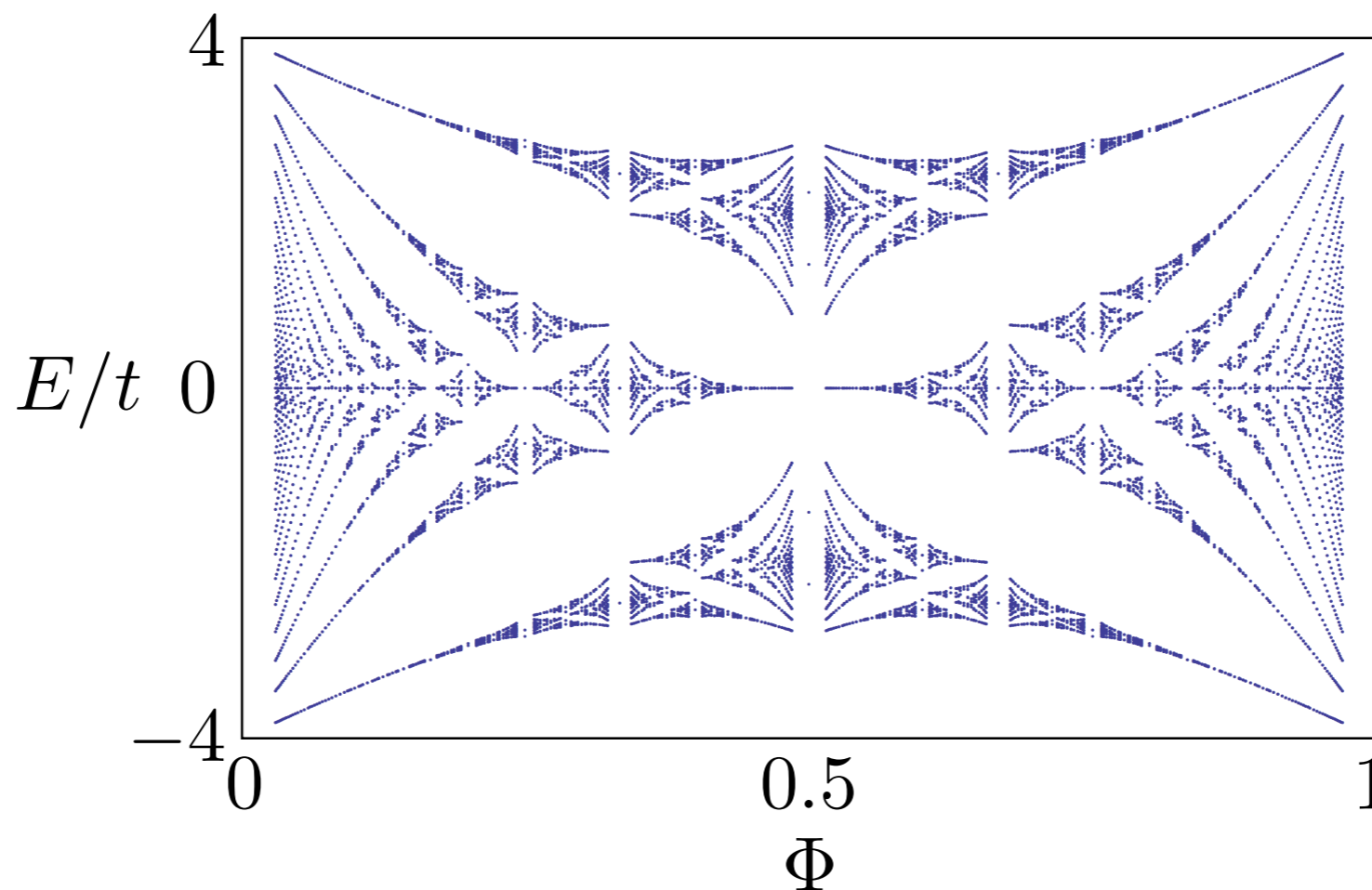


# Energy spectrum of spin-1 atoms with closed b.c. and non-zero flux

Periodic boundary conditions along  $\mathbf{e}_m$  can be induced with an extra coupling  $|m = 1\rangle \leftrightarrow |m = -1\rangle$  accompanied by a momentum recoil  $k_R$  along  $\mathbf{e}_x$ . The system becomes periodic IFF  $\Phi = P/Q$  with  $P, Q$  co-prime integers.

# of loops needed for periodicity:  $l/W$  where  $l = \text{LCM}(W, Q)$  [thus  $Q$  or  $Q/3$  for  $W=3$ ]

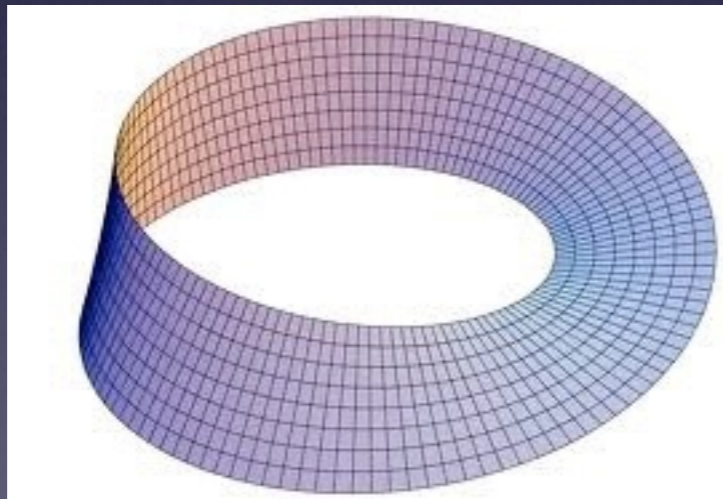
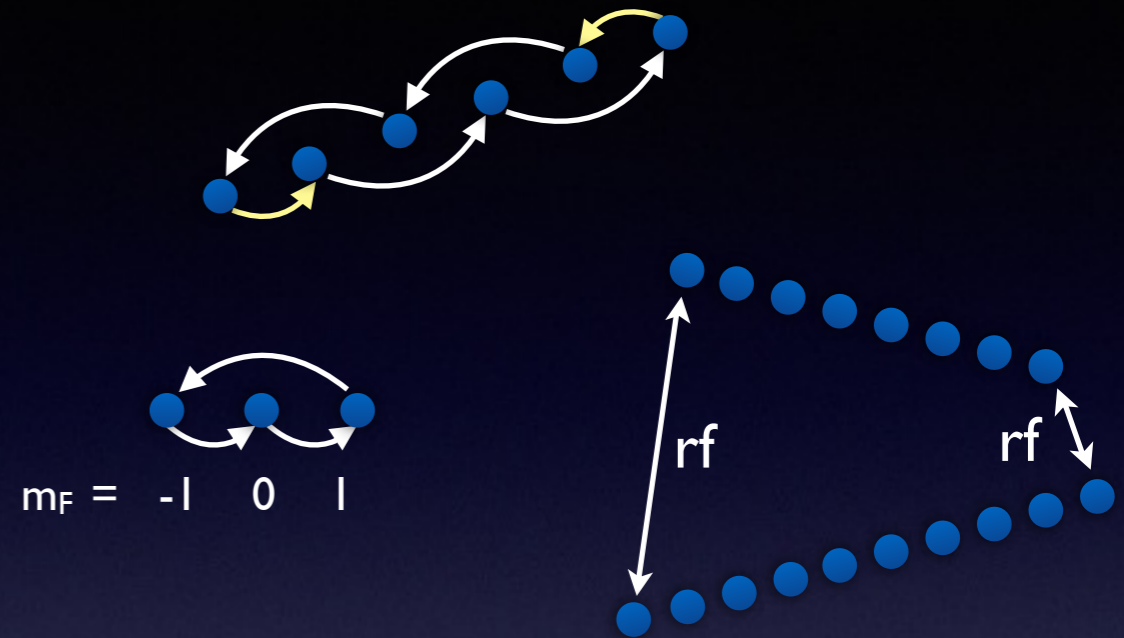
The spectrum has the fractal structure of a **Hofstadter butterfly**:



# Interesting topologies

possible boundary conditions  
along the spin direction:

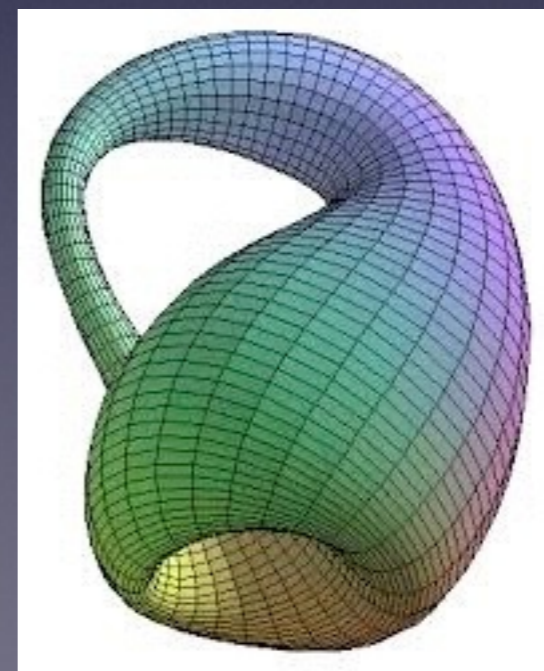
- open
- closed
- twisted (a closed loop encircles a phase)



**Möbius strip**  
linear chain in the spatial dir.,  
 $\pi/2$  twist in spin

## Klein bottle

ring in the spatial dir.,  
 $\pi/2$  twist in spin





# Interactions: $SU(W)$ -invariant case

$$H_{\text{int}} = \frac{\mathcal{U}}{2} \sum_n \mathcal{N}_n (\mathcal{N}_n - 1) \quad \text{with} \quad \mathcal{N}_n \equiv \sum_m a_{n,m}^\dagger a_{n,m}$$

Interactions are local along  $\mathbf{e}_m$ , but infinite in range along  $\mathbf{e}_m$ !

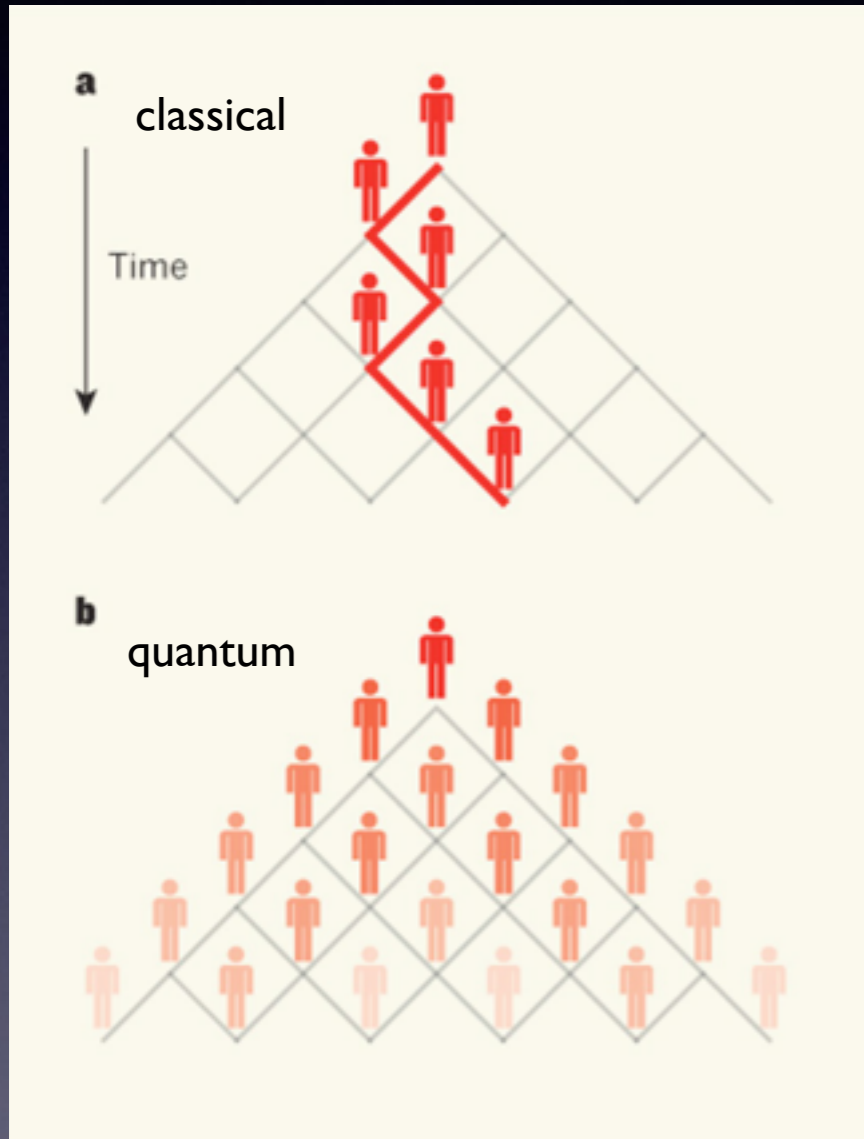
For  $SU(W)$ -invariant interactions, there exists a basis  $\{c_{n,j}\}$  in which the Hamiltonian is diagonal in the spin direction. Denoting its eigenvalues by  $\{\epsilon_{n,j}\}$  we can minimize the energy for fixed  $\langle H_{\text{int}} \rangle$  by populating only the states with lowest  $\epsilon_{n,j_n}$ , as this minimizes the kinetic term  $\langle H \rangle$ .

Ground state of the interacting system:

- if the local (spin) ground state is not degenerate, the global ground state can be mapped to the one of a 1D uniform Bose-Hubbard chain
- if  $\epsilon_{n,j}$  is minimal for two of the  $W$  values of  $j$ , the ground state of the 1D chain will possess a primitive cell containing  $Q$  consecutive lattice points

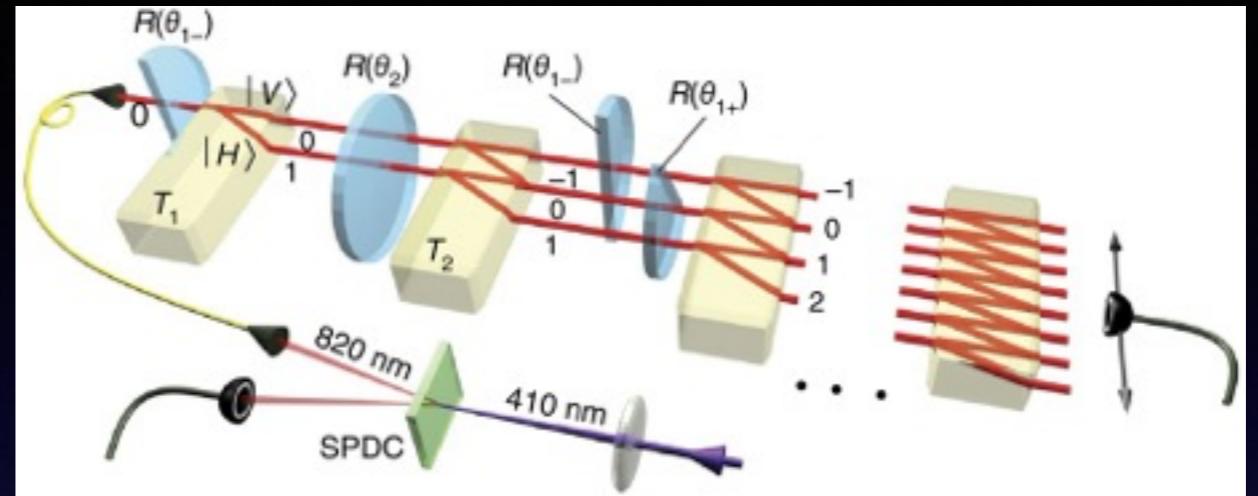
(degeneracy is possible only for closed spin-b.c. and rational  $\Phi$ . For open b.c., the eigenvalues are independent of  $\gamma$  and of  $n$ , and never degenerate)

# Quantum walks

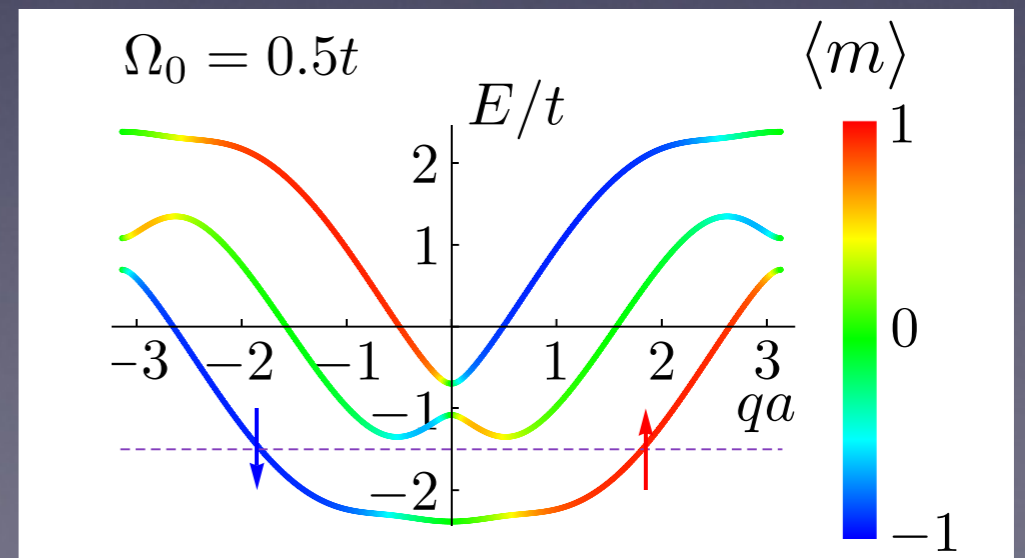
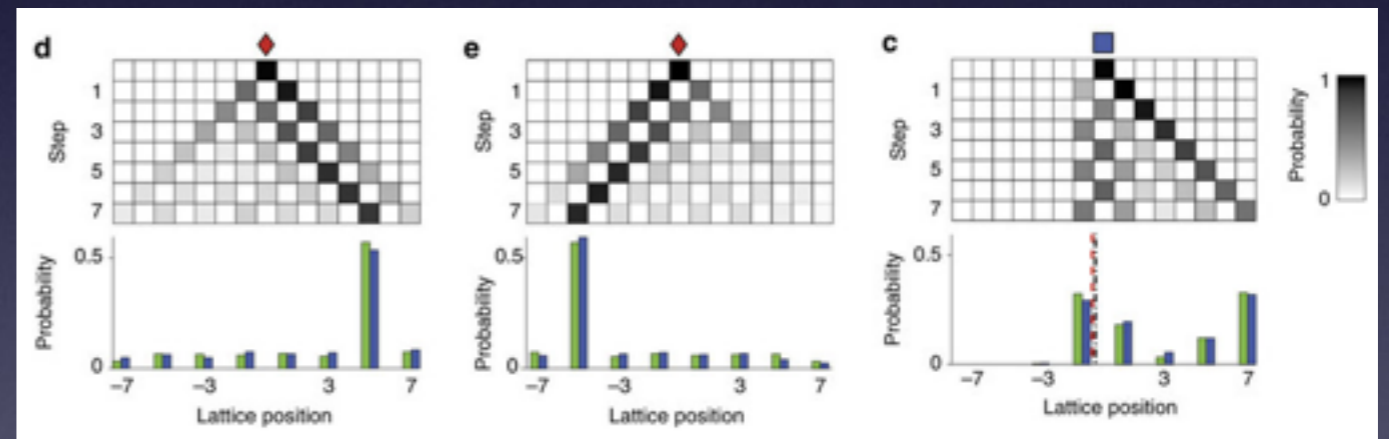


spin  $\uparrow$  move right,  
spin  $\downarrow$  move left

use edge states  
as q. walkers?



Experimental realization in photonic systems  
T. Kitagawa et al., *Nat. Comm.* **3**, 882 (2012)



# Conclusions

- Synthetic gauge fields yield non-trivial hopping phases
- Using an internal d.o.f. as an extra-dimension:
  - ★ uniform fluxes and robust edge states
  - ★  $\infty$ -ranged interactions
  - ★ novel cooling schemes possible?
  - ★ quantum simulation of high-energy theories, and  $D > 3$  systems?  
(e.g., critical exponents of phase transitions)

*Synthetic gauge fields in synthetic dimensions*

A. Celi, P. Massignan, J. Ruseckas, I. Spielman, G. Juzeliunas, and M. Lewenstein, arXiv:1307.8349