Measuring Chern numbers in narrow Hofstadter strips

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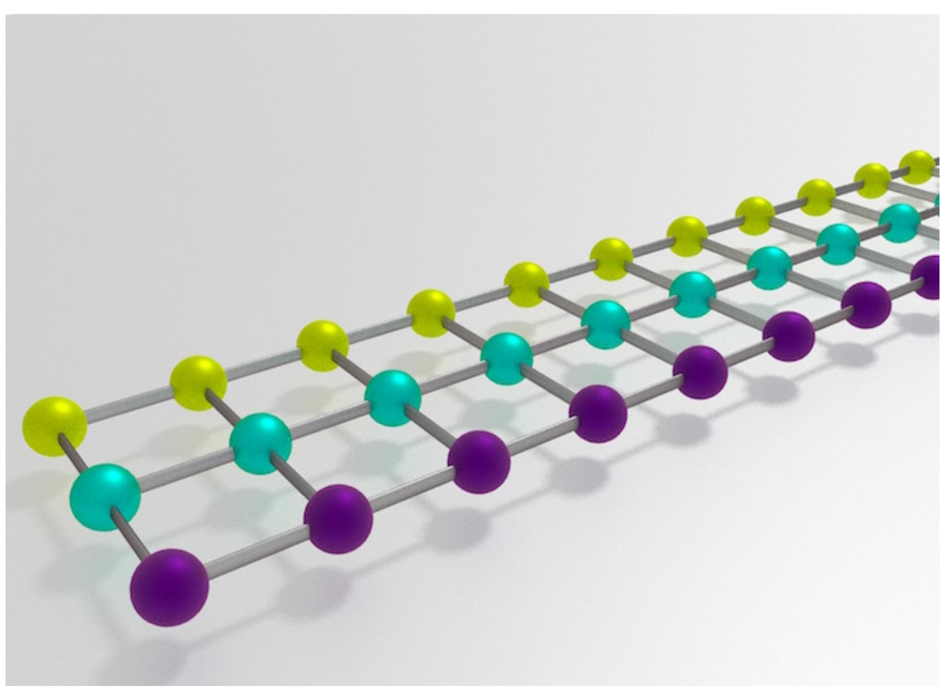
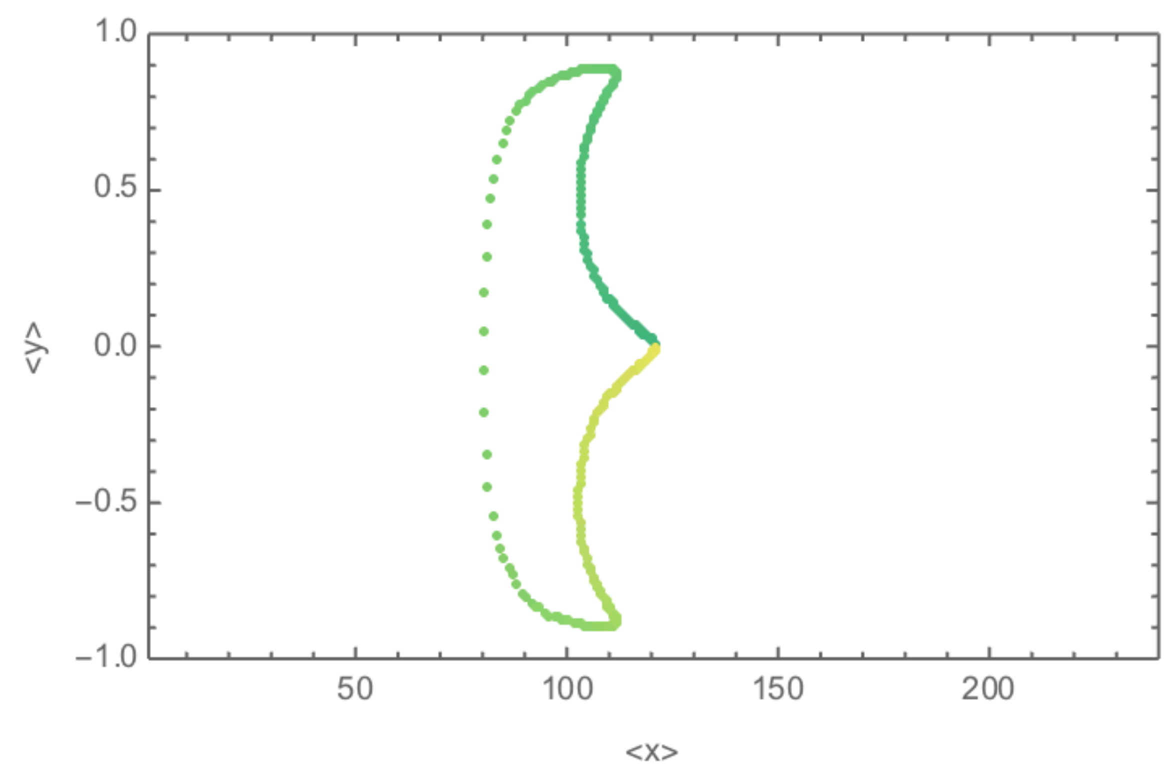


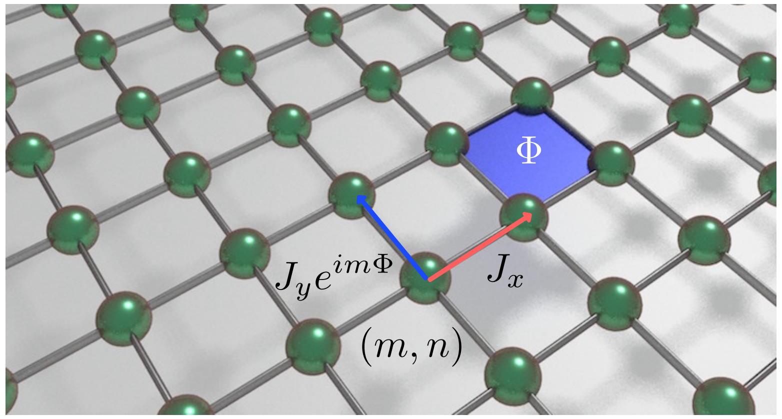


- Hofstadter model
- Synthetic dimensions and Hofstadter strips
- Laughlin pumping
- Measuring Chern numbers in narrow strips
- Robustness (disorder/trapping)

Hofstadter model

- A square lattice pierced by a uniform magnetic flux $\Phi = 2\pi \frac{p}{q}$

$$\hat{\mathrm{H}}_0 = -\sum_{m,n} J_x \hat{c}^\dagger_{m+1,n}\hat{c}_{m,n} + J_y e^{im\Phi}\hat{c}^\dagger_{m,n+1}\hat{c}_{m,n} + \mathrm{H.c.}$$



- Spectrum: q bands of Bloch eigenstates $|u_j(\mathbf{k})\rangle$

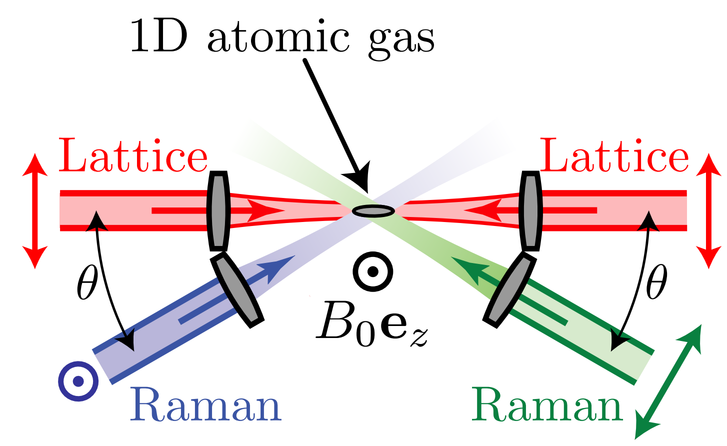
- Brillouin zone: $(2\pi/(qd)) \times (2\pi/d)$

- Topology characterized by: $\mathcal{C}_j = \frac{1}{2\pi}\int_{BZ} \mathcal{F}_j(\mathbf{k})\mathrm{d}^2\mathbf{k}$

- Berry curvature: $\mathcal{F}_j(\mathbf{k}) = 2\,\mathrm{Im}\langle \partial_{k_y} u_j(\mathbf{k}) | \partial_{k_x} u_j(\mathbf{k})\rangle$

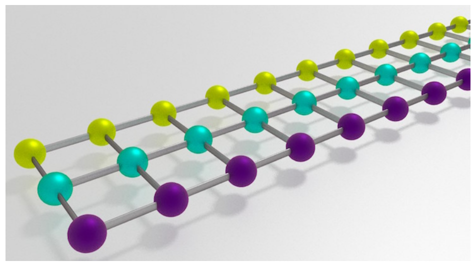
Hofstadter strips

- Synthetic gauge fields in synthetic dimensions:



$$\Phi = k_R \frac{a}{\pi} = \frac{2\pi\cos\theta}{\lambda_R}\frac{a}{\pi}$$

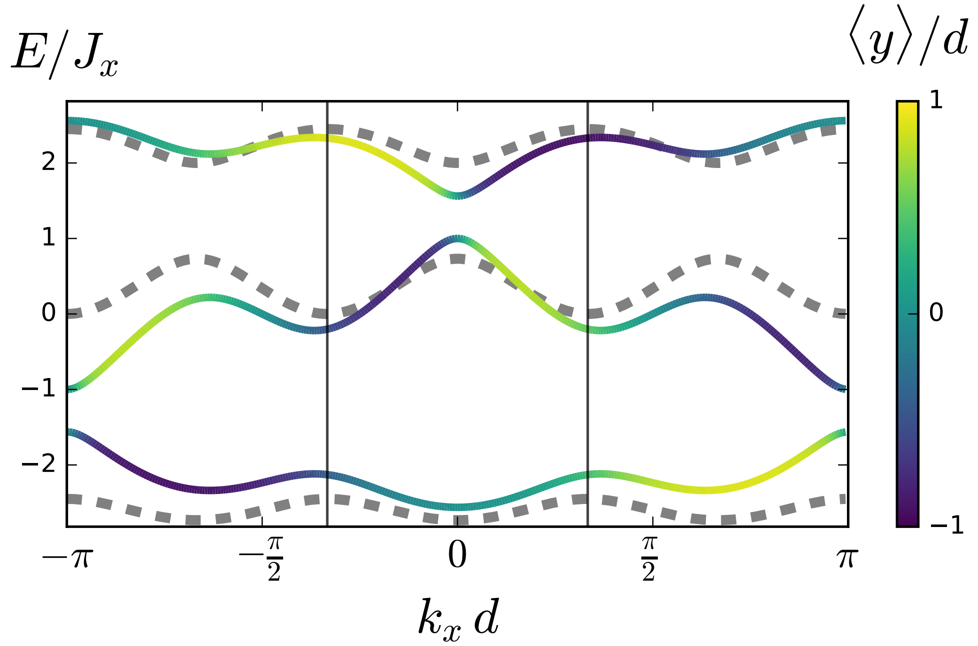
[Celi, Massignan, Lewenstein et al., PRL (2014)]



- Gauge change: $\hat{c}_{n,m} \to e^{imn\Phi}\hat{c}_{n,m}$

$$\hat{\mathrm{H}}_0(k_x) = -\sum_n 2J_x \cos(k_x d - n\Phi)\hat{c}^\dagger_{k_x,n}\hat{c}_{k_x,n} + (J_y \hat{c}^\dagger_{k_x,n+1}\hat{c}_{k_x,n} + \mathrm{H.c.})$$

- Spectrum:



grey: periodic b.c. along y

colored: open b.c.

Laughlin pumping

- Add a constant force along x: $\hat{\mathrm{H}} = \hat{\mathrm{H}}_0 + \hat{F}_x = \hat{\mathrm{H}}_0 + F_x \sum_m m \, \hat{c}^\dagger_{m,n} \hat{c}_{m,n}$

- Take its effect into account through the gauge transf. $\hat{U} = \exp(i\hat{F}_x t/\hbar)$

- Within the single-band and adiabatic approximations, the momenta of Bloch states evolve as $\mathbf{k}(t) = \mathbf{k}_0 + \frac{t}{\hbar d} F_x \mathbf{e}_x$

- Linear response: $\mathbf{v}_j(\mathbf{k}) = \frac{1}{\hbar} \partial_{\mathbf{k}} E_j(\mathbf{k}) + \frac{F_x}{\hbar d} \mathcal{F}_j(\mathbf{k}) \mathbf{e}_y$

  group vel. anomalous vel.

- For a filled band: $\langle \mathbf{v}(t) \rangle = \frac{2\pi F_x}{\hbar d} \mathcal{C}_j \mathbf{e}_y$

[Thouless, Kohmoto, Nightingale, and den Nijs, PRL (1982)]

Single particles?

- Mean velocity: $\langle \mathbf{v}(t) \rangle = \sum_j \int_{BZ} \mathbf{v}_j(\mathbf{k}) \rho_j(\mathbf{k}, t) \mathrm{d}^2\mathbf{k}$

  where $\rho_j(\mathbf{k}, t) = |\langle u_j(\mathbf{k}) | \psi(\mathbf{k}, t) \rangle|^2$ is the overlap with the Bloch states

- Prepare a wavepacket extended along x, but concentrated along y; its Fourier transform is uniform along y: $\rho_j(\mathbf{k}, t) \approx \rho_j(k_x, t)$

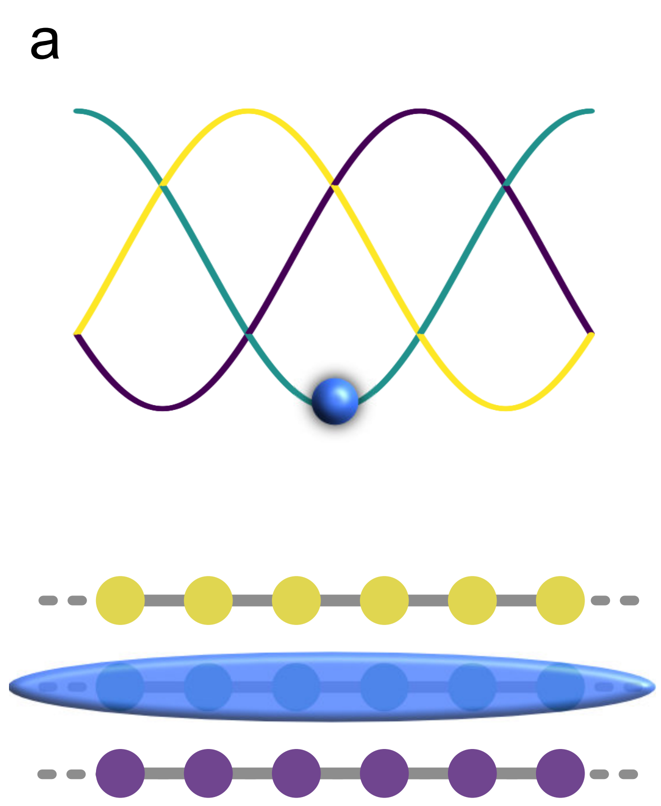
- Support on a single band: drop the sum

- A complete Bloch oscillation is performed over a period $T = \dfrac{2\pi\hbar}{q|F_x|}$

- Net displacement: $\langle \mathbf{r}(T) - \mathbf{r}(0) \rangle = \int_0^T \langle \mathbf{v}_j(t) \rangle \, \mathrm{d}t = \mathrm{sgn}(F_x) \mathcal{C}_j d \, \mathbf{e}_y$

during a Bloch period, the particle is “pumped” over a number of sites equal to the Chern number!

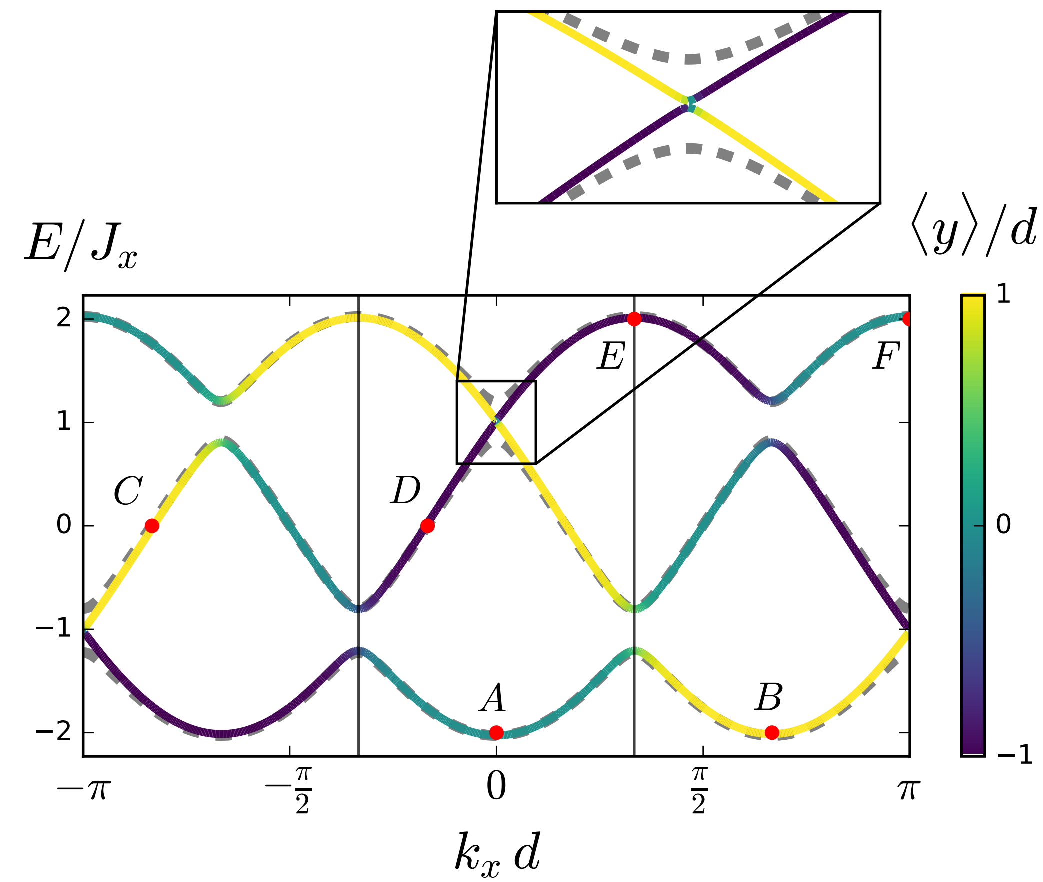
Proposal

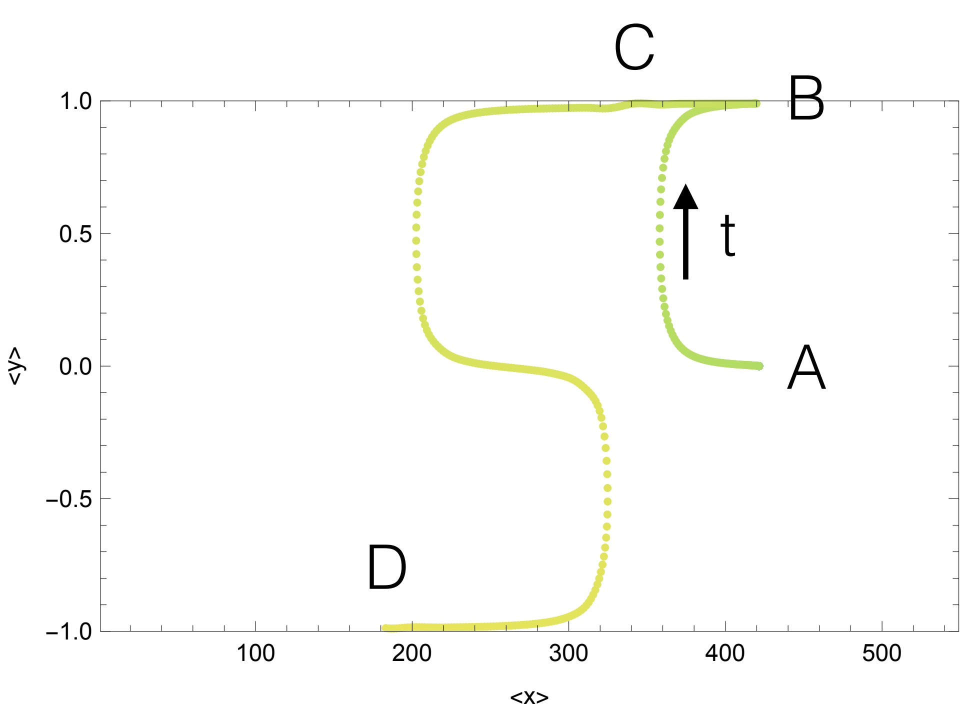


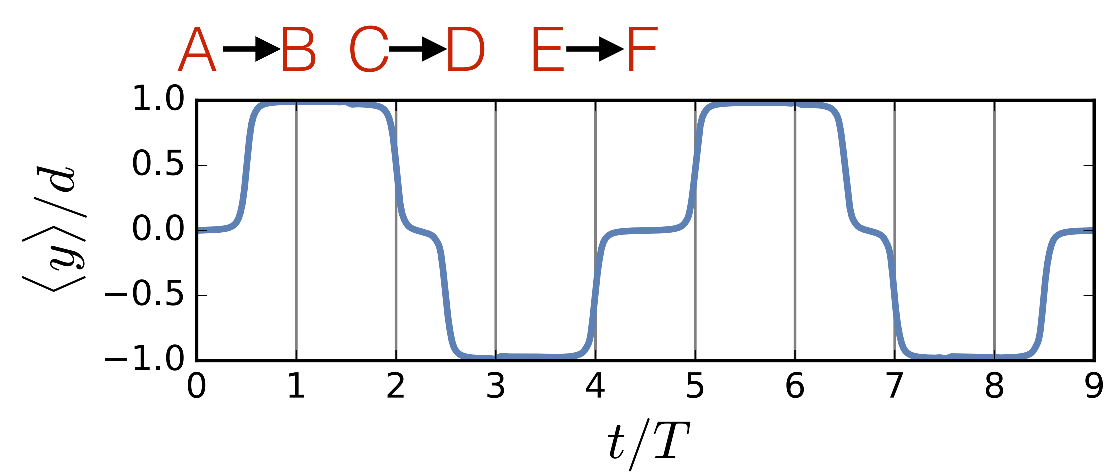
- a) prepare a wavepacket centered at y=0
- b) ramp up adiabatically the Raman coupling (“J_y”)
- c) turn on the force, and measure the transverse displacement at t=T

Pumping dynamics

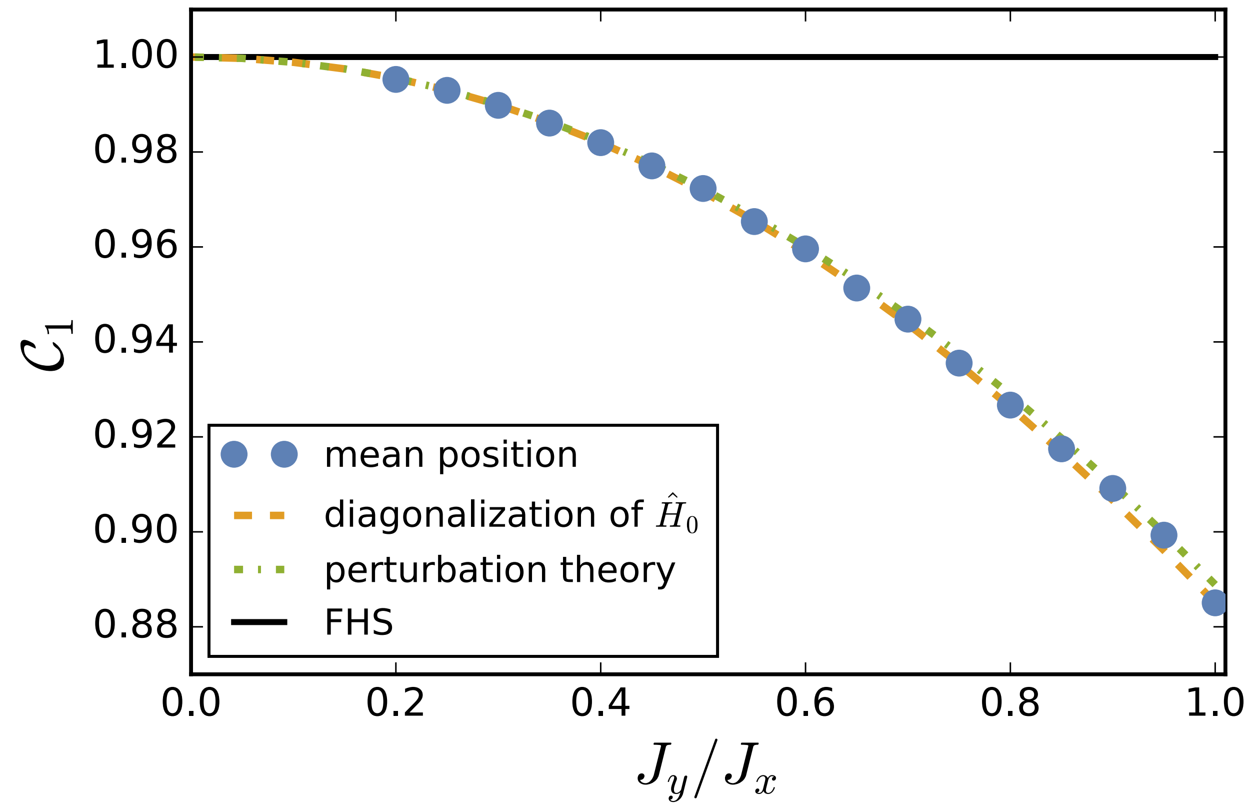
- Three-legged ladder ($N_y = 3$)
- Small Raman coupling: $J_y = 0.2 J_x$
- Force: $F_x = 0.02\,\Delta E_1/d$







Stronger Raman coupling



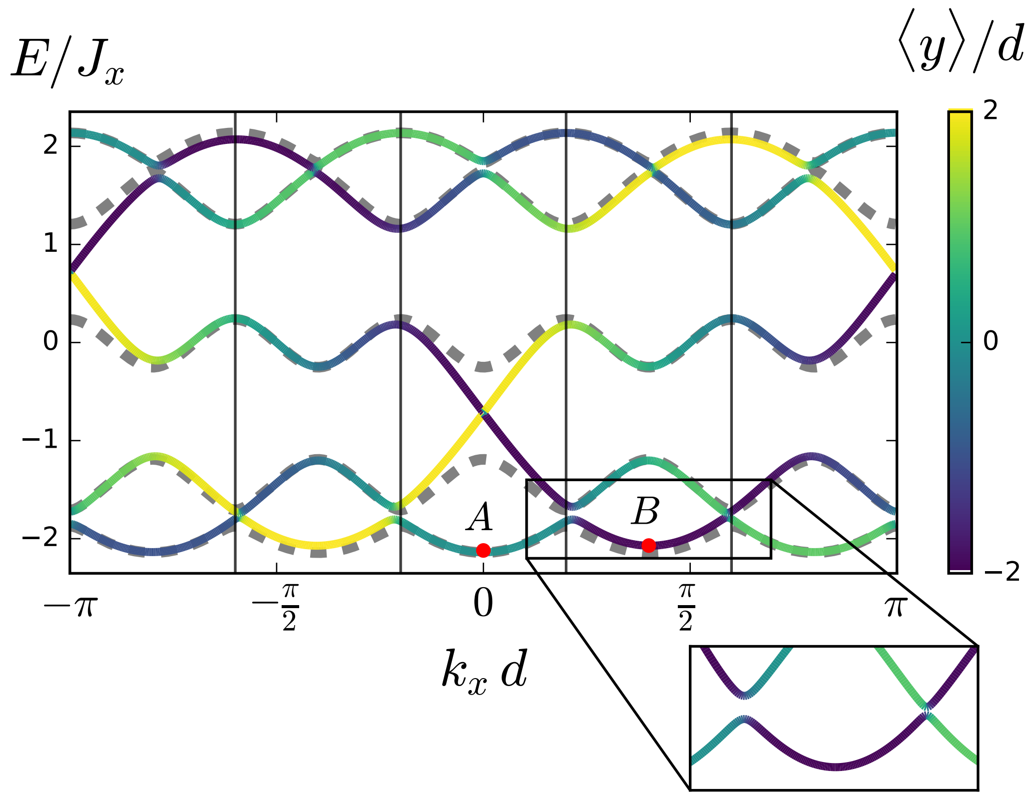
- Transverse displacement: $\langle \Delta y \rangle = \langle \Psi_T | \, \hat{n} \, | \Psi_T \rangle - \langle \Psi_0 | \, \hat{n} \, | \Psi_0 \rangle$

- Perturbation theory: $|\varphi_1\rangle \propto \left(|n=1\rangle + \frac{J_y/2J_x}{\cos(k_x d - \Phi) - \cos(k_x d)} \, |n=0\rangle \right) + \mathcal{O}\left(\lambda^2\right)$

- Displacement: $\langle \Delta y \rangle = \langle \varphi_1 | \, \hat{n} \, | \varphi_1 \rangle \Big|_{k_x = \frac{2\pi}{3}} = 1 - \left(\frac{J_y}{3J_x} \right)^2 + \mathcal{O}\left(\frac{J_y}{J_x} \right)^3$

Higher Chern numbers

- The ground band with $\Phi = \frac{4\pi}{5}$ has $\mathcal{C}_1 = -2$

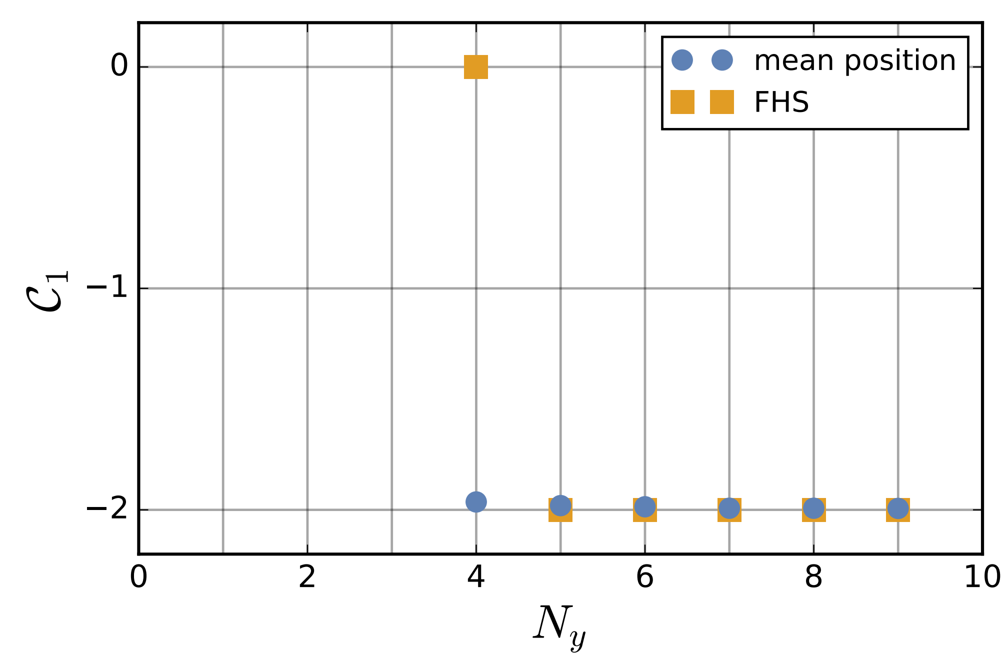


grey: periodic b.c. along y

colored: open b.c.

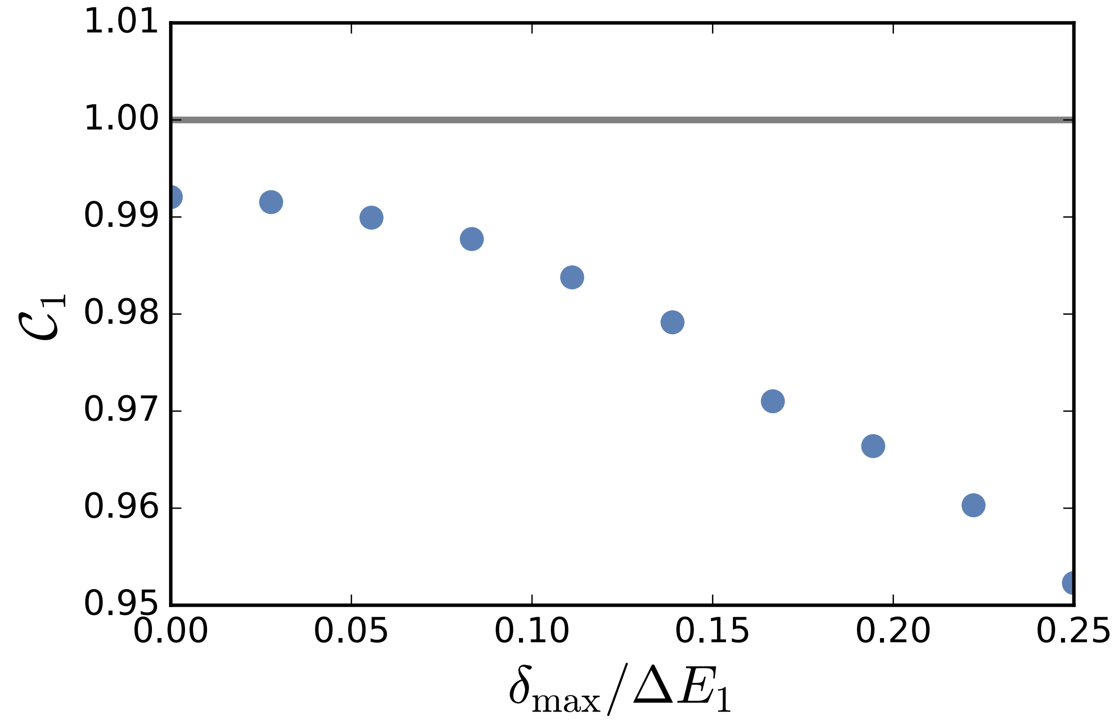
N_y=5

our method works
even when
the FHS algorithm fails!

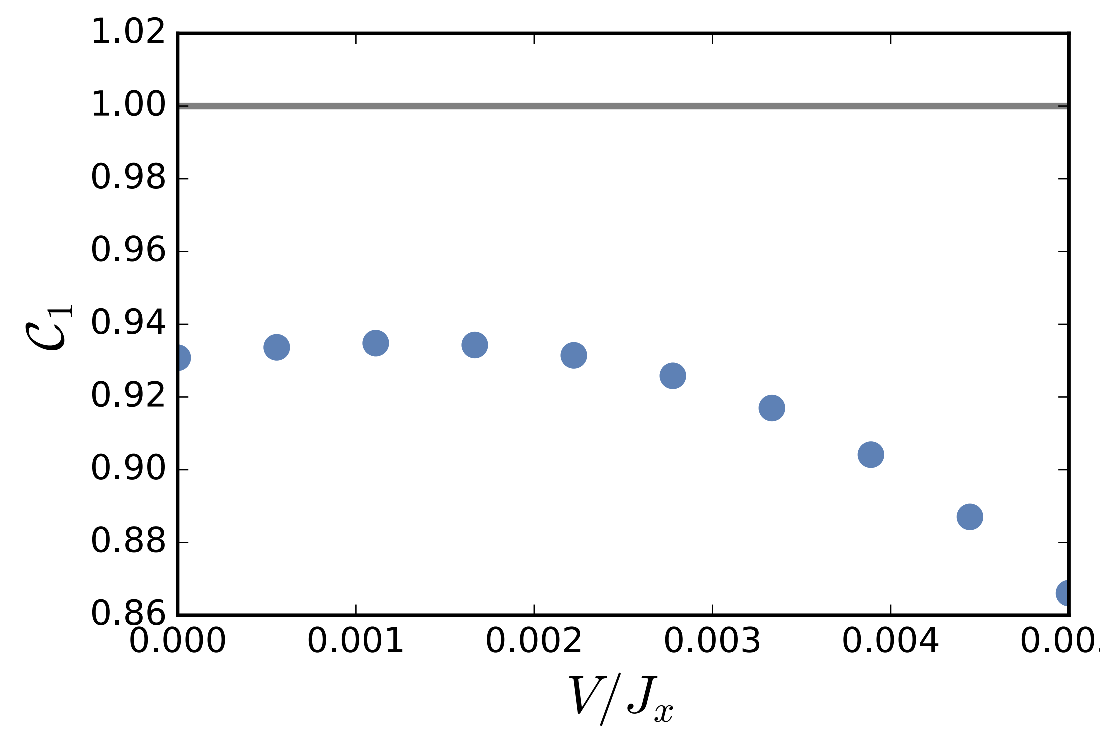


Robustness

- Static onsite disorder:



- Harmonic trapping:



Conclusions

- Transverse conductance remains “almost quantized” even in very narrow strips (which have tiny bulk regions)

- Powerful method to read out the Chern number, which:
  - works for all bands in the limit $J_y \ll J_x$
  - may be applied whenever $N_y < 2q$
  - is resistent to disorder
  - may be applied in harmonic traps