# Bose polarons at finite temperature and strong coupling


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## Introduction

Mobile impurities in a quantum bath play a fundamental role in a wide range of systems including metals, semiconductors, Helium mixtures, and high-$T_c$ super-conductors. The case of impurities in a Fermi sea is by now relatively well understood (see, e.g., the recent [1,2], and refs. therein).

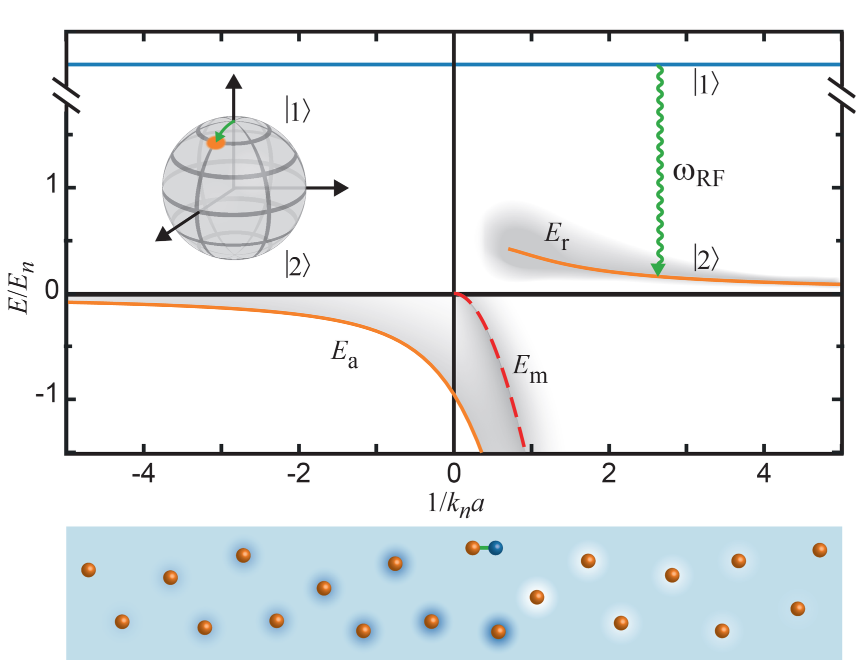
Recently, two experimental groups embedded impurities in a Bose-Einstein condensate (BEC) and observed long-lived quasiparticles coined *Bose polarons* [3,4].

On general grounds, the properties of Bose polarons should depend significantly on temperature, since the low-energy density of states in a Bose gas changes dramatically below the BEC transition.

Indeed, we show that the attractive polaron fragments into two quasiparticles for *0<T<T*$_c$ whenever $|a| \gtrsim a_B$. The upper quasiparticle disappears at $T_c$, while the lower one remains well-defined across the critical temperature.

## System

- A **single impurity** of mass $m$
- Bath: bosons of mass $m_B$, density $n$, and temperature $T$
- $\hbar = k_B = 1$
- Momenta and energies in units of $k_n = (6\pi^2 n)^{1/3}$, $E_n = k_n^2/2m_B$
- Weakly-interacting bath $(0 < k_n a_B \ll 1)$ — Aarhus: $k_n a_B = 0.01$
- Impurity-bath scattering length: $a$
- Spectrum at *T=0*:




Source: Jørgensen *et al.*, Ref. [3]

## Model

- Bath treated with Bogoliubov theory

» critical temperature: $T_c = \frac{2\pi}{m_B}\left(\frac{n}{\zeta(\frac{3}{2})}\right)^{2/3} \approx 0.436 E_n$

» condensate density: $n_0 = n[1-(T/T_c)^{3/2}]$

» bath chemical potential: $\mu_B = \mathcal{T}_B n_0$

» bath scattering matrix: $\mathcal{T}_B = 4\pi a_B/m_B$

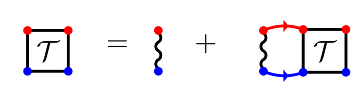
» dispersion of the excitations: $E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^B(\epsilon_{\mathbf{k}}^B + 2\mu_B)}$

» free bosons: $\epsilon_{\mathbf{k}}^B = k^2/2m_B$

- Impurity-bath coupling treated **non-perturbatively** by means of **finite temperature Green's functions**.
- Polaron energy: $\omega_{\mathbf{p}} = \epsilon_{\mathbf{p}} + \mathrm{Re}[\Sigma(\mathbf{p}, \omega_{\mathbf{p}})]$
- Polaron residue: $Z_{\mathbf{p}} = \dfrac{1}{1 - \partial_\omega \mathrm{Re}[\Sigma(\mathbf{p},\omega)]|_{\omega_{\mathbf{p}}}}$

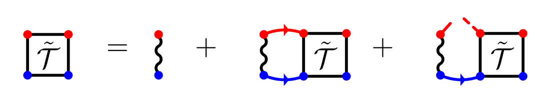
## *Extended ladder* approximation

Impurity Green's function: $\mathcal{G}(\mathbf{p}, i\omega_j) = \dfrac{1}{\mathcal{G}_0(\mathbf{p}, i\omega_j)^{-1} - \Sigma(\mathbf{p}, i\omega_j)}$

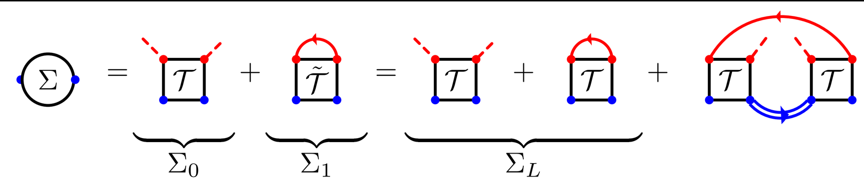


$\mathcal{T}_v = \dfrac{2\pi a}{m_r}$

Ladder T-matrix: $\mathcal{T}(\mathbf{p}, i\omega_j)^{-1} = \mathcal{T}_v^{-1} - \Pi(\mathbf{p}, i\omega_j)$



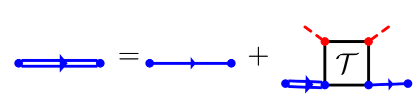
Extended T-matrix: $\tilde{\mathcal{T}}(\mathbf{p}, i\omega_j)^{-1} = \mathcal{T}_v^{-1} - \Pi(\mathbf{p}, i\omega_j) - n_0\mathcal{G}_0(\mathbf{p}, i\omega_j)$




Extended self-energy: $\Sigma = \Sigma_0 + \Sigma_1$

$$\Sigma_0(\mathbf{p}, i\omega_j) = n_0 \mathcal{T}(\mathbf{p}, i\omega_j)$$

$$\Sigma_1(\mathbf{p}, i\omega_j) = \int \frac{d^3k}{(2\pi)^3}\left[u_{\mathbf{k}}^2 f_{\mathbf{k}} \tilde{\mathcal{T}}(\mathbf{k}+\mathbf{p}, i\omega_j + E_{\mathbf{k}}) + v_{\mathbf{k}}^2(1+f_{\mathbf{k}})\tilde{\mathcal{T}}(\mathbf{k}+\mathbf{p}, i\omega_j - E_{\mathbf{k}})\right]$$

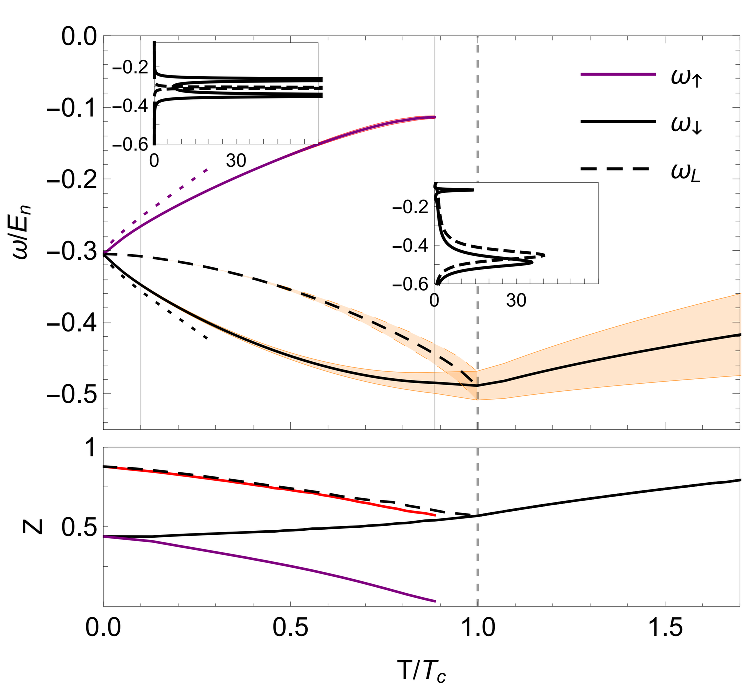


(impurity dressed by BEC only)

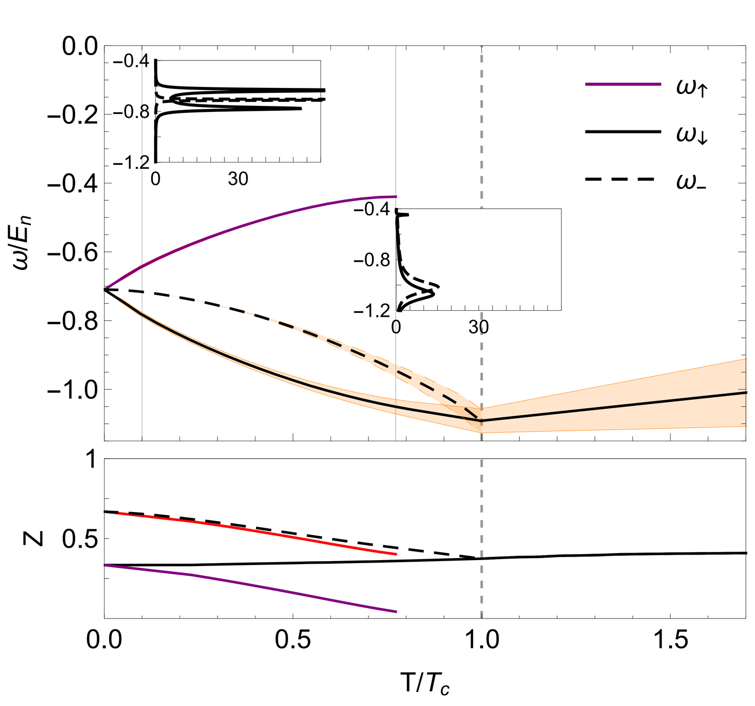
(the ladder approximation at T=0 was considered in Ref. [5])

## The attractive polaron fragments into two quasiparticles for 0<T<T$_c$

- Weak attraction: $(k_n a = -1)$




- Unitarity:

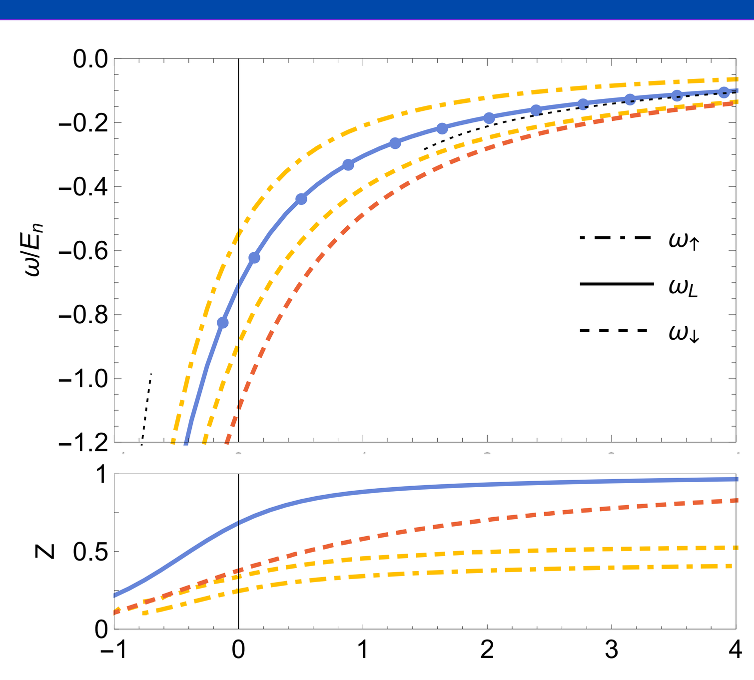



## Attractive polaron(s) across the Feshbach resonance

*T=0*

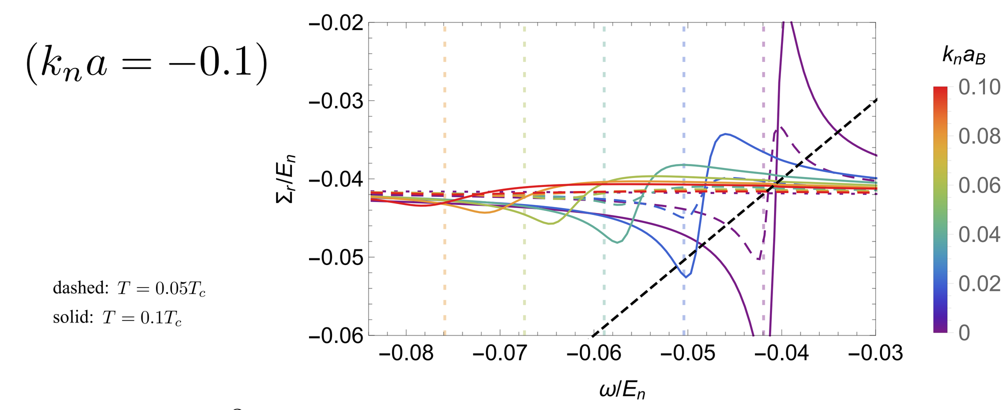
*T=0.5T*$_c$

*T=T*$_c$




## Understanding fragmentation

- Weak coupling, low temperature behavior of $\Sigma$:




$$\Sigma_1(\omega) \approx \int \frac{d^3k}{(2\pi)^3}\frac{f_{\mathbf{k}}}{\mathcal{T}_v^{-1} - \Pi(\mathbf{k}, \omega + E_{\mathbf{k}}) - \frac{n_0}{\omega + E_{\mathbf{k}} - \epsilon_{\mathbf{k}}}}$$

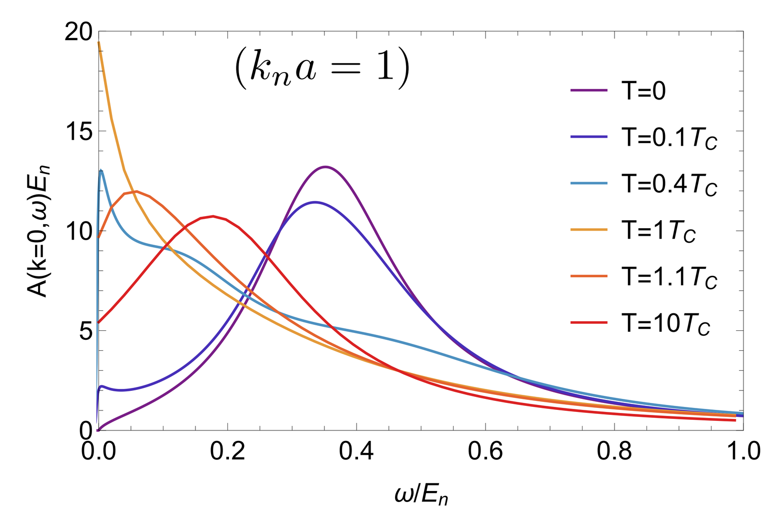
$$\approx \frac{\omega + n_0\mathcal{T}_B}{\omega - n_0(\mathcal{T}_v - \mathcal{T}_B)} n_{\mathrm{ex}}\mathcal{T}_v$$

on-shell for: $\omega + E_{\mathbf{k}} = \epsilon_{\mathbf{k}} + \Sigma_0(\mathbf{k}, \omega + E_{\mathbf{k}})$

| $\|a\| \gtrsim a_B$ : equal splitting | $\|a\| \lesssim a_B$ : single polaron |
|---|---|
| $\omega_{\uparrow,\downarrow} \simeq \omega_0[1 \pm (Z_0 n_{\mathrm{ex}}/n_0)^{1/2}]$ | $\omega_\uparrow \simeq n\mathcal{T}_v$ |
| $Z_{\uparrow,\downarrow} \simeq Z_L/2$ | in accord with perturbation theory [6] |

## Repulsive polarons

- At this level of approximation, in a neighborhood of $T_c$ the repulsive polaron is replaced by a very broad peak centered at $\omega$=0




- The continuum however should start at the energy of the repulsive polaron; self-consistent theory is needed…

## Outlook and conclusions

- Upon increasing the temperature, the attractive polaron present at *T=0* fragments into two quasiparticles
- Purely non-perturbative effect, due to the presence of a dressed propagator inside $\Sigma$
- Physically, the effect arises due to the large low-energy density of states available at small $k_n a_B$ and finite $T$
- The fragmentation should be observable in state-of-the-art experiments
- Open question: does the fragmentation of the attractive polaron at *T>0* arise also in lower dimensions?

## References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**[this work]** N.-E. Guenther, P. Massignan, M. Lewenstein, and G. M. Bruun, *Bose polarons at finite temperature and strong coupling*, arXiv:1708.08861.


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