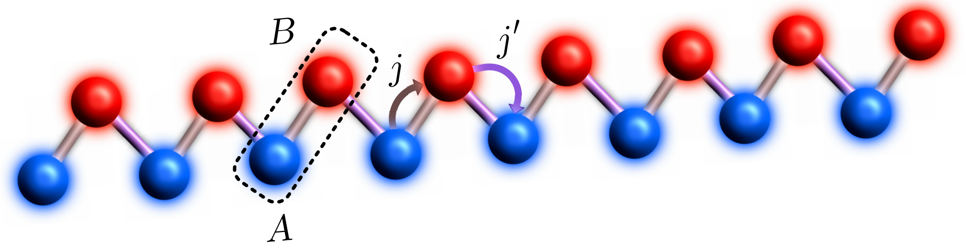
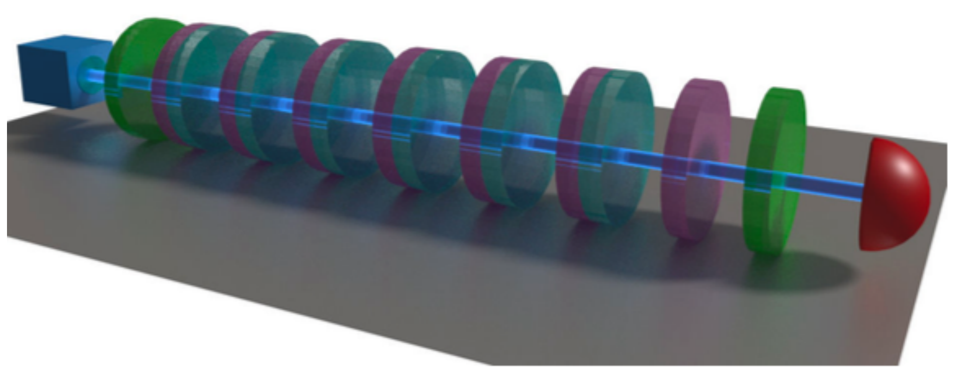
Detection of bulk topological features in real time

Pietro Massignan





UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

ICFO The Institute of Photonic Sciences

Main collaborators

Theory
(Barcelona)

ICFO
Institut
de Ciències
Fotòniques


Alessio D'Errico


Filippo Cardano


Lorenzo Marrucci


Maria Maffei


Alexandre Dauphin


Maciej Lewenstein

Experiment
(Naples)

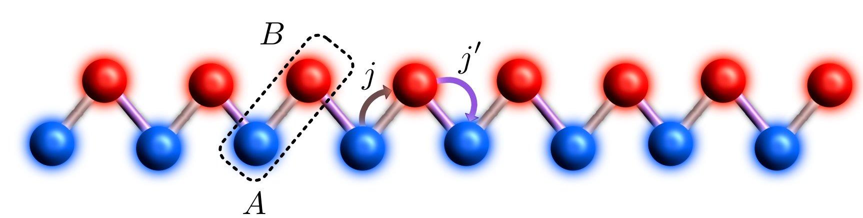
SLAM group

Outline

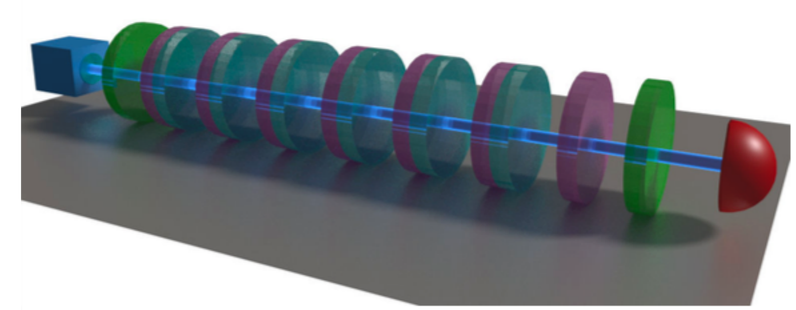
- Topology in condensed matter systems

- One-dimensional chiral models

  - static (SSH)

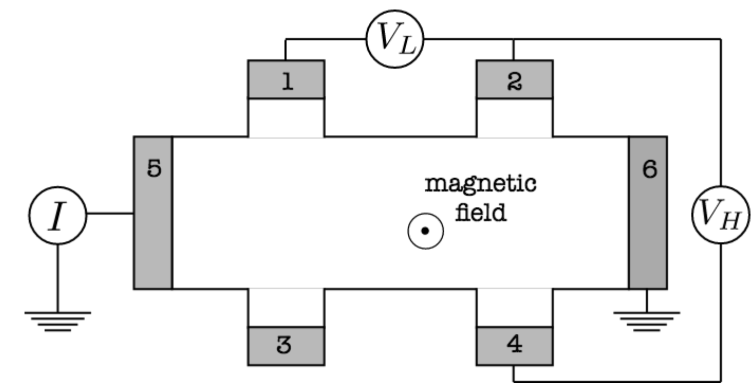


  - periodically-driven
    (photonic quantum walk)

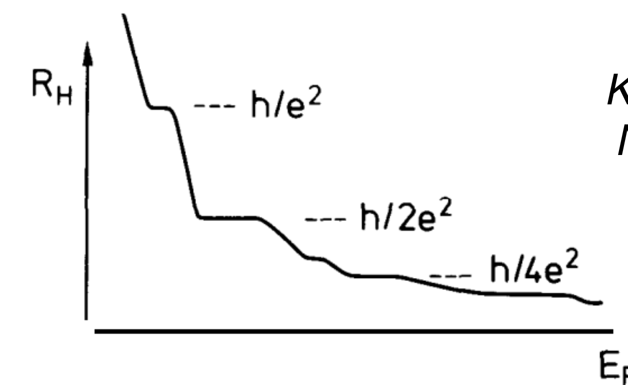


Hall effect

- Classical Hall effect (1879):
  when current flows in a 2D material,
  in presence of an out-of-plane B field,
  there appears a transverse (Hall) current

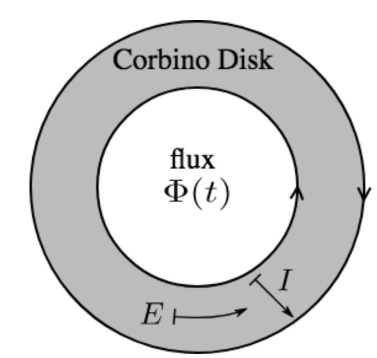


- Quantum Hall effect (1980):
  at low temperatures and high-B,
  the Hall current is quantized!



K. Von Klitzing
Nobel lecture

- Laughlin (1982): robustness due to **topology**



- TKNN (1982): Kubo formula links conductivity
  to the Chern number, a topological invariant
  defined on the occupied bands

Thouless, Kohmoto, Nightingale & den Nijs
Phys. Rev. Lett. (1982)

Topological insulators

- Insulators in the bulk, but have robust current-carrying edge states
- Protected by the topology of bulk bands against local perturbations, like *disorder* and *defects*
- Enormous progresses in the last 10 years (QSH, 3D TIs., 4D QH, …)
- Characterization non-interacting TIs in terms of <u>discrete symmetries</u>
  T: time-reversal
  C: charge-conjugation
  S: chiral

IQHE, Hofstadter, Chern insulators → A

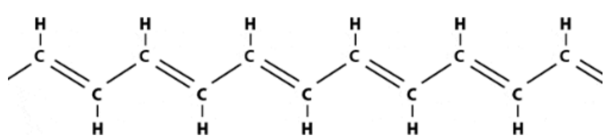
chiral → AIII, BDI

Class	T	C	S	# of dimensions 0	1	2	3	4	5	6	7
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	Chern number			$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z}$	Winding		0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

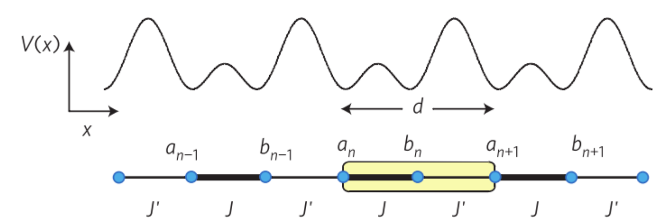
- Beyond the periodic table:
  Mott / Anderson / crystalline / Floquet TIs, …

Chiu, Teo, Schnyder & Ryu, Rev. Mod. Phys. (2016)

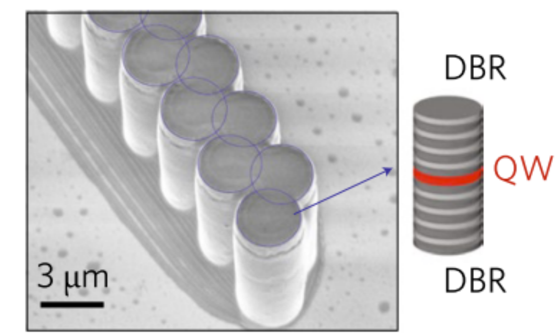
1D chiral systems



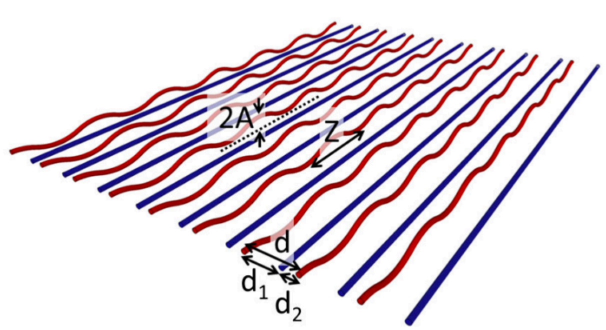
polyacetilene
[Nobel prize in Chemistry 2000]



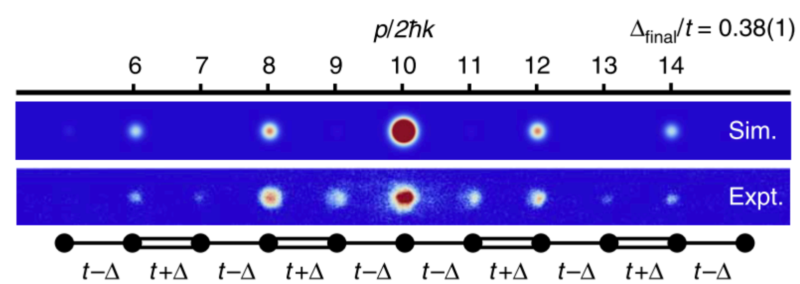
ultracold atoms
in superlattices
[M. Atala *et al.*, Nat. Phys. 2013]



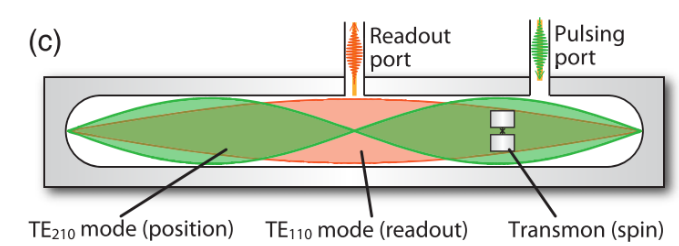
cavity polaritons
[St. Jean *et al.*, Nat. Phot. 2017]



optical waveguides
[Zeuner *et al.*, PRL 2015]



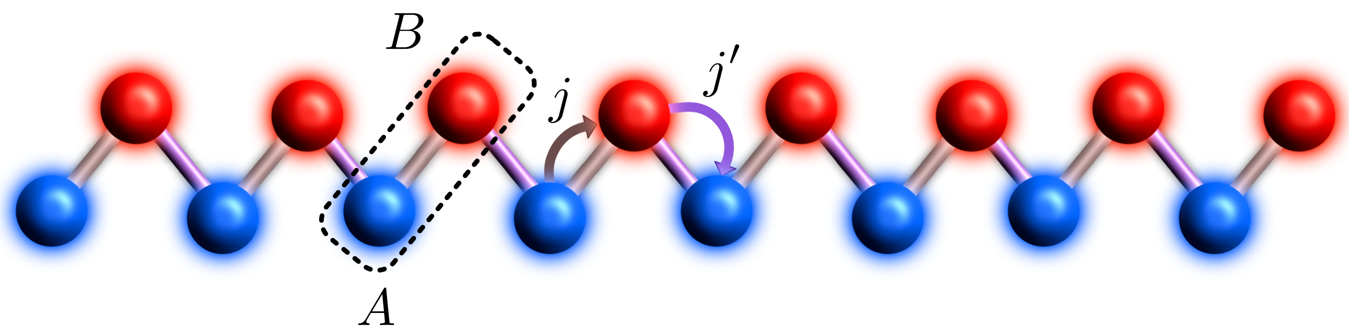
ultracold atoms
in k-space lattices
[Meier *et al.*, Nat. Comm. 2016]



SC qubits
in mw-cavities
[Flurin *et al.*, PRX 2017]

SSH model

- Spinless fermions with staggered tunnelings:



Su, Schrieffer & Heeger
Phys. Rev. Lett. (1979)

Asbóth, Oroszlány, & Pályi
Lecture Notes in Physics (2016)

- $\exists$ two sublattices

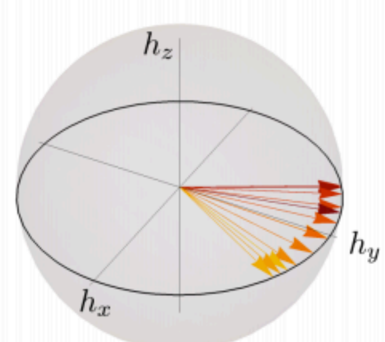
  $\exists$ a “canonical” basis where H is purely off-diag: $H = \begin{pmatrix} 0 & h^\dagger \\ h & 0 \end{pmatrix}$

- Chiral symmetry: $\Gamma H \Gamma = -H$ (Γ: unitary, Hermitian, local)

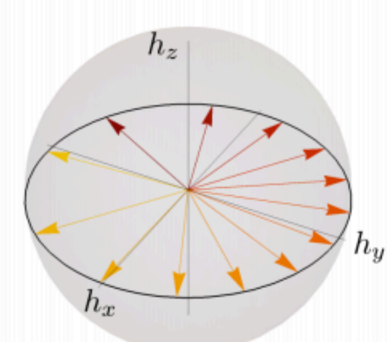
- In mom. space the Hamiltonian is 2*2, $H_k = E_k \mathbf{n}_k \cdot \boldsymbol{\sigma}$

- In the canonical basis, $\mathbf{n}_k \perp \hat{\mathbf{z}} \quad \forall k$ and $\Gamma = \sigma_z$

- Winding:



$\mathcal{W} = 0$



$\mathcal{W} = 1$

The winding $\mathcal{W}$

- $\mathcal{W}$ may be calculated:

$$H_k = E_k \mathbf{n}_k \cdot \boldsymbol{\sigma}$$

  - from $\mathbf{n}$: $\mathcal{W} = \oint \frac{\mathrm{d}k}{2\pi} \left(\mathbf{n} \times \partial_k \mathbf{n}\right)_z$
  - from the *eigenstates*: $\mathcal{W} = \oint \frac{\mathrm{d}k}{\pi} \mathcal{S},$ $\quad \mathcal{S} = i\langle \psi_+ | \partial_k \psi_- \rangle$

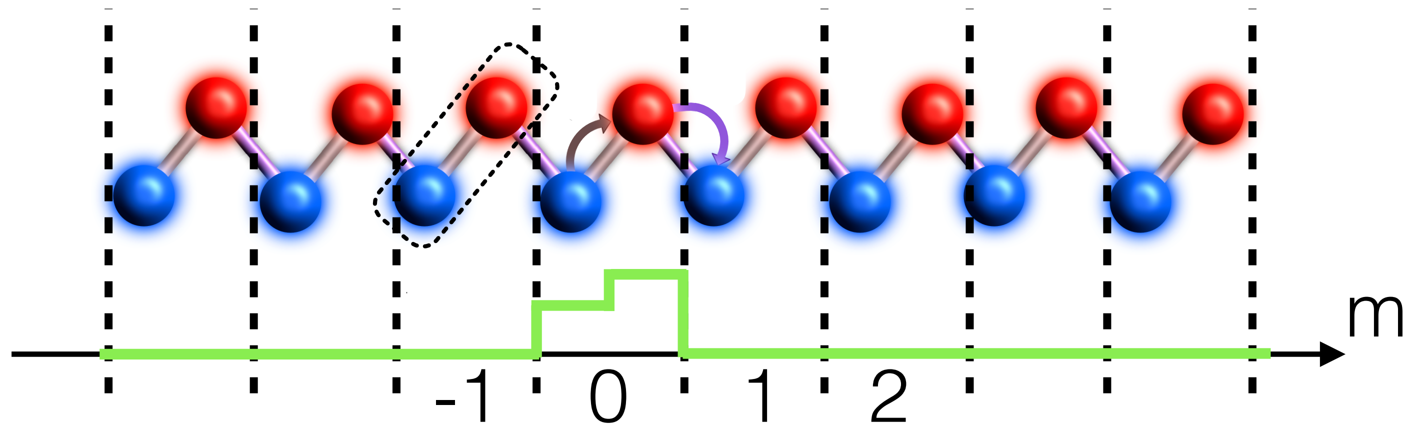
skew polarization

- What if the Hamiltonian is not known?
  Can one *measure* the winding?

  Yes, and it's simple!

Evolution in real time

- Initial condition
  localized on the $m\!=\!0$ cell:

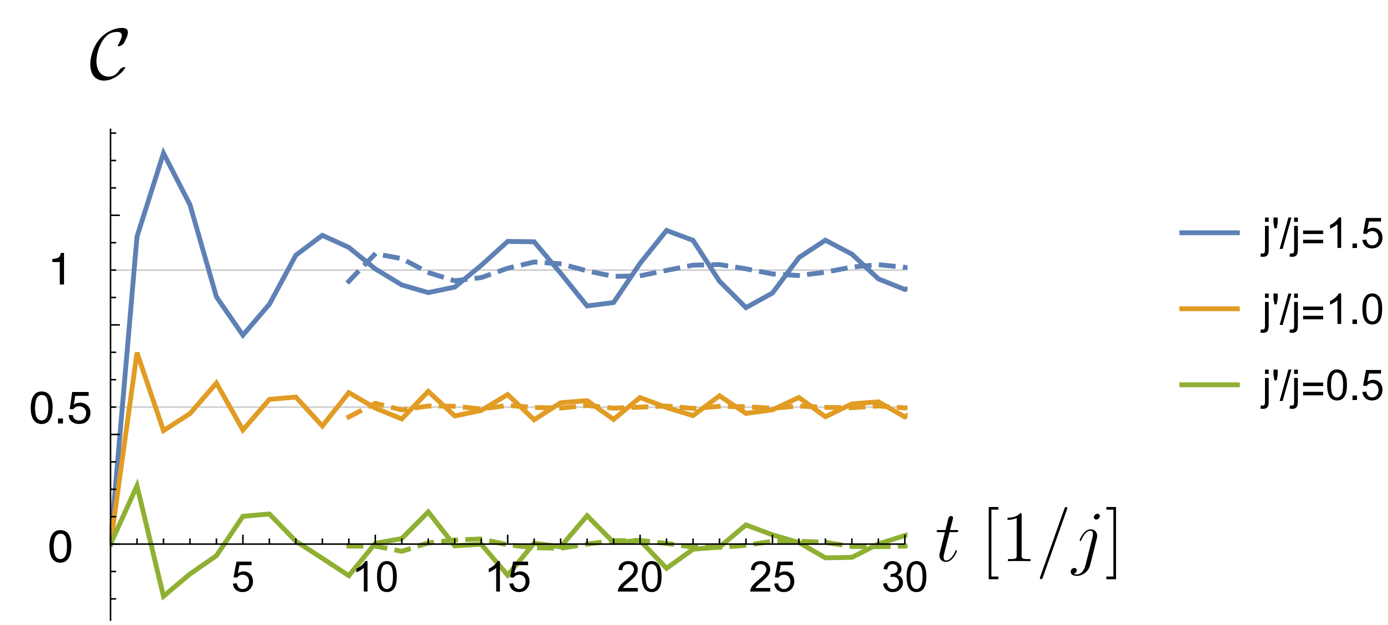


- **Mean Chiral Displacement:**

$$\mathcal{C}(t) \equiv 2\langle\widehat{\Gamma m}(t)\rangle = 2\int_{-\pi}^{\pi}\frac{\mathrm{d}k}{2\pi}\,\langle U^{-t}\sigma_z(i\partial_k)U^t\rangle_{\psi_0} = 2\int_{-\pi}^{\pi}\frac{\mathrm{d}k}{2\pi}\sin^2(Et)\,|\mathbf{n}\times\partial_k\mathbf{n}| \quad \xrightarrow{t\to\infty} \quad \mathcal{W}$$

- Easy to measure:

$$\mathcal{C}(t) = 2\Big[\langle m_A(t)\rangle - \langle m_B(t)\rangle\Big]$$



- Fast convergence

- Bulk measurement!

Cardano, D'Errico, Dauphin, Maffei, … Marrucci, Lewenstein & PM
Nature Comm. (2017)

Resistance to disorder

SSH model in the topological phase $\quad j' = 2j \quad \rightarrow \quad \begin{cases} \mathcal{W} = 1 \\ \Delta_{\text{gap}} = 2j \end{cases}$

+

independent disorder of amplitude Δ on ***all*** tunnelings

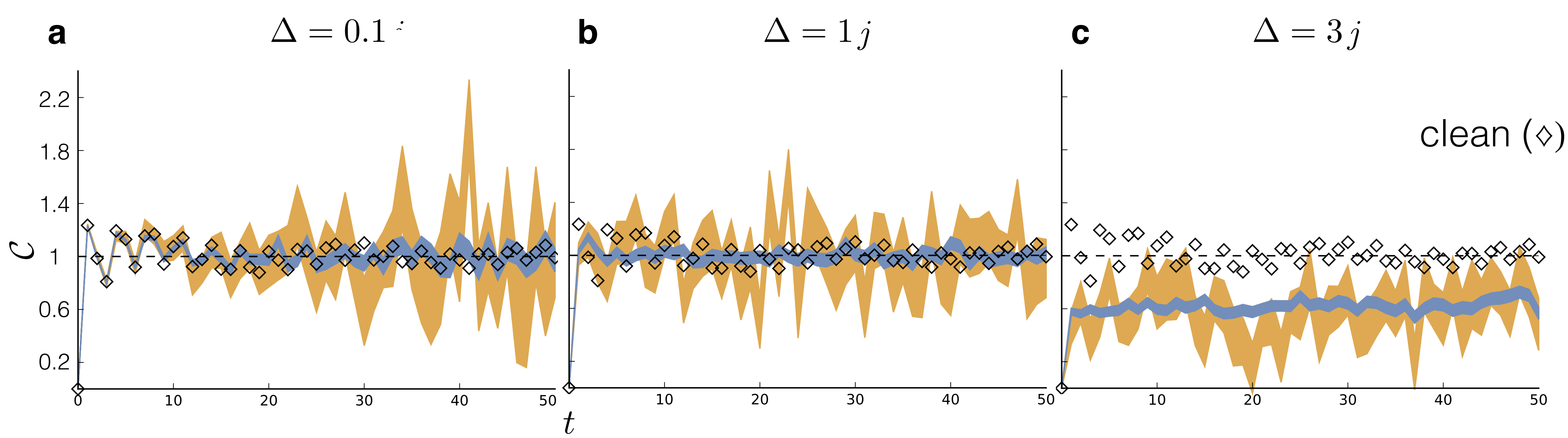
+

randomly-polarized localized initial condition

+

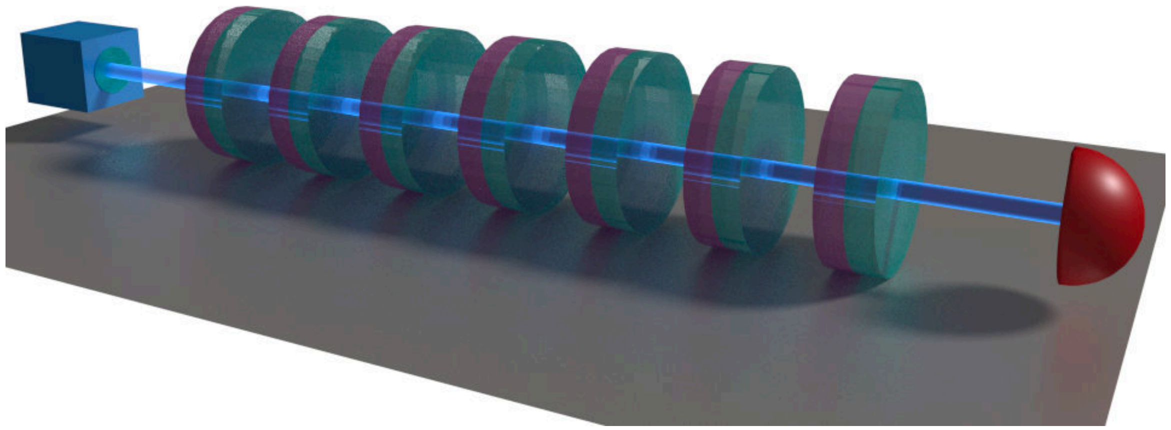
average over 50 (1000) disorder realizations

⬇



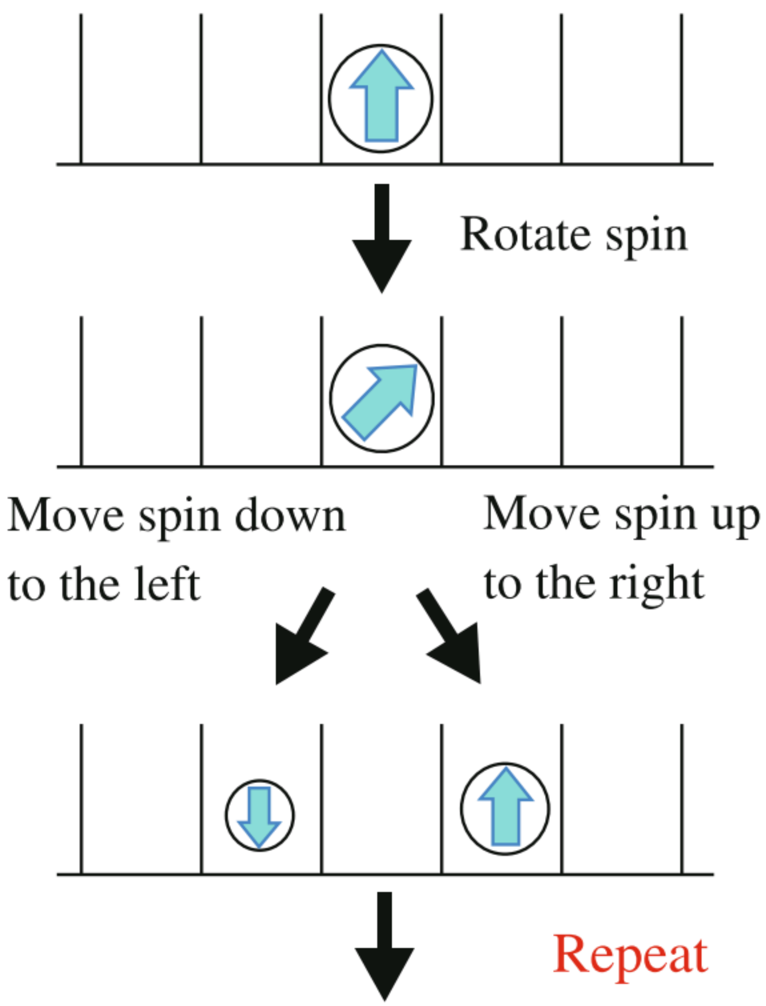
the MCD stays locked to the topological invariant as long as $\Delta < \Delta_{\text{gap}}$

Floquet 1D chiral models



photonic quantum walk of *twisted* photons

Discrete-Time Quantum Walk

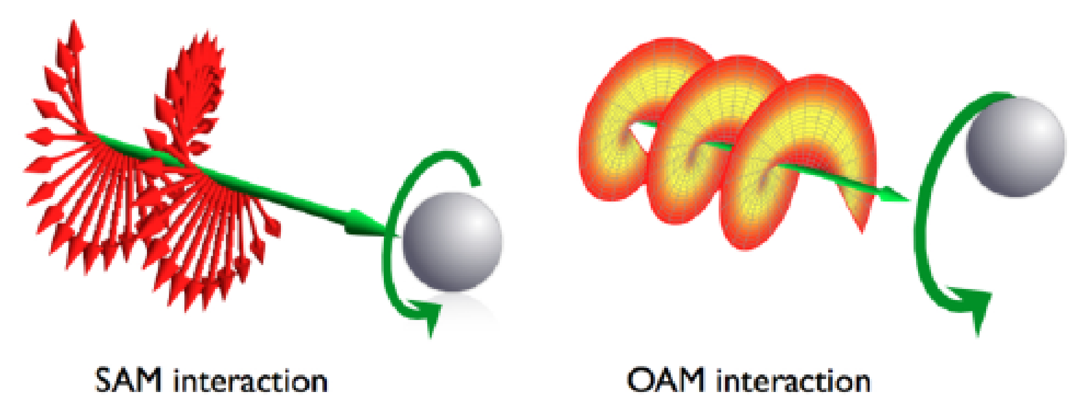


[Kitagawa, QIP (2012)]

Twisted photons

25th anniversary: Allen et al., PRA (1992)

- Collimated monochromatic beam propagating along $\hat{\mathbf{z}}$
- Light has linear momentum $\mathbf{p} \propto \mathbf{E}^* \times \mathbf{B}$ (“push”)
- But it can also carry also *angular momentum*
- In the “paraxial approximation”, $\hat{J}_z = \hat{S}_z + \hat{L}_z$
- “Spin” AM: $\hat{S}_z = \hbar \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
- Orbital AM: $\hat{L}_z = -i\hbar(\mathbf{r} \times \boldsymbol{\nabla})_z$



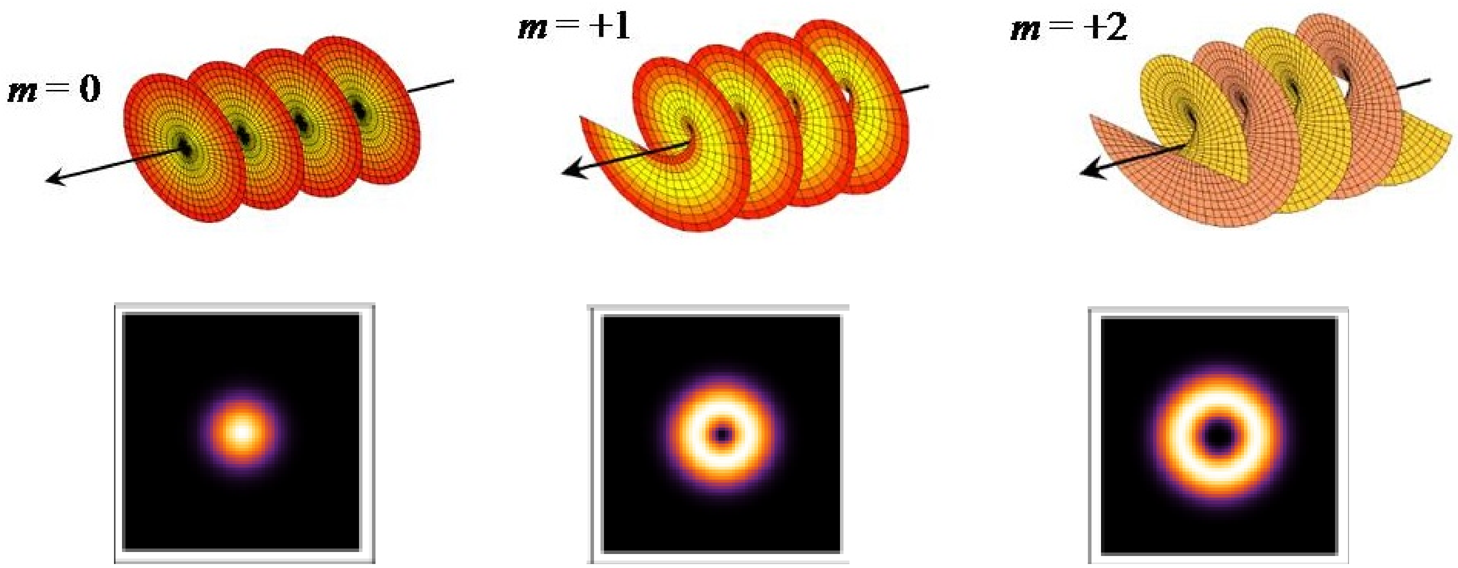
circularly polarized light interacts with the particle’s spin

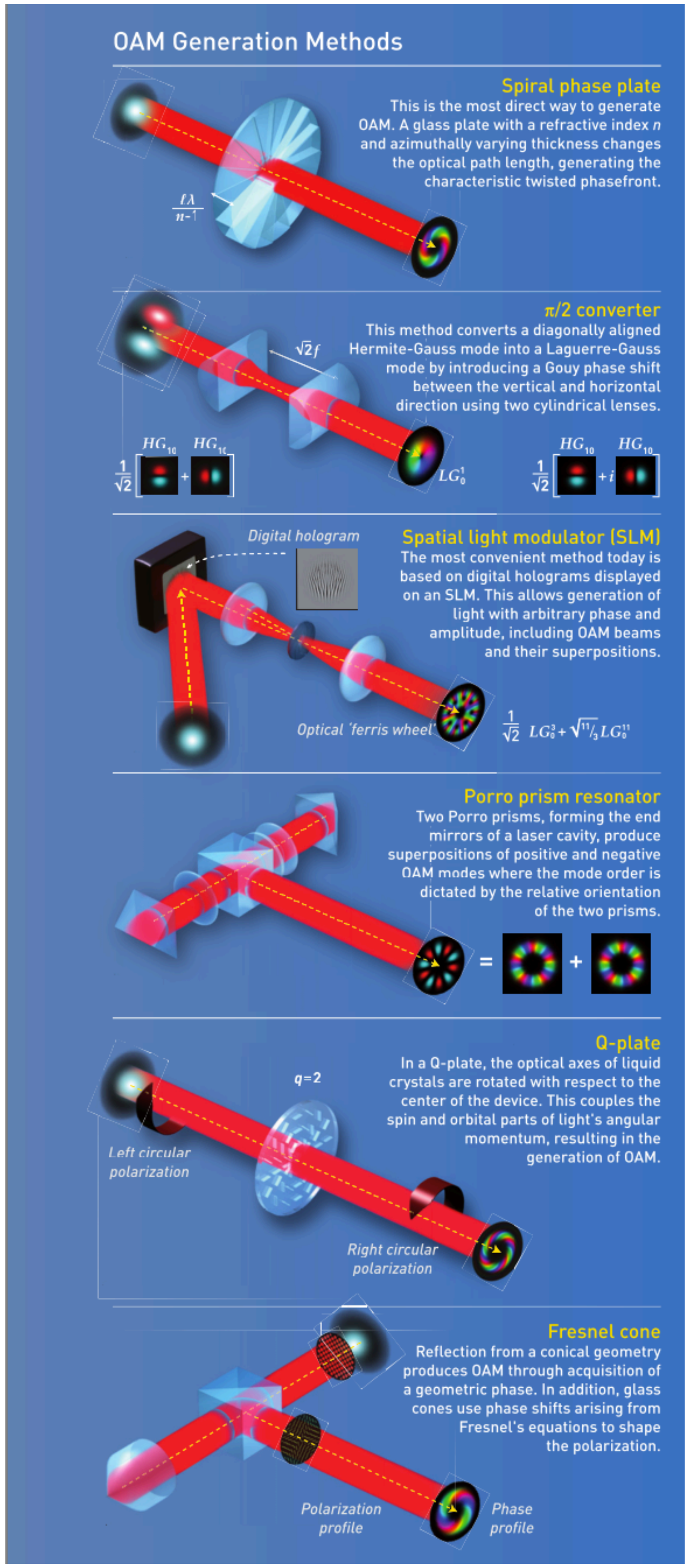
light with OAM rotates a particle around the beam axis

Twisting light

- Helical modes have a phase pattern $e^{im\phi}$

- Their OAM is quantized, $\hbar m$

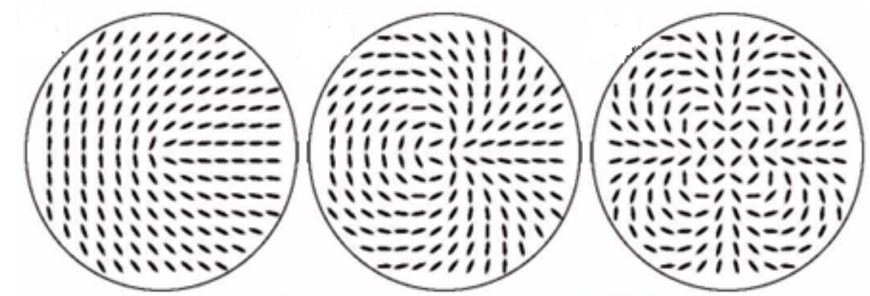




Franke-Allen & Radwell
Optics&Photonics News (2017)

Q-plates

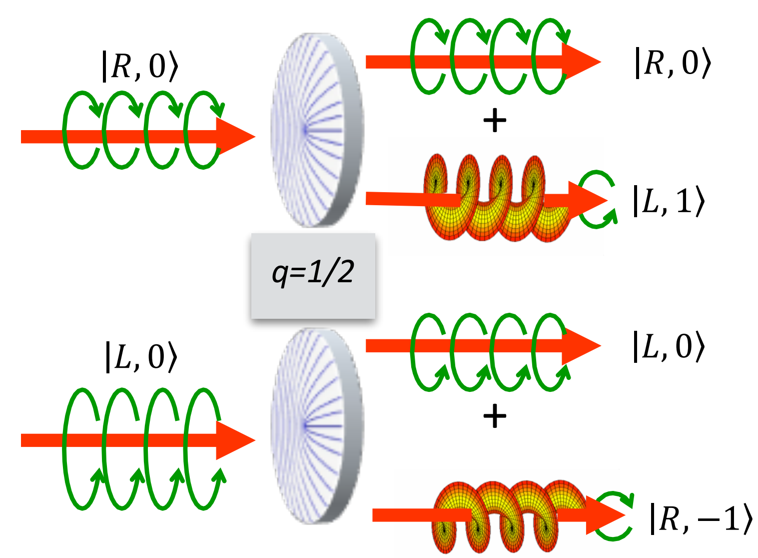
- Liquid crystals deposited on glass plates along singular patterns cause phase retardation of the beam



q=1/2 q=3/2 q=3

- Q-plates mix OAM and SAM:

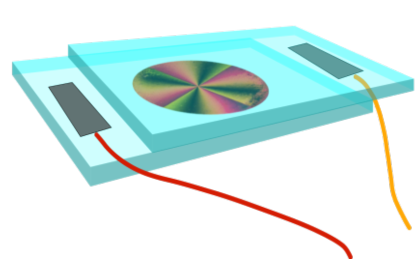
  (“spin-dependent translation”)



$\cos^2(\delta/2)$

$\sin^2(\delta/2)$

- An external voltage controls the orientation of the LCs, and therefore the mixing parameter δ



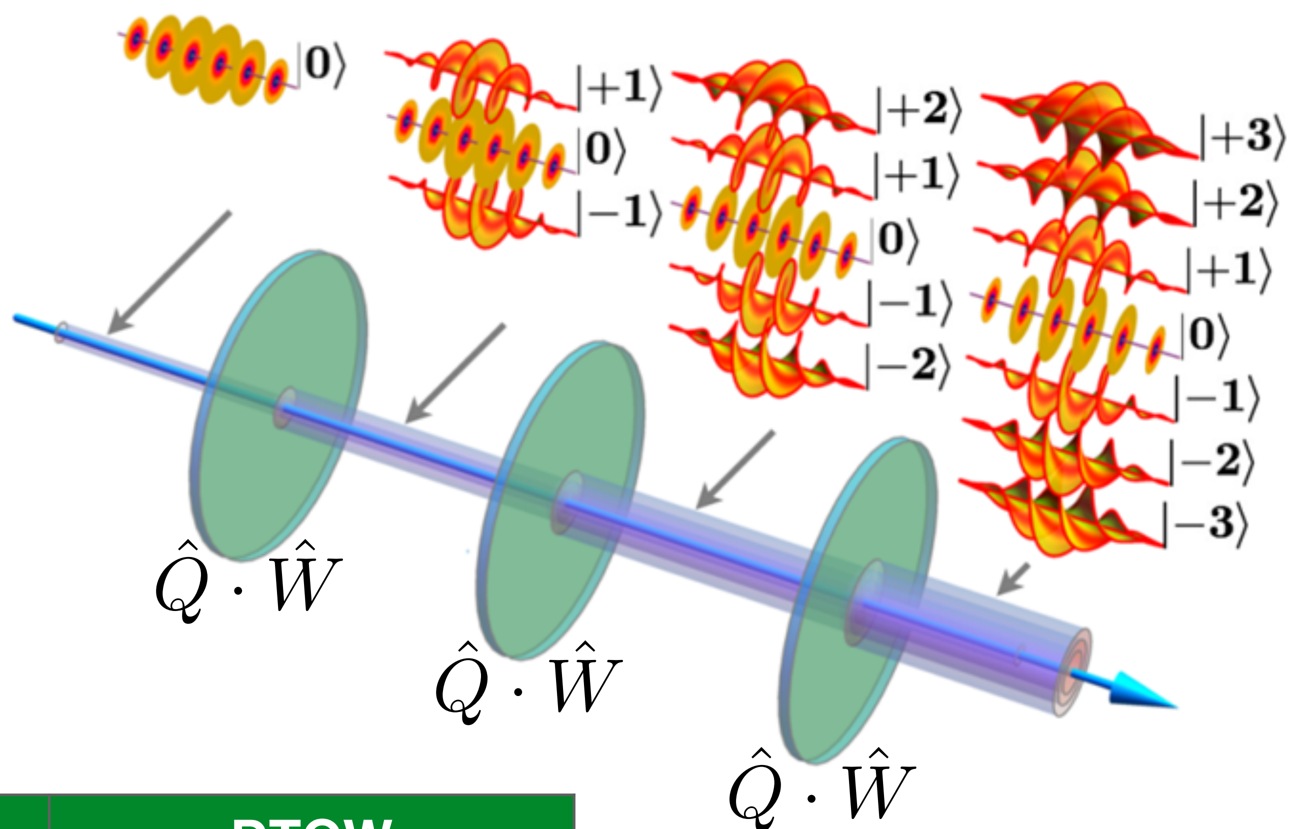
[Marrucci *et al.*, Phys. Rev. Lett.(2006)]

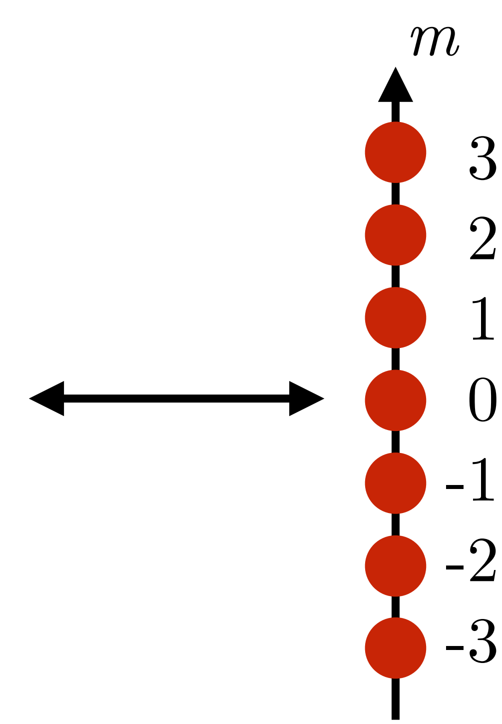
Discrete-Time Quantum Walk with twisted photons

- Cascade of Q-plates and quarter-wave plates W

$$\hat{W} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

- Initial state: $m=0$ OAM, and a given polarization



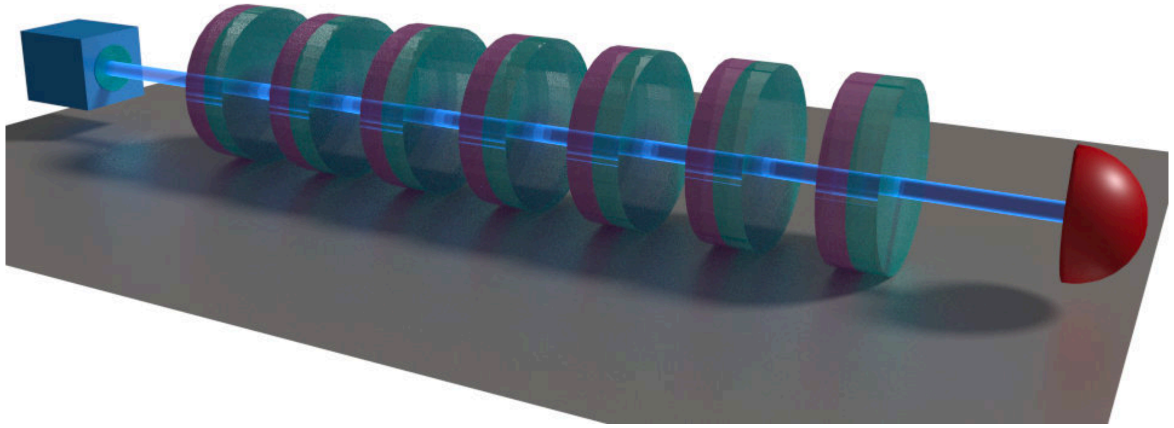


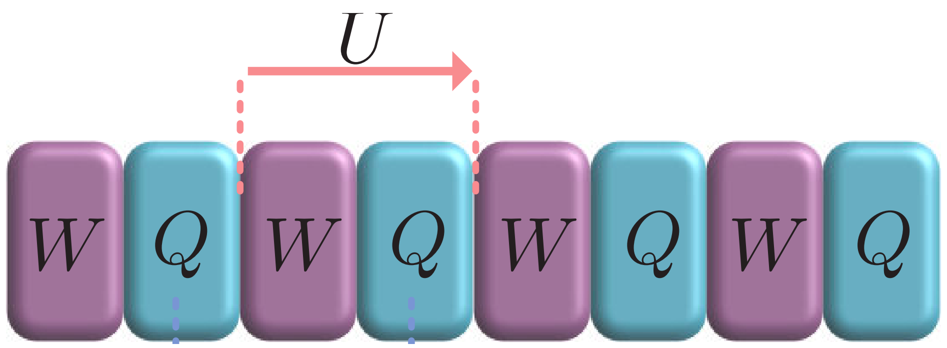
a synthetic dimension!

Twisted photons	DTQW
OAM (m)	walker's position
polarization (⟳/⟲)	coin state (↑/↓)
Q-plate	conditional displacement
$\hat{\mathbf{z}}$	time

[Cardano et al., Science Advances (2015)]

Discrete-Time Quantum Walk



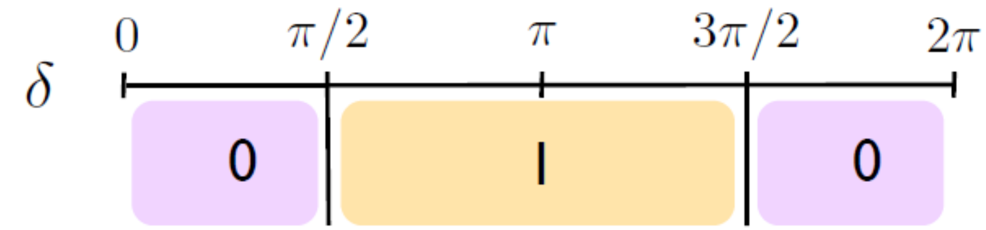


- Periodic evolution: may be treated via Floquet theory
- One-step evolution operator U ➙ $H_{\text{eff}} \equiv i(\log U)/T$
- In momentum space, $H_{\text{eff}}(k) = E_k \hat{\mathbf{n}}_k \cdot \boldsymbol{\sigma}$
- The spectrum of H_{eff} is 2π-periodic (quasi-energies E_k)
- T+C+S symmetries: BDI class —> same invariant as the static SSH model

Cardano, D'Errico, Dauphin, Maffei, … Marrucci, Lewenstein & PM
Nature Comm. (2017)

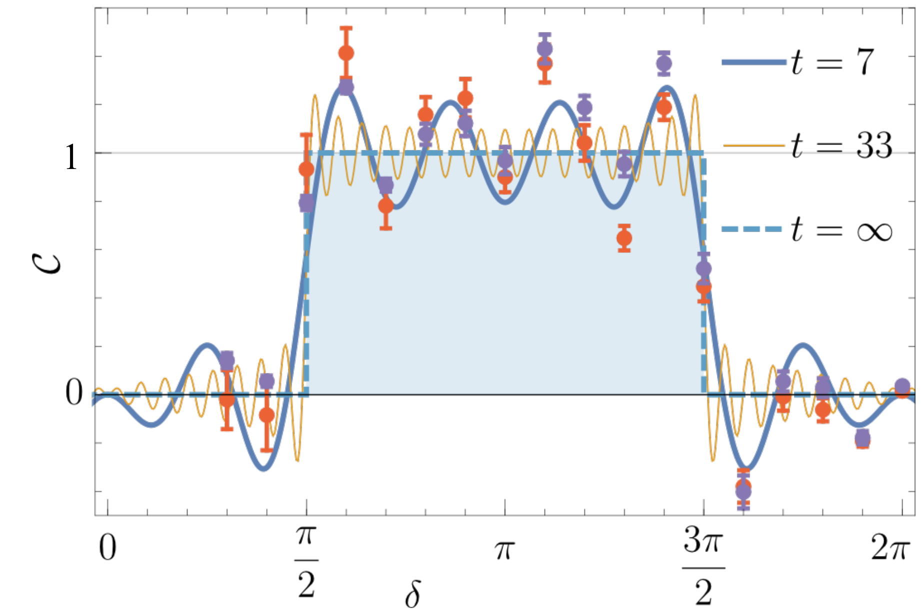
Detecting the invariant

- Winding: $\mathcal{W} = \oint \frac{\mathrm{d}k}{2\pi} \left(\mathbf{n} \times \partial_k \mathbf{n} \right)_z$



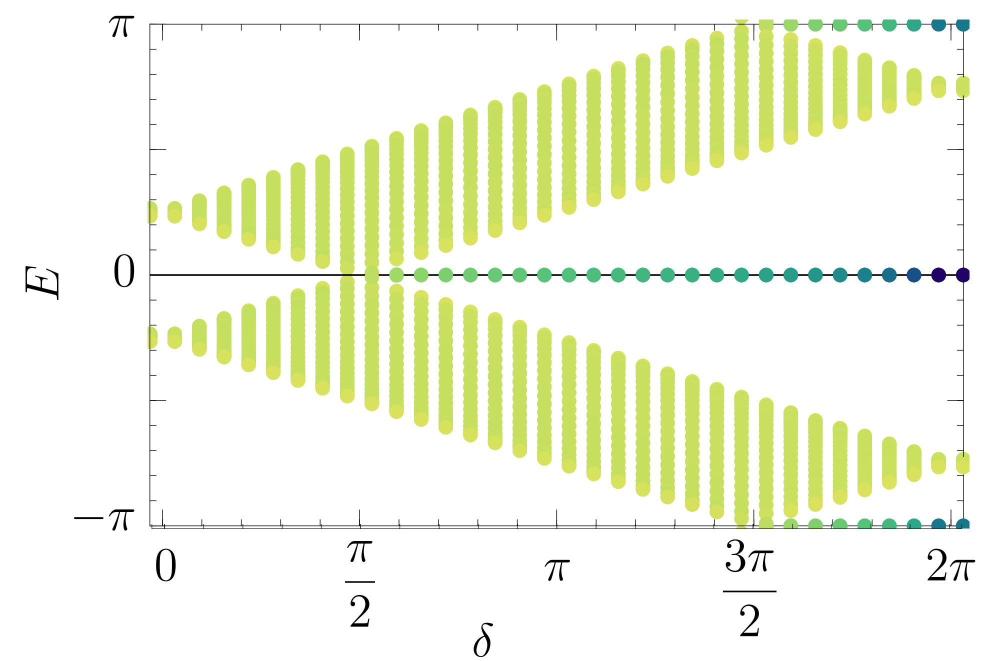
- Experimental measurement of the MCD after 7 timesteps of the DTQW with twisted photons:

(●/●): different initial polarizations



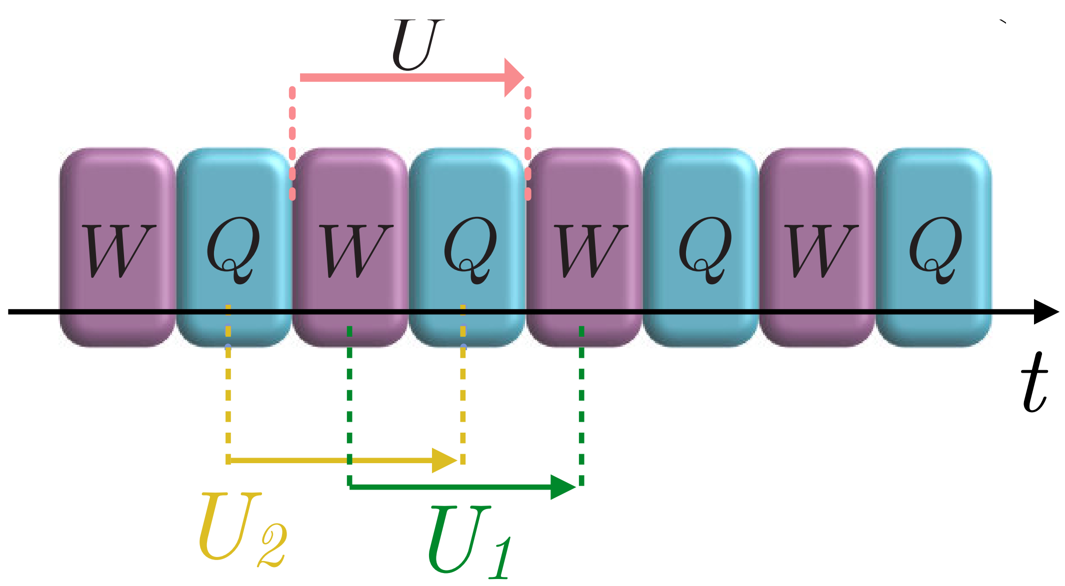
- Check bulk-boundary correspondence
- Spectrum with edges:

darker colors: “edgier” states



- Bulk-boundary correspondence violated?

Timeframes

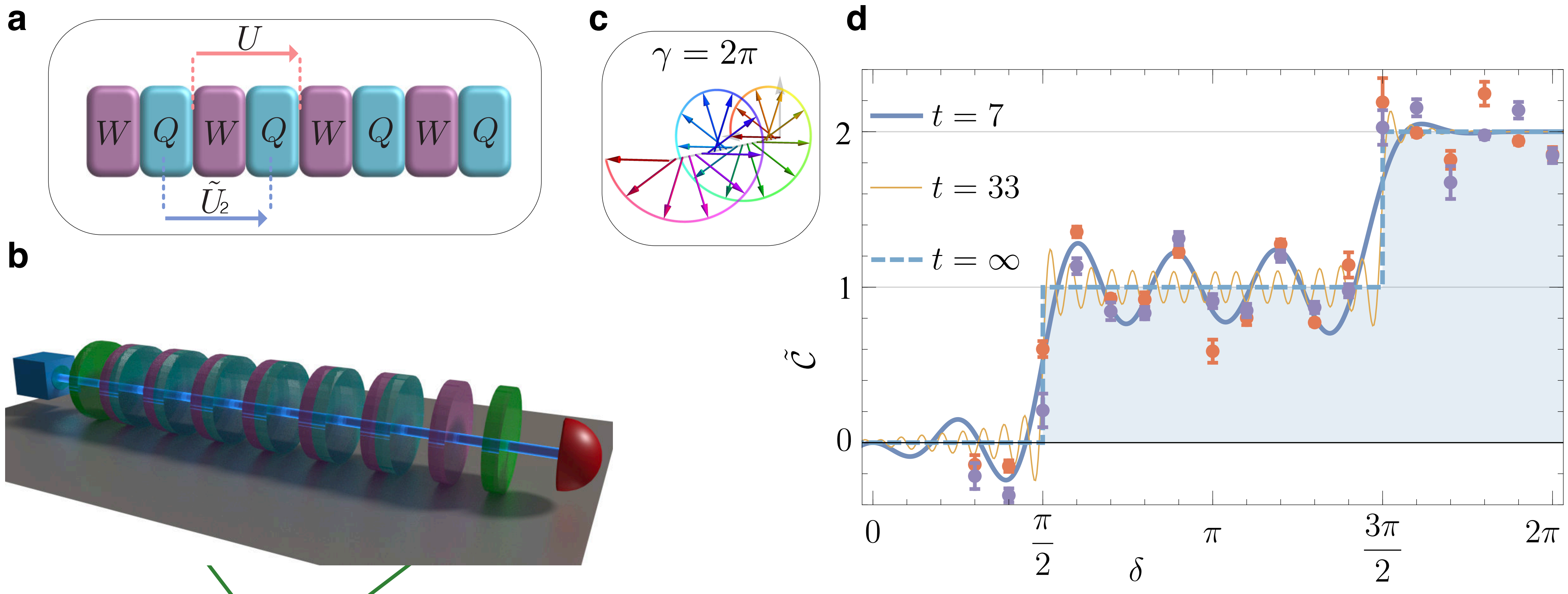


- Different initial t_0 lead to different U
- Eigenvalues of H_{eff} don't depend on t_0
- Eigenstates instead do! And so does the winding $\mathcal{W} = \mathcal{W}_1 \neq \mathcal{W}_2$
- Timeframes invariant under time-reflection (U_1 and U_2) are special
- # of 0-energy edge states: $C_0 = (\mathcal{W}_1 + \mathcal{W}_2)/2$
- # of π-energy edge states: $C_\pi = (\mathcal{W}_1 - \mathcal{W}_2)/2$

[Asboth and Obuse, PRB (2013)]

Winding in an alternative timeframe

Measurement of the MCD with protocol U_2:



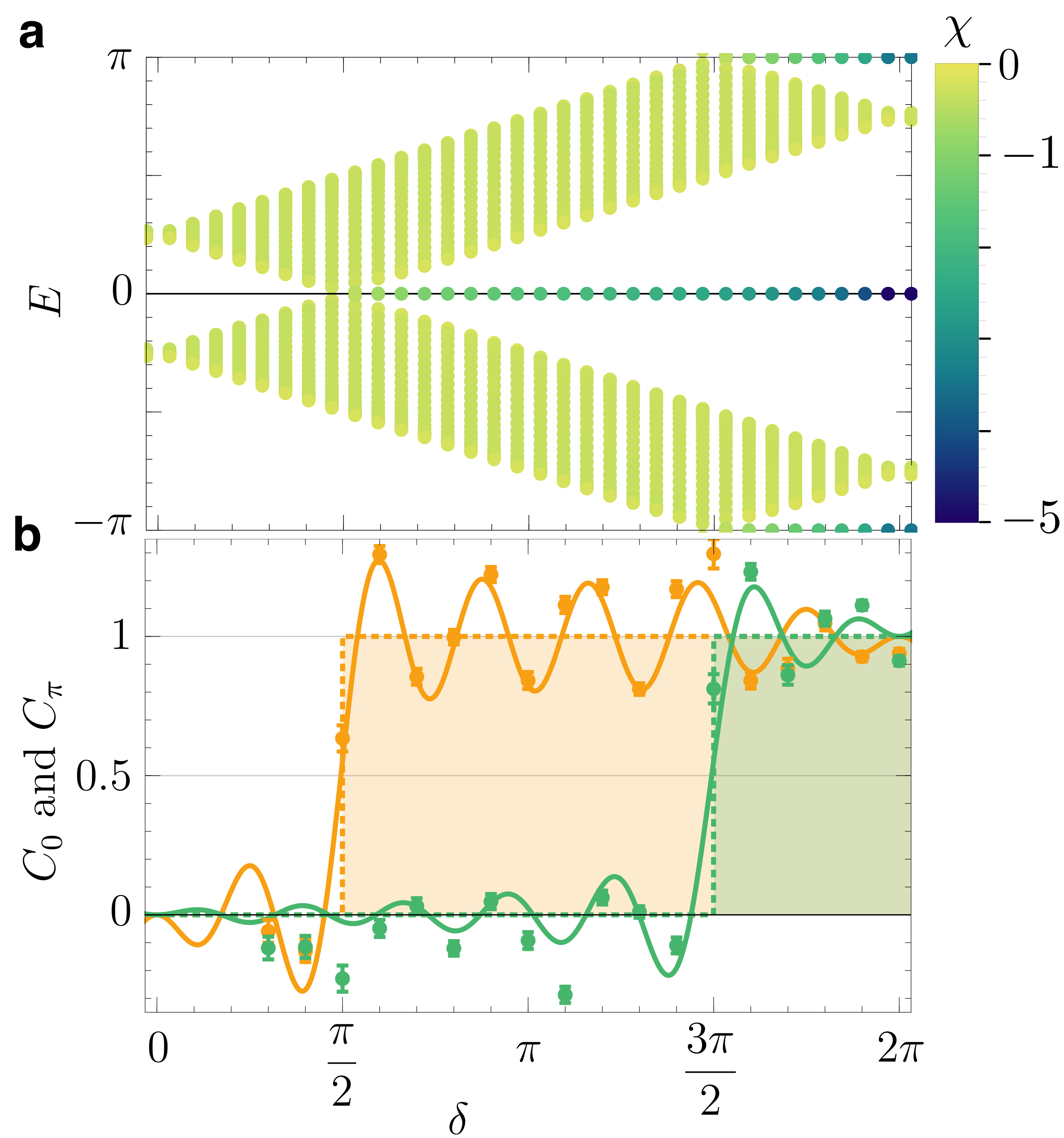
$$\sqrt{Q_\delta} = Q_{\delta/2}$$

(●/●): different initial polarizations

Bulk-boundary correspondence

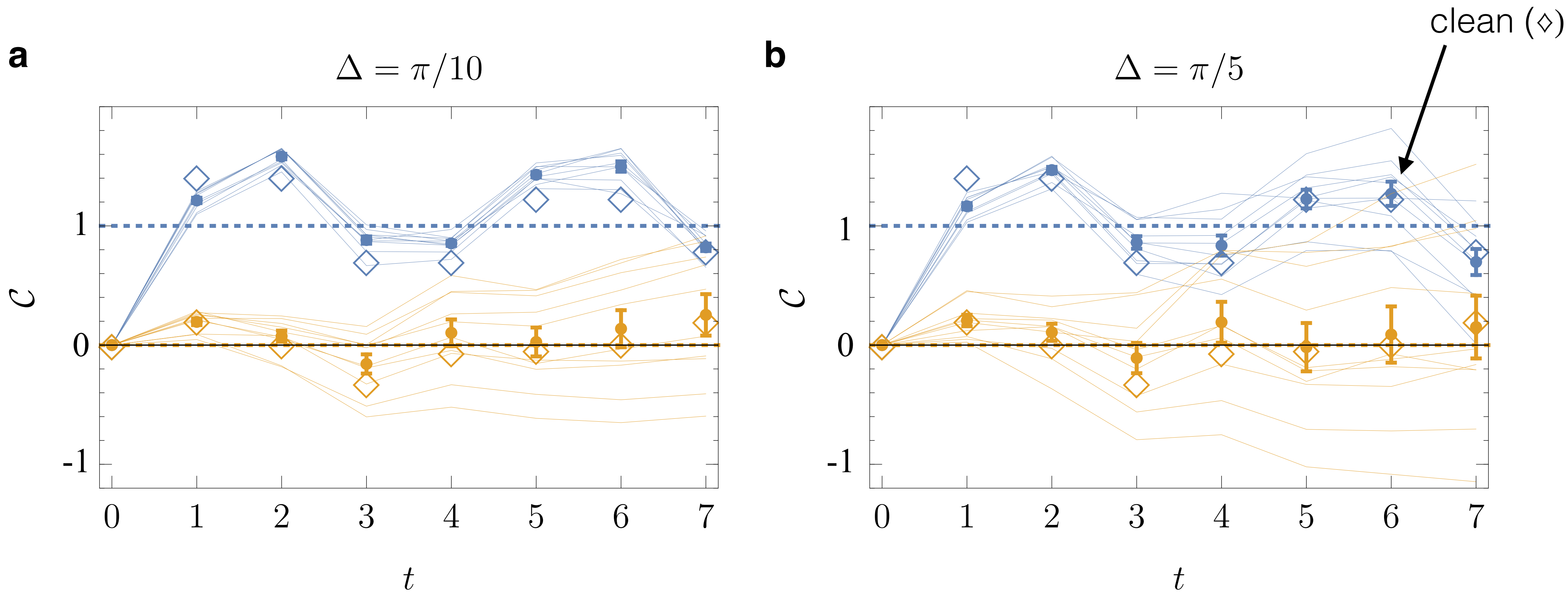
Theory:

Measurements:



Robustness to noise

- Adding noise to a trivial/non-trivial configuration:

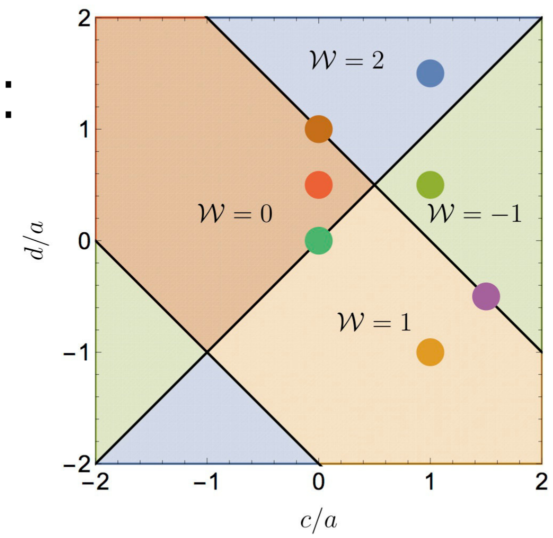


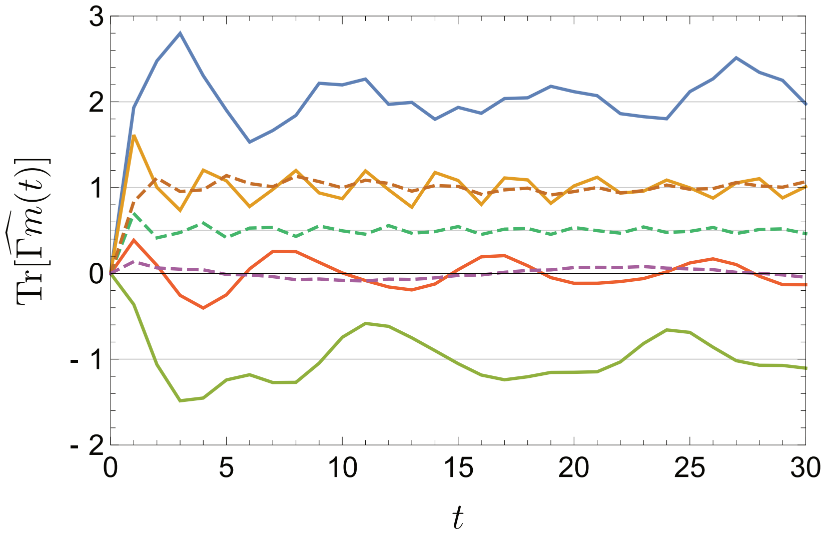
(●/●): averages over 10 disorder realizations

Recent developments

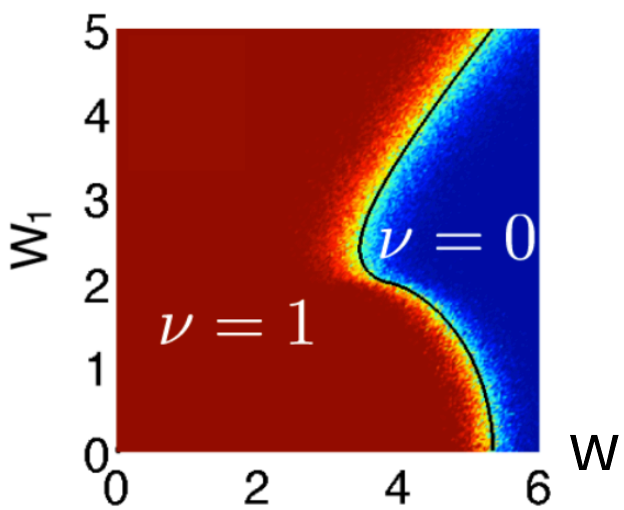
- Extension to multi-band models:

Maffei, Dauphin, ..., and PM
New J. Phys, in press (arXiv 2017)



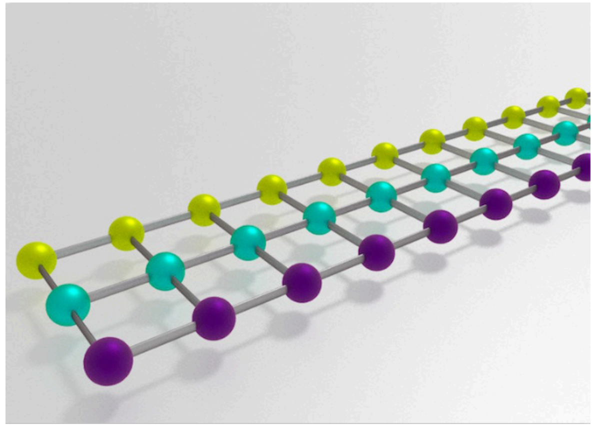


- Topological transitions driven by disorder:



[work in progress]

- 2D Hofstadter strips (ladders)



Mugel, Dauphin, PM *et al.*
SciPost Physics **3**, 012 (2017)

Conclusions

- The *mean chiral displacement* captures the winding of 1D chiral systems (both static and periodically driven)
- Detection of MCD is *simple, rapid, and robust* to disorder and noise
- Topological characterization of Floquet systems by studying *different timeframes*

- Extending the MCD to *other topological classes*?
- Interacting systems?

Thank you!